

# MEASURING THE MASSES OF STARS

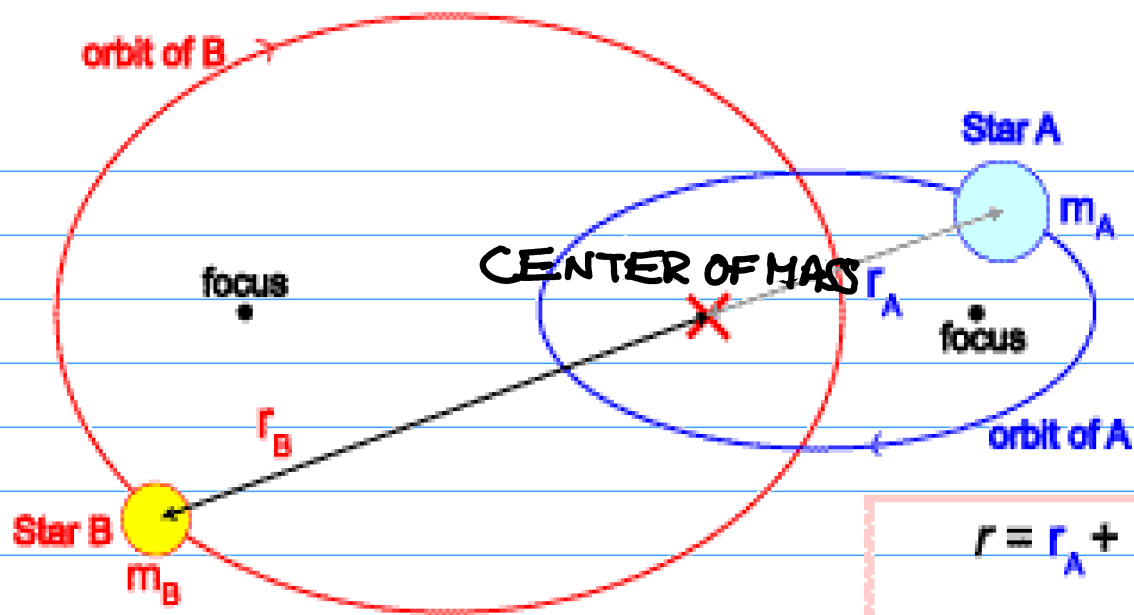
OVER 50% OF THE STARS WHICH ARE CLOSE TO THE SUN ARE MEMBERS OF BINARY SYSTEMS (BINARIES) - THEY ORBIT AROUND EACH OTHER HELD TOGETHER BY THEIR MUTUAL GRAVITATIONAL ATTRACTION.



BINARY SYSTEM KRUGER 60

THIS IS AN EXAMPLE OF VISUAL BINARIES - WE CAN DIRECTLY SEE THE ORBITAL MOTION OF ONE STAR ABOUT THE OTHER.

THEN WE CAN APPLY THE THIRD KEPLER'S LAW AS FORMULATED BY NEWTON TO DETERMINE THE TOTAL MASS OF THE BINARY SYSTEM FROM THE ORBITAL DATA:



$$r = r_A + r_B$$

$$M = m_A + m_B$$

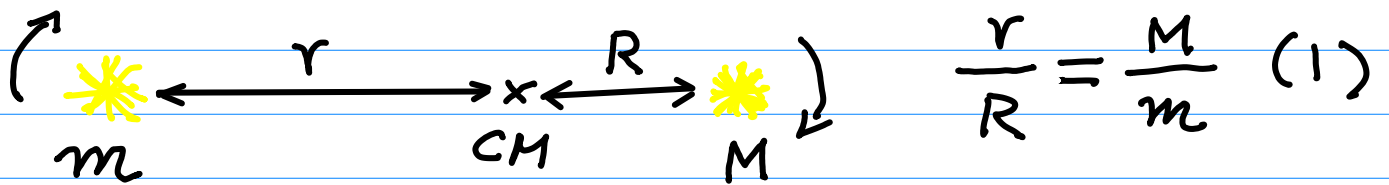
$$m_A r_A = m_B r_B$$

### Mass of a Binary System

THE TWO STARS REVOLVE AROUND THEIR COMMON CENTER OF MASS (CM).

Binary stars orbit their "center of mass", a point between the two stars

CENTER OF MASS IS CLOSER TO LARGER MASS



THE THIRD KEPLER'S LAW AS FORMULATED BY NEWTON:

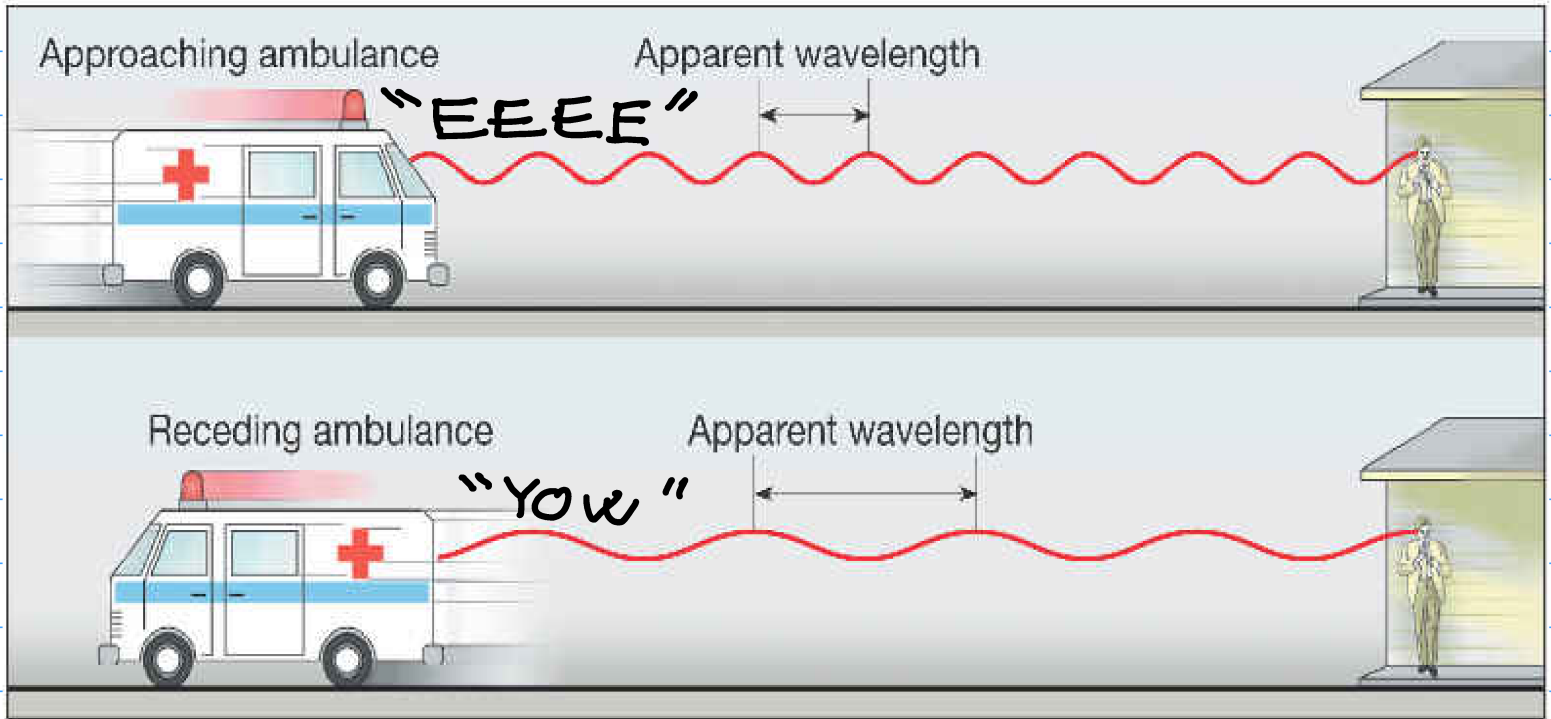
$$\frac{a^3}{P^2} = M + m \quad (2)$$

MEASURE  $a$ ,  $P$ ,  $r$  AND  $R$  AND DEDUCE  $M$  AND  $m$  FROM (1) AND (2).

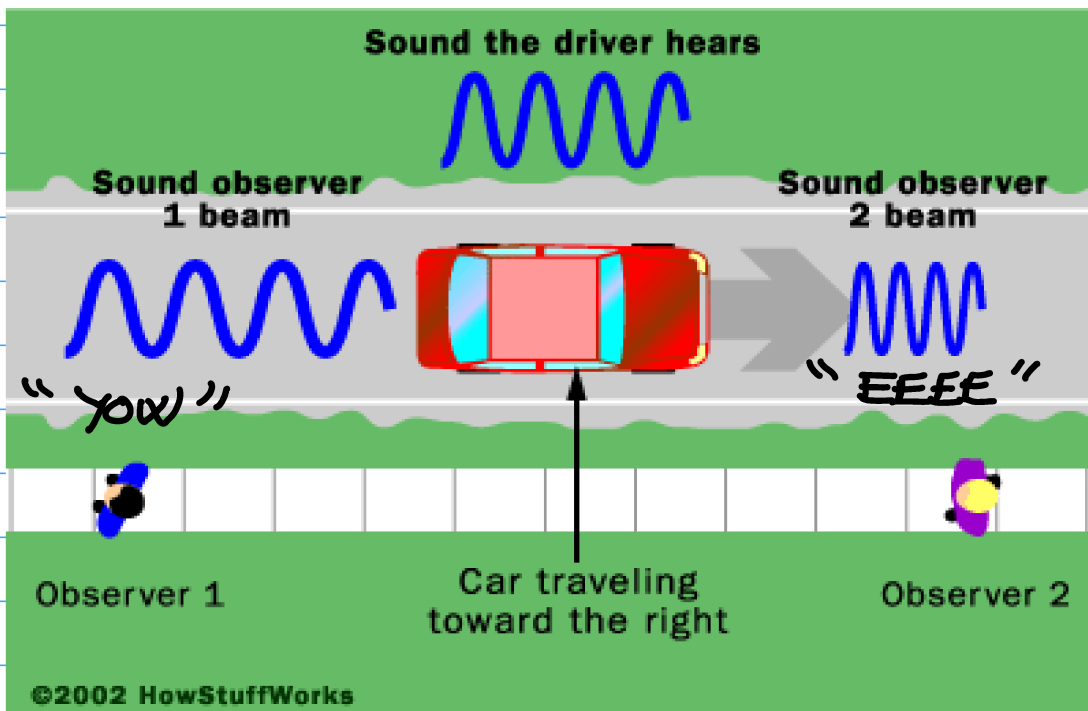
RESULTS: STAR MASSES FALL WITHIN A FAIRLY NARROW RANGE FROM  $0.1 M_{\odot}$  TO  $30 M_{\odot}$ . VERY FEW STARS HAVE MASSES FROM  $60 M_{\odot}$  TO  $100 M_{\odot}$ .

HOWEVER, THERE ARE BINARY SYSTEMS WHICH ARE TOO DISTANT FOR THE TWO STARS TO BE RESOLVED AS SEPARATE STARS BY A TELESCOPE. WE DETECT THEM USING THE DOPPLER EFFECT AND THEY ARE CALLED THE SPECTROSCOPIC BINARIES.

# THE DOPPLER EFFECT :

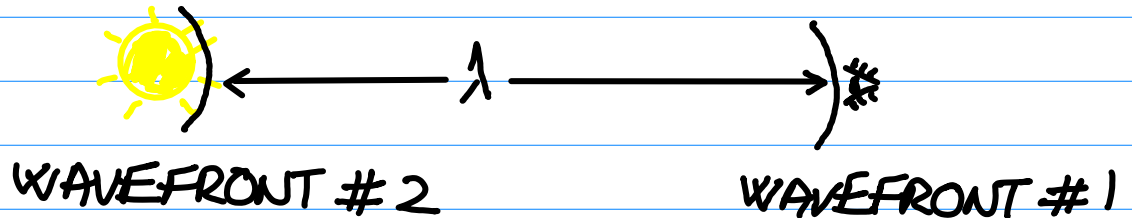


CHANGE IN PITCH (FREQUENCY/WAVELENGTH)  
CAUSED BY RELATIVE MOTION OF THE SOURCE  
AND THE OBSERVER.

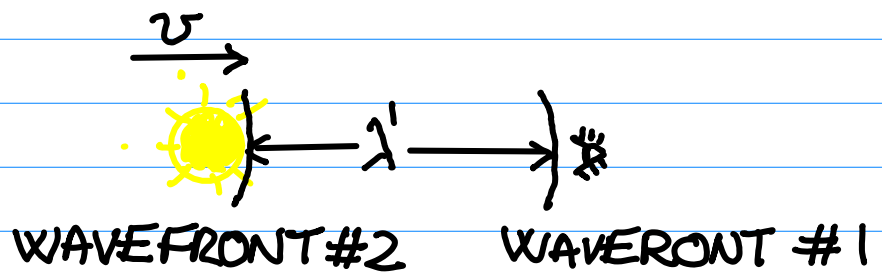


THE SAME EFFECT IS OBSERVED WITH LIGHT:

1) BOTH THE SOURCE AND THE OBSERVER ARE STATIONARY:

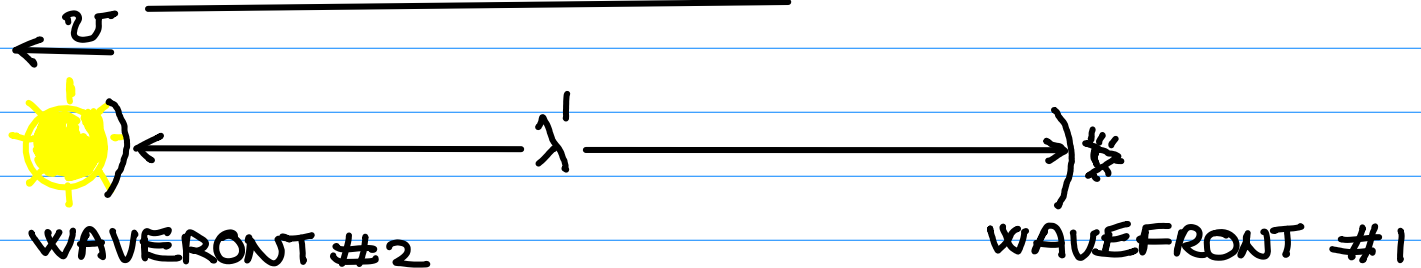


2) THE SOURCE APPROACHING THE OBSERVER:



$\lambda'$  IS SHORTER THAN  $\lambda$  - THE WAVELENGTH IS BLUESHIFTED.

3) THE SOURCE RECEEDING FROM THE OBSERVER:



$\lambda'$  IS LONGER THAN  $\lambda$  - THE WAVELENGTH IS REDSHIFTED.

THE CHANGE IN WAVELENGTH IS

$$\Delta \lambda = \lambda' - \lambda,$$

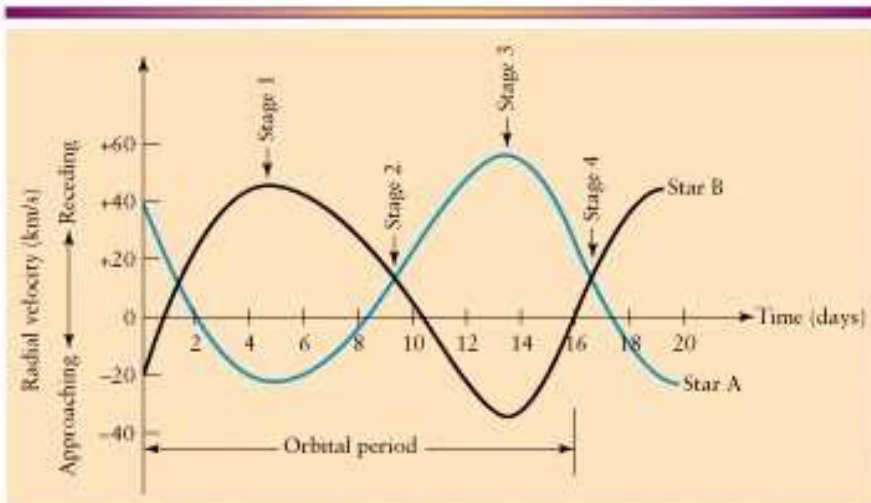
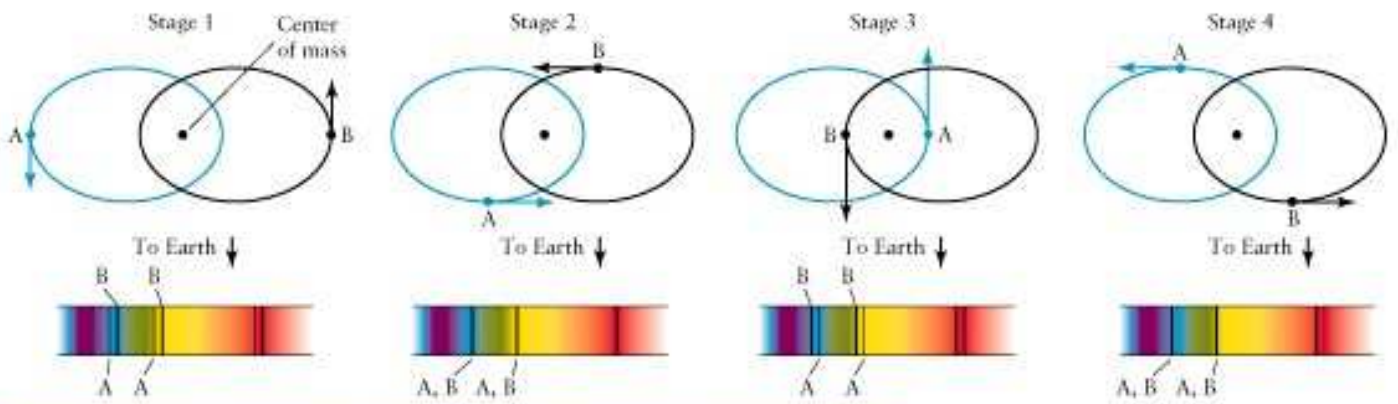
AND IT TURNS OUT THAT

$$Z = \frac{\Delta \lambda}{\lambda} = \frac{v}{c}$$

← SPEED OF THE SOURCE  
RELATIVE TO THE OBSERVER

← SPEED OF LIGHT IN  
VACUUM

By measuring Z one can deduce the relative speed v of the source.



ONE CAN DEDUCE THE ORBITAL PERIOD  $P$  FROM THE PERIOD OF OSCILLATION OF THE SPECTRAL LINES.

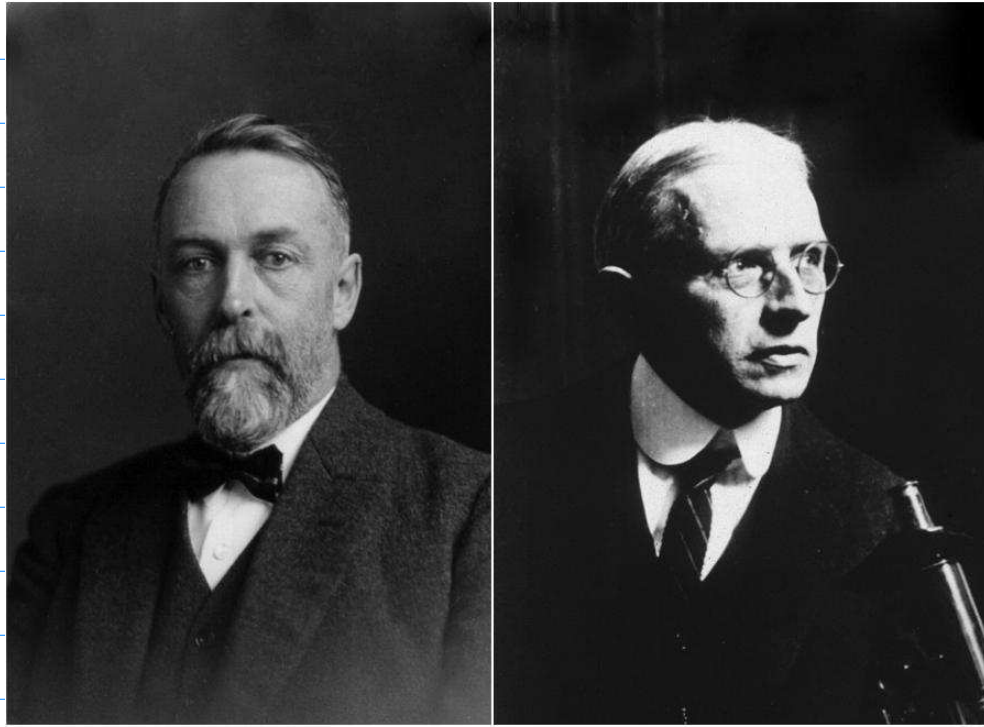
FROM THE DOPPLER SHIFTS ONE CAN FIND OUT THE RADIAL VELOCITIES.

FROM  $P$  AND THE MAXIMUM VALUE OF THE RADIAL VELOCITY ONE CAN DEDUCE THE AVERAGE VALUE OF THE SEMI-MAJOR AXIS  $a$ .

THEN, THE (AVERAGE) TOTAL MASS IS FOUND FROM THE THIRD KEPLER'S LAW AS FORMULATED BY NEWTON

$$\frac{a^3}{P^2} = M + m.$$

# HERTZSPRUNG - RUSSELL (H-R) DIAGRAM



EJNAR HERTZSPRUNG  
(in 1911)

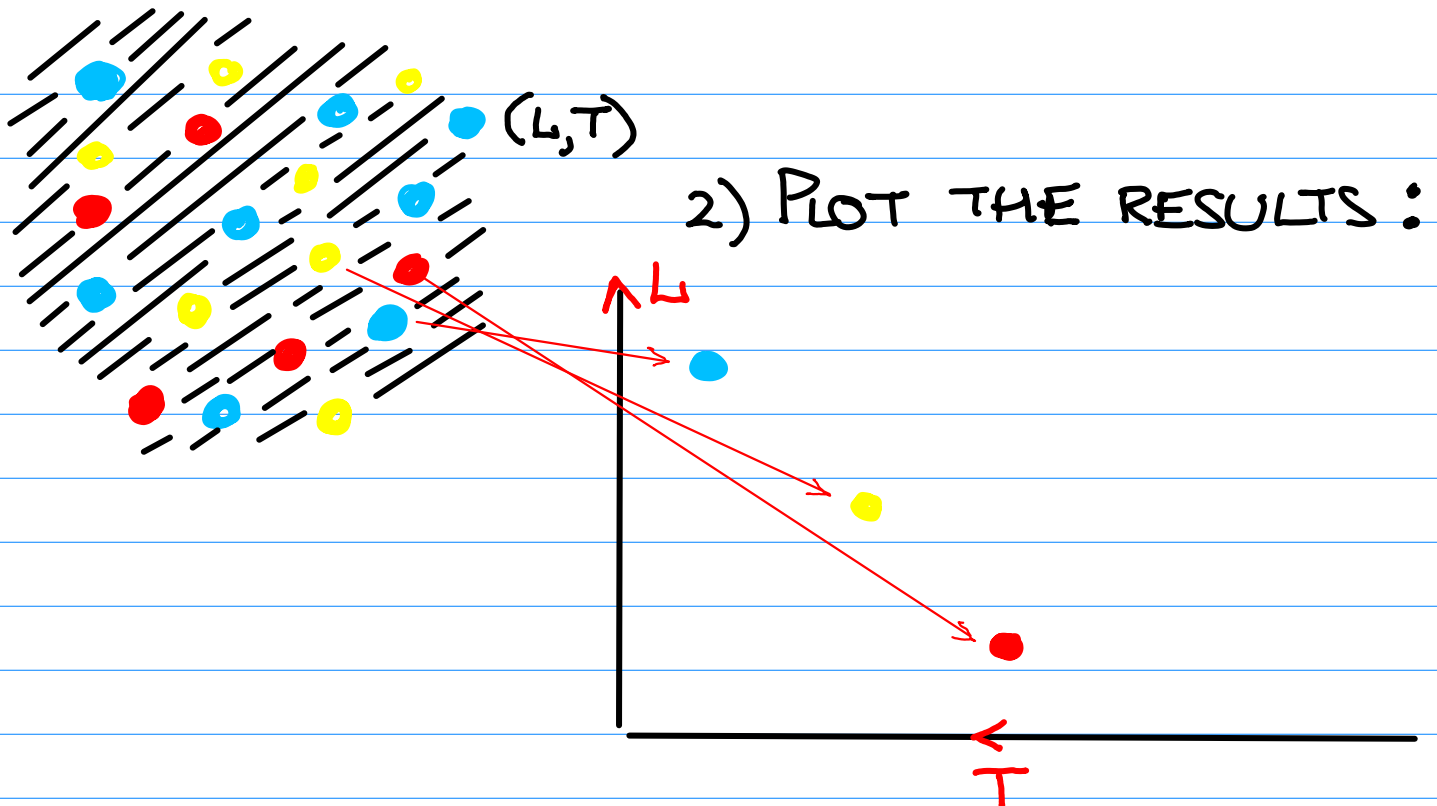
HENRY NORRIS RUSSELL  
(in 1913)

THEY PLOTTED STARS ACCORDING TO THEIR LUMINOSITY ( $L$ ) AND SURFACE TEMPERATURE ( $T$ ), i.e. THE SPECTRAL CLASS.

## PROCEDURE:

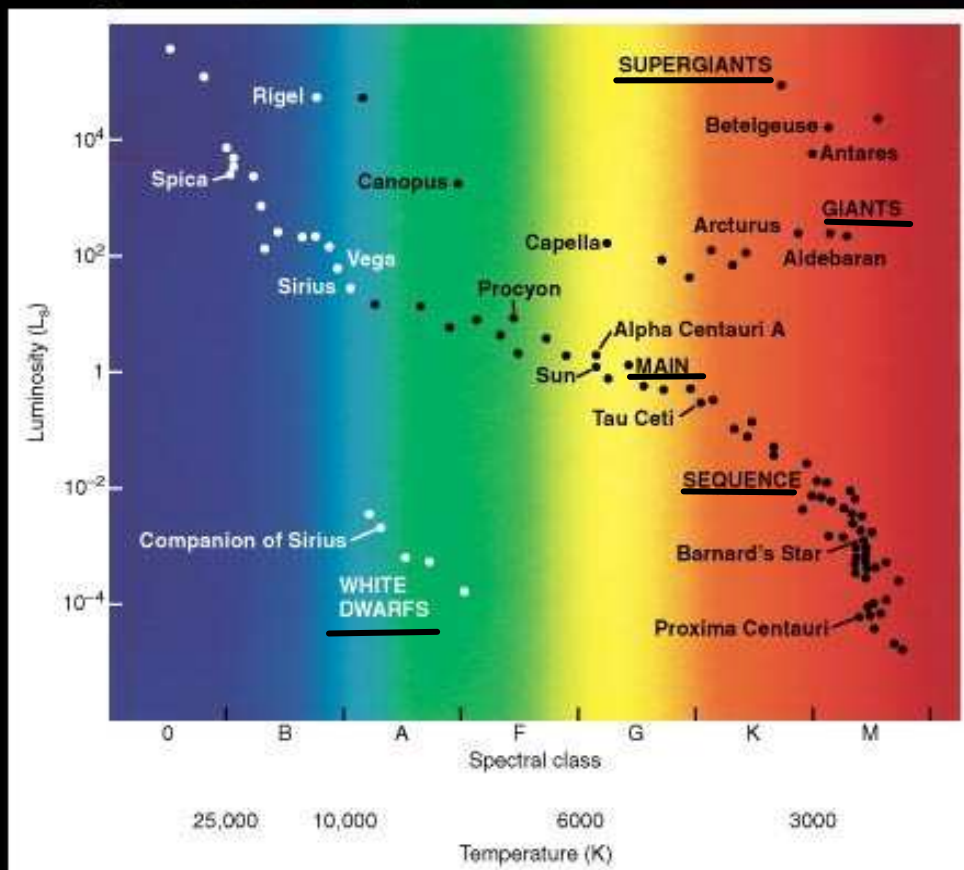
- 1) TAKE A CLUSTER (OR SEVERAL CLUSTERS) OF STARS AND FOR EACH STAR MEASURE ITS LUMINOSITY ( $L$ ) AND THE SURFACE TEMPERATURE ( $T$ ):





## A TYPICAL H-R DIAGRAM:

Fraknoi/Morrison/Wolff, Voyages Through the Universe, 2/e  
Figure 17.15 H-R diagram for a Selected Sample of Stars



THERE THREE BASIC GROUPS OF STARS IN HR DIAGRAM:

1) THE MAIN SEQUENCE STARS - ABOUT 90% OF STARS ARE THE MAIN SEQUENCE STARS.

AT THE UPPER LEFT CORNER OF THE MAIN SEQUENCE ARE HOT, LUMINOUS STARS.

AT THE LOWER RIGHT CORNER OF THE MAIN SEQUENCE ARE COOL, DIM STARS.

OUR SUN IS NEAR THE MIDDLE OF THE MAIN SEQUENCE.

2) WHITE DWARFS - A GROUP OF VERY HOT BUT VERY DIM STARS.

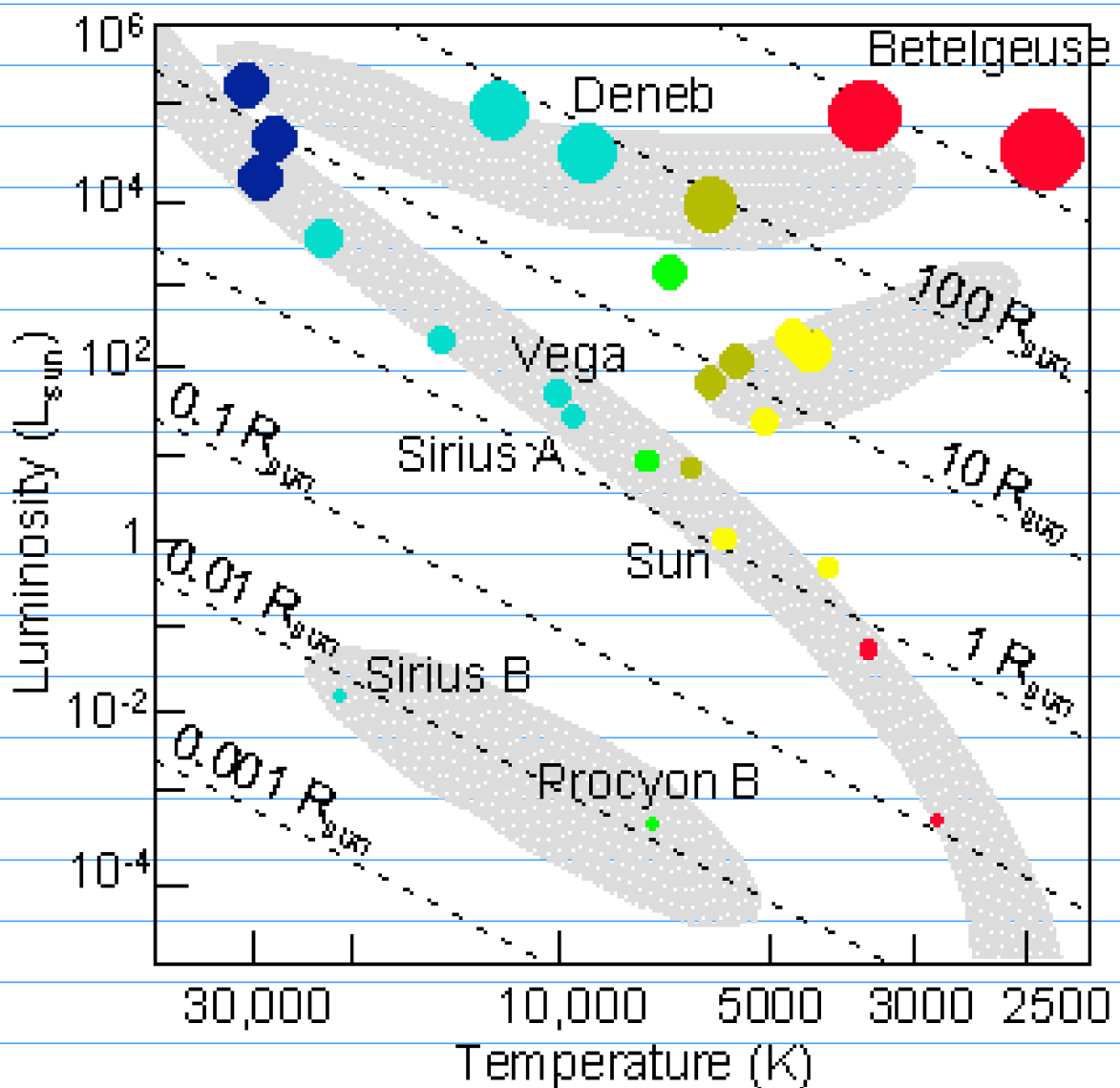
THE STEFAN - BOLTZMANN LAW:

$$L = 4\pi R^2 T^4$$

IF  $L$  IS LOW IN SPITE OF HIGH  $T$  (WHITE HOT) THEN THE RADIUS  $R$  MUST BE VERY SMALL (DWARFS).

3) RED GIANTS/SUPERGIANTS - A GROUP OF VERY COOL BUT VERY LUMINOUS STARS.

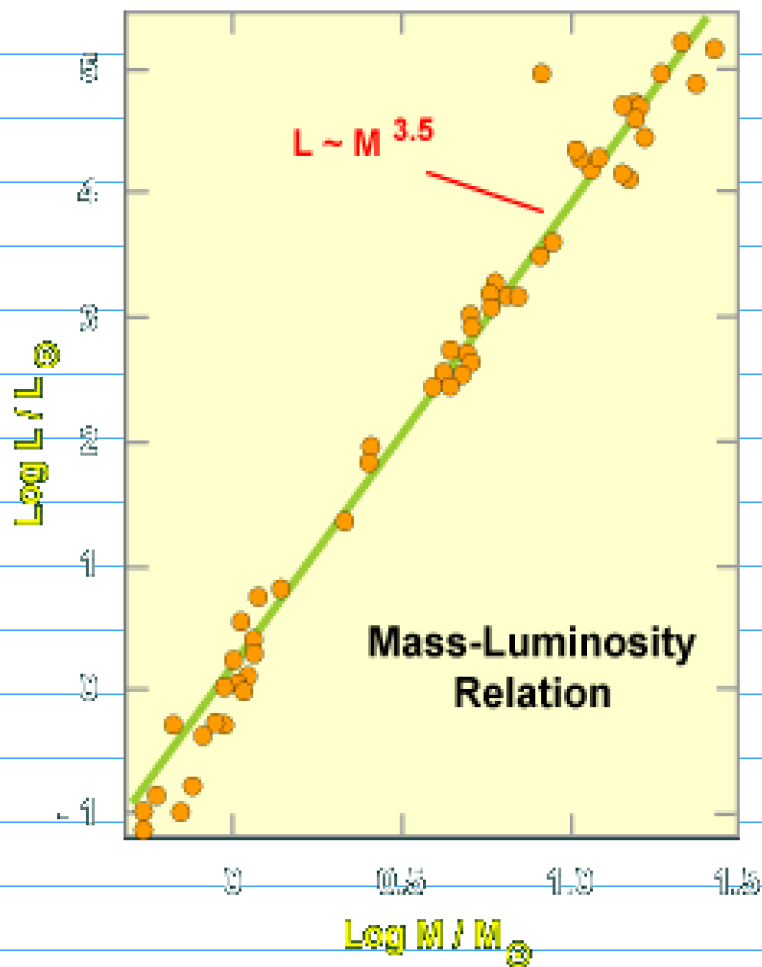
IF  $L$  IS HIGH IN SPITE OF LOW  $T$  (RED) THE STEFAN - BOLTZMANN LAW IMPLIES THAT THE RADIUS  $R$  MUST BE VERY LARGE (GIANTS/SUPERGIANTS).



THE DASHED LINES ARE OBTAINED BY PLOTTING  $L = 64\pi R^2 T^4$  FOR SEVERAL FIXED VALUES OF  $R$ .

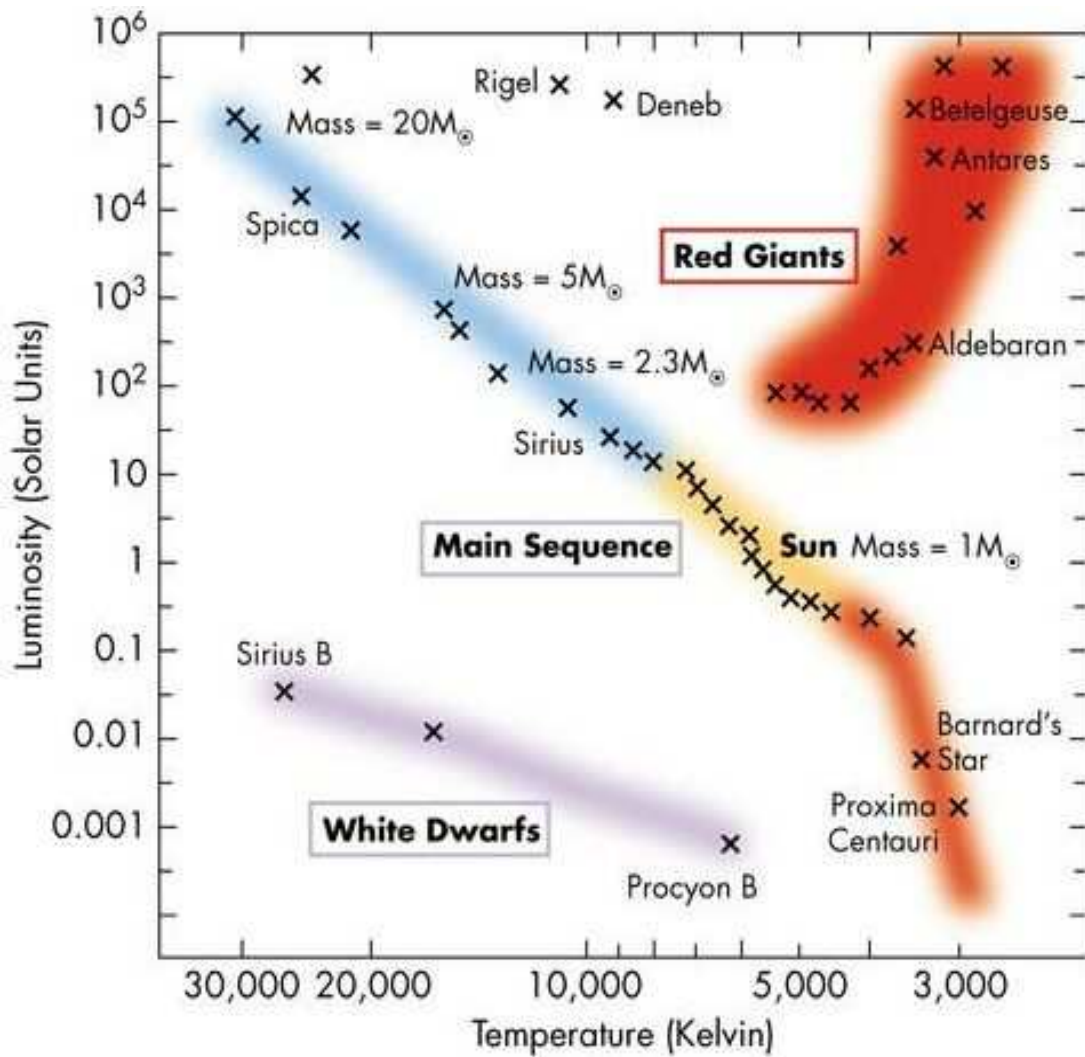
THE RADII OF MAIN SEQUENCE STARS SPAN A RANGE FROM  $0.1 R_{\odot}$  TO  $10 R_{\odot}$ .

IN 1924 A.S. EDDINGTON FOUND AN IMPORTANT RELATION BETWEEN THE LUMINOSITY AND MASS FOR THE MAIN SEQUENCE (AND ONLY FOR THE MAIN SEQUENCE STARS!):



$$L \propto M^{3.5}$$

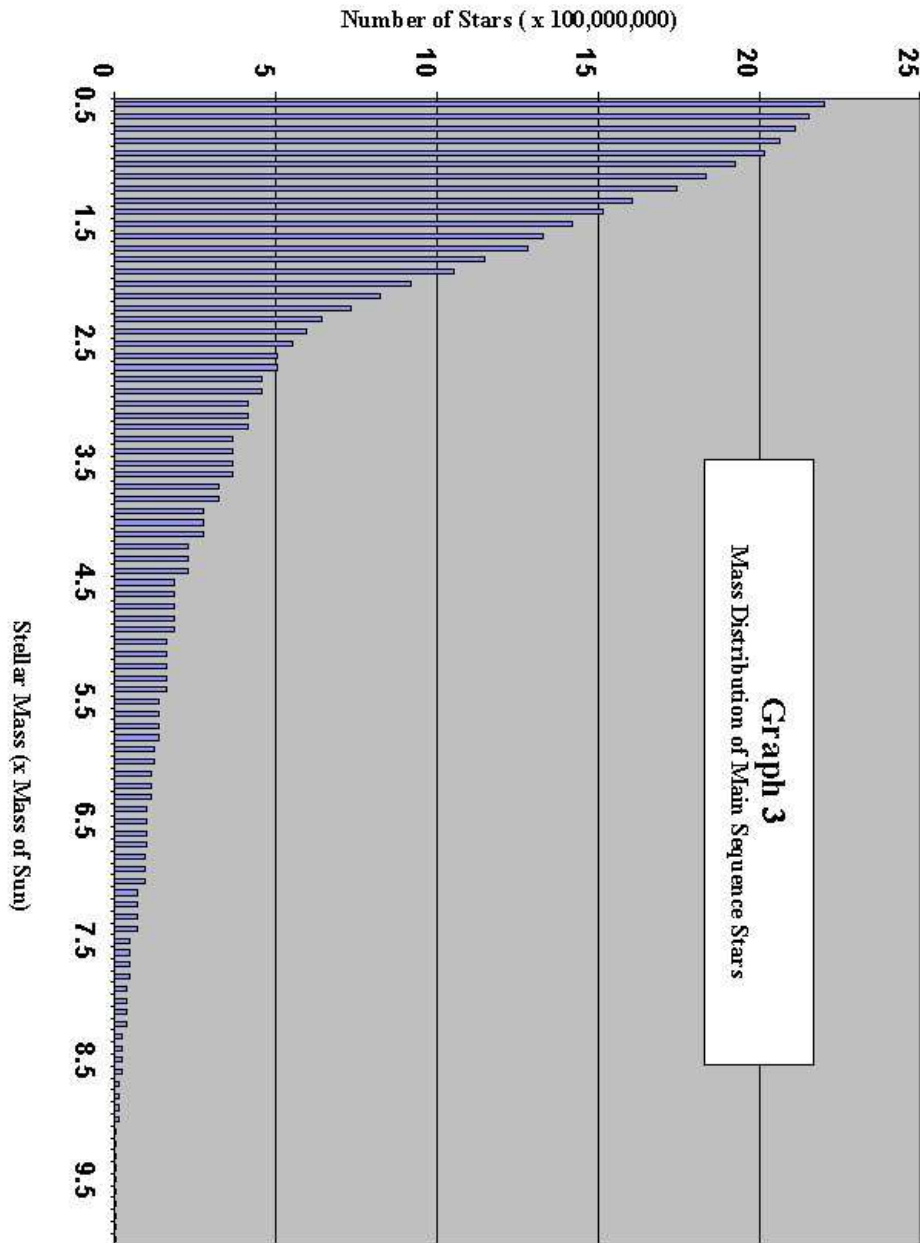
MASS-LUMINOSITY  
RELATION FOR THE  
MAIN SEQUENCE STARS



THE MASS-LUMINOSITY RELATION PROVIDES INFORMATION ABOUT THE LIFETIMES OF THE MAIN SEQUENCE STARS:

$$\text{LIFETIME} = \frac{\text{FUEL AVAILABLE}}{\text{ENERGY OUTPUT}} = \frac{M}{L} \propto \frac{M}{M^{3.5}} = \frac{1}{M^{2.5}}$$

THUS, MORE LUMINOUS/MASSIVE MAIN SEQUENCE STARS HAVE SHORTER LIFETIMES. HENCE, VERY MASSIVE STARS ARE NOT VERY COMMON.



THE GIANTS, THE WHITE DWARFS, AND THE MAIN SEQUENCE STARS DIFFER DRAMATICALLY IN THEIR AVERAGE DENSITIES:

$$\text{AVERAGE DENSITY} = \frac{\text{MASS}}{\text{VOLUME}} ;$$

SUN (A MAIN SEQUENCE STAR):

$$\text{AVERAGE DENSITY} \approx 1 \frac{\text{g}}{\text{cm}^3};$$

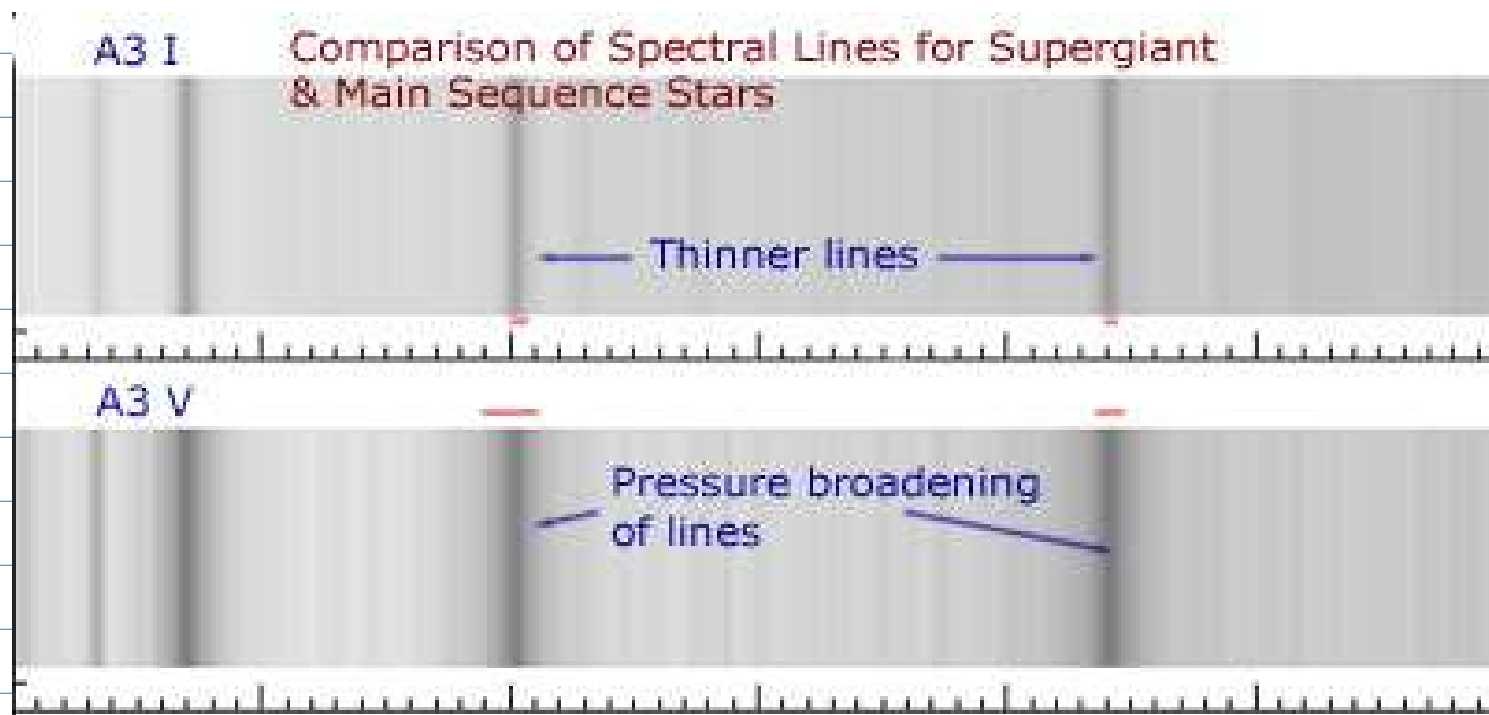
A TYPICAL GIANT STAR:

$$\text{AVERAGE DENSITY} \approx 10^{-6} \frac{\text{g}}{\text{cm}^3};$$

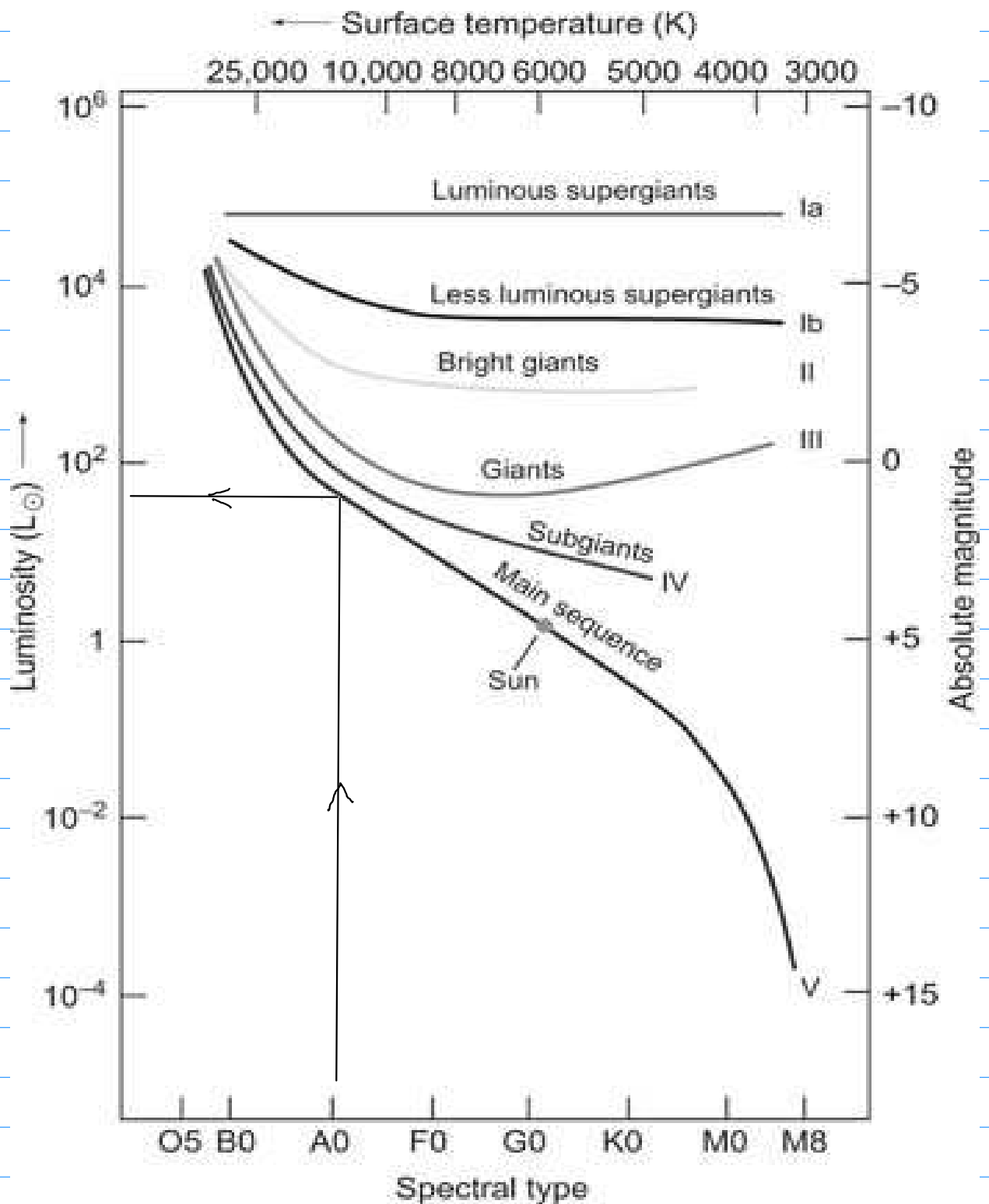
A TYPICAL WHITE DWARF:

$$\text{AVERAGE DENSITY} \approx 5 \times 10^5 \frac{\text{g}}{\text{cm}^3}.$$

THE WIDTH OF THE SPECTRAL LINES DEPENDS ON THE DENSITY (PRESSURE) IN THE STAR'S ATMOSPHERE:



BASED ON THE DETAILS OF THEIR SPECTRA THE STARS COULD BE DIVIDED INTO SEVERAL LUMINOSITY CLASSES:





SPECTROSCOPIC PARALLAX IS A METHOD OF DETERMINING STAR'S DISTANCE ( $d$ ) FROM ITS SPECTRUM USING THE H-R DIAGRAM:

- 1) DETERMINE THE SPECTRAL CLASS (O, B, A, F, G, K, M) OF A STAR, I.E. ITS SURFACE TEMPERATURE.
- 2) DETERMINE THE STAR'S LUMINOSITY CLASS (E.G. THE MAIN SEQUENCE).
- 3) READ OFF THE H-R DIAGRAM THE LUMINOSITY ( $L$ ) OF THE STAR.
- 4) MEASURE THE BRIGHTNESS OF THE STAR ( $B$ ) AND DEDUCE ITS DISTANCE ( $d$ ) FROM

$$B = \frac{L}{4\pi d^2} .$$