

NEWTONIAN REVOLUTION



ISAAC NEWTON (1642-1727) DISCOVERED THE THREE LAWS OF MOTION (OF MACROSCOPIC OBJECTS) AND THE LAW OF GRAVITY. HE DEVELOPED CALCULUS SO THAT HE COULD APPLY THESE LAWS TO THE REAL WORLD.



IN SUMMARY, THE NEWTON'S LAWS OF MOTION STATE THAT IN ORDER TO CHANGE THE SPEED OF AN OBJECT OR ITS DIRECTION OF MOTION A NET (I.E. UNBALANCED) FORCE IS NEEDED; IN THE ABSENCE OF A NET FORCE THE OBJECT MOVES ALONG A STRAIGHT LINE AT A CONSTANT SPEED OR IS AT REST (ZERO SPEED).



A COMMON MISCONCEPTION IS THAT A FORCE IS NEEDED TO MOVE AN OBJECT AT A CONSTANT SPEED:



FORCE OF
FRICTIONAL
RESISTANCE

PULLING FORCE

THE PULLING FORCE IS EQUAL IN MAGNITUDE BUT OPPOSITE IN DIRECTION TO THE FORCE OF FRICTION AND THE NET FORCE IS ZERO. AS A RESULT THE TIRES MOVE AT A CONSTANT SPEED (WHICH COULD BE ZERO).

FOR LINEAR MOTION THE ACCELERATION (a) IS THE RATE OF CHANGE IN SPEED AND THE UNIT IS THE UNIT FOR SPEED (E.G. m/s) OVER THE UNIT OF TIME (E.G. s) (AND THE UNIT FOR ACCELERATION IS $m/s/s = m/s^2$).

THE SECOND NEWTON'S LAW RELATES THE ACCELERATION OF A BODY (a) TO THE NET FORCE ON IT (F), AND THE BODY'S MASS (m):

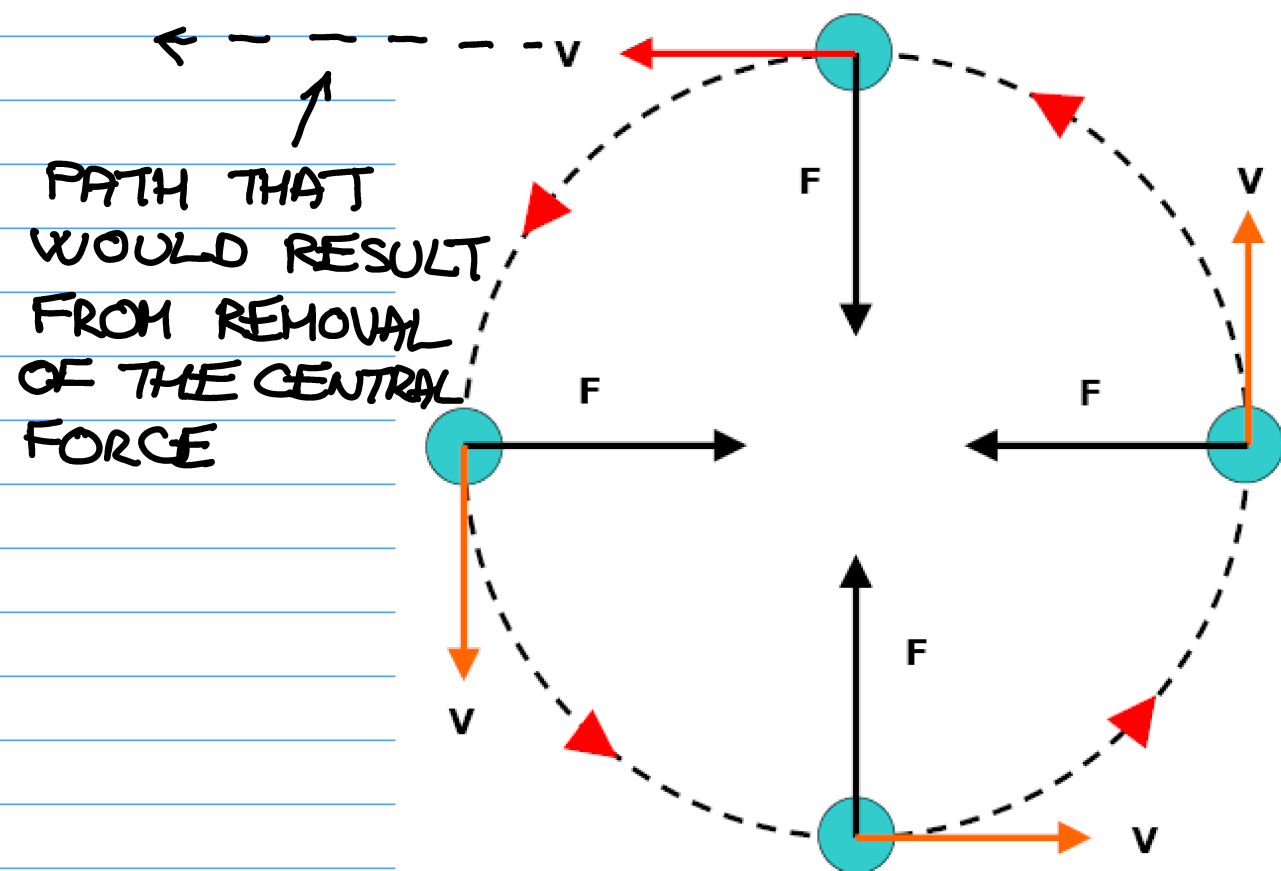
$$\boxed{a = \frac{F}{m}} \quad \text{OR} \quad \boxed{F = ma}$$

THUS, IF FOR A GIVEN BODY (I.E. A GIVEN m) THE NET FORCE IS DOUBLED THE RESULTING ACCELERATION IS DOUBLED.

IF A GIVEN NET FORCE (I.E. A GIVEN F) IS APPLIED TO A TWICE MORE MASSIVE OBJECT ($m \rightarrow 2m$) THE RESULTING ACCELERATION IS HALVED ($a \rightarrow \frac{a}{2}$).

THE VELOCITY OF AN OBJECT SPECIFIES BOTH ITS INSTANTANEOUS SPEED AND THE DIRECTION OF MOTION (IT IS A VECTOR).

IN A UNIFORM CIRCULAR MOTION THE INSTANTANEOUS SPEED (v) IS CONSTANT BUT THE DIRECTION OF MOTION CONSTANTLY CHANGES:



THE CHANGE IN THE DIRECTION OF MOTION IS CAUSED BY THE CENTRAL FORCE \vec{F} (IT IS ALSO A VECTOR) WHICH POINTS TOWARD THE CENTRE OF THE CIRCULAR TRAJECTORY.

THE MOTION IS ACCELERATED BECAUSE THE DIRECTION OF VELOCITY IS CHANGING.

USING CALCULUS NEWTON SHOWED THAT THE ACCELERATION (a) OF A BODY IS RELATED TO ITS SPEED (v) AND THE RADIUS (R) OF THE CIRCULAR TRAJECTORY BY

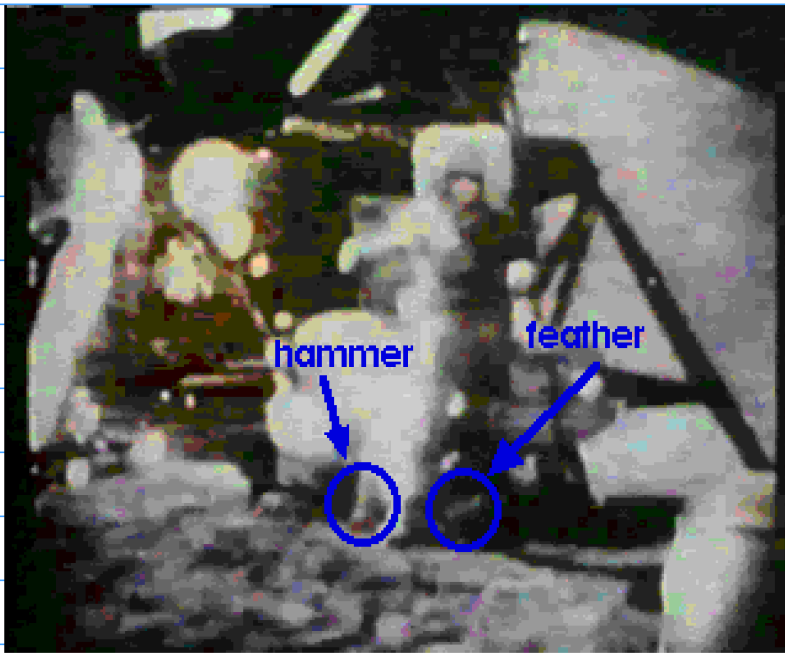
$$a = \frac{v^2}{R}$$

HENCE, IF THE SPEED v IS DOUBLED THE ACCELERATION a AND THE NECESSARY FORCE $F (=ma)$ ARE QUADRUPLED:



F (IN THIS CASE IT IS THE FORCE OF FRICTION BETWEEN THE TIRES AND THE ROAD)

GALILEO REALIZED THAT REGARDLESS OF THEIR SIZE (I.E. MASS) ALL OBJECTS ACCELERATE DOWNWARD WITH THE SAME ACCELERATION OF ABOUT 9.8 m/s^2 UNDER THE INFLUENCE OF GRAVITY, PROVIDED THAT THE AIR RESISTANCE CAN BE IGNORED:



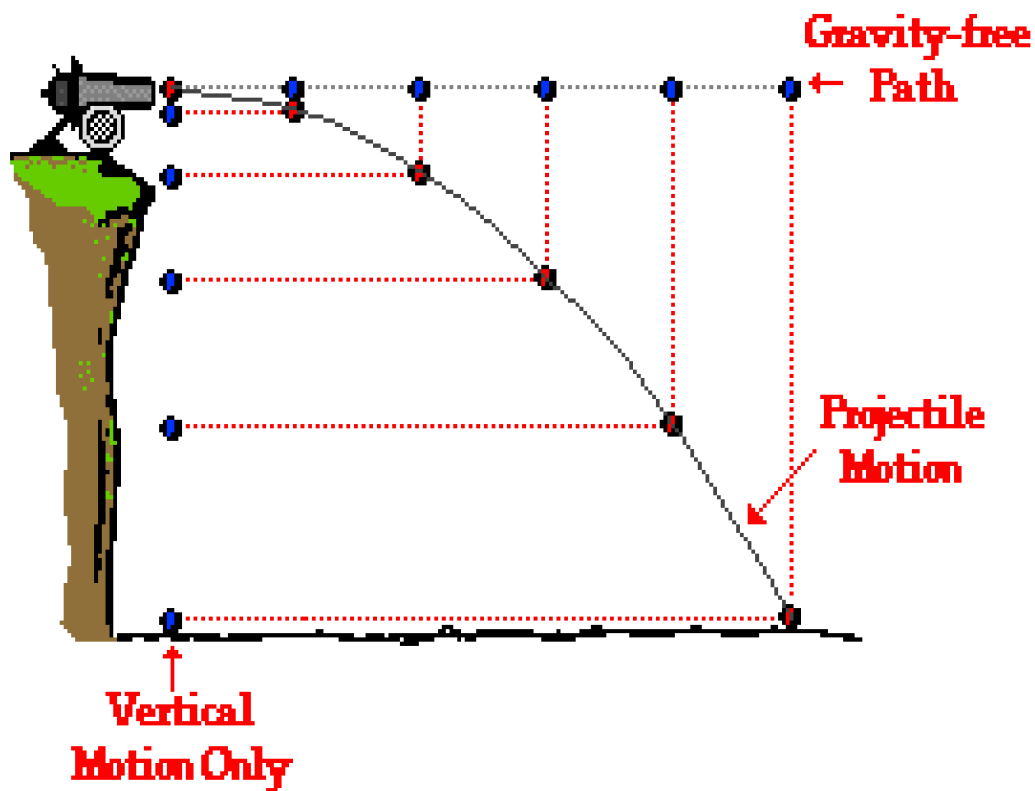
APOLLO 15 EXPERIMENT ON THE MOON:
A HAMMER AND A FEATHER IN A FREE FALL WITHOUT AIR RESISTANCE

THE NEWTON'S SECOND LAW IMPLIES THAT THE FORCE OF GRAVITY F_g ON A BODY OF MASS m NEAR THE SURFACE OF THE EARTH IS GIVEN BY

$$F_g = m \times 9.8 \frac{\text{m}}{\text{s}^2}$$

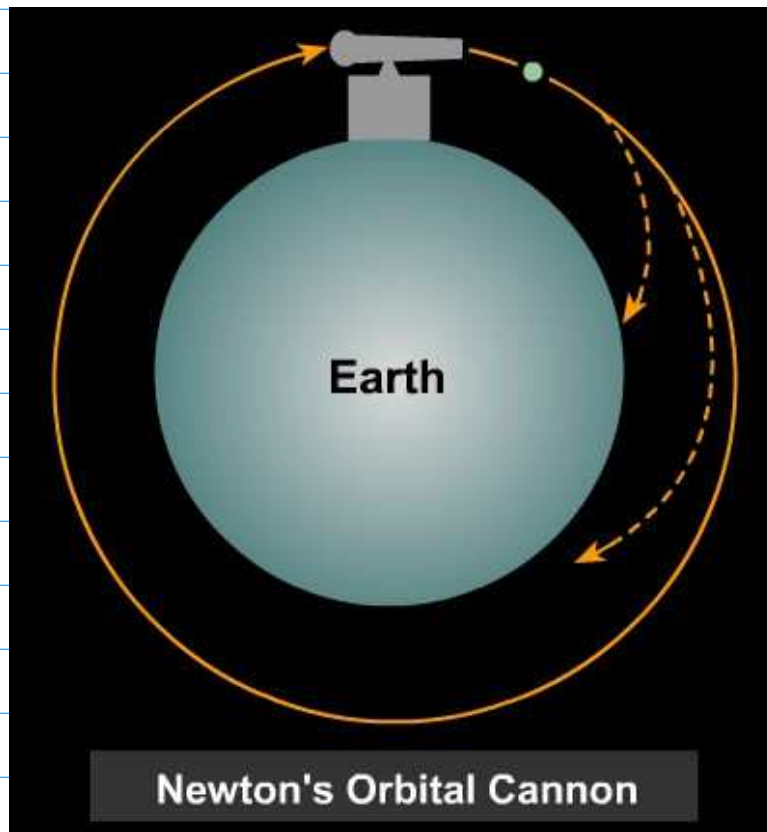
HENCE THE FORCE OF GRAVITY ON A BODY IS PROPORTIONAL TO ITS MASS.

MOREOVER, GALILEO KNEW THAT THE DISTANCE BY WHICH AN OBJECT HAS FALLEN IN A GIVEN TIME DOES NOT DEPEND ON ITS HORIZONTAL SPEED :



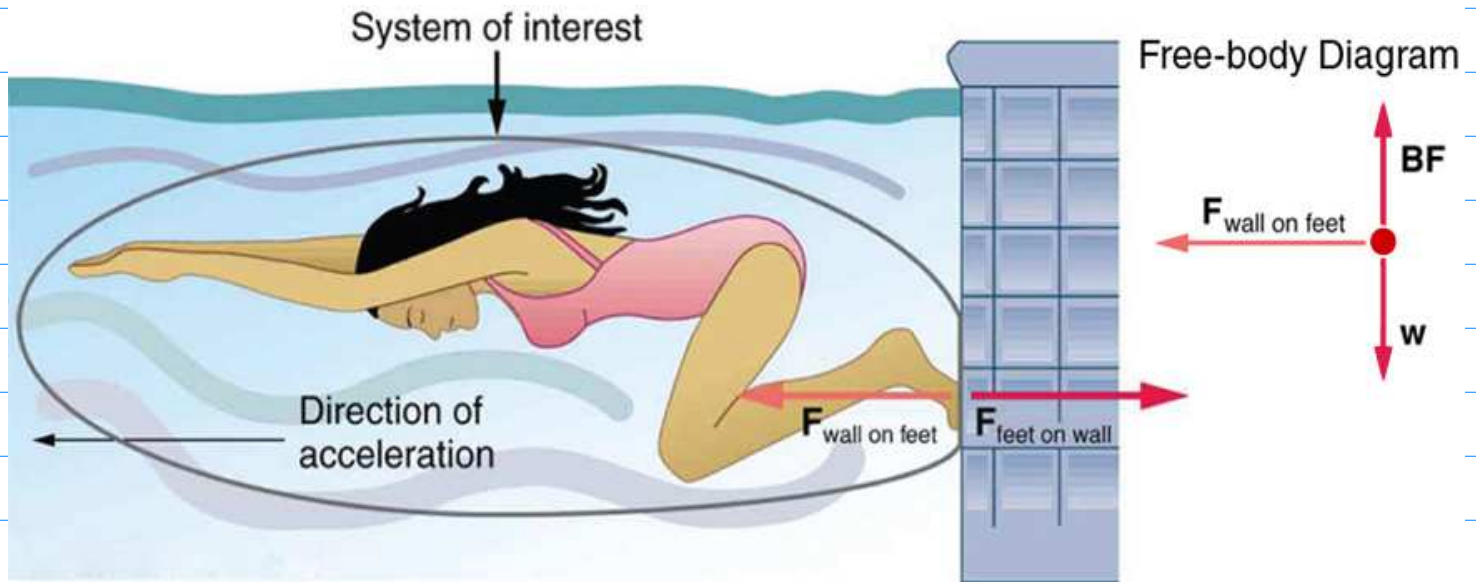
ONLY THE RANGE DEPENDS ON HORIZONTAL SPEED.

NEWTON'S THOUGHT EXPERIMENT (AN EXPERIMENT NOT CARRIED OUT IN PRACTICE BUT POSSIBLE IN PRINCIPLE):



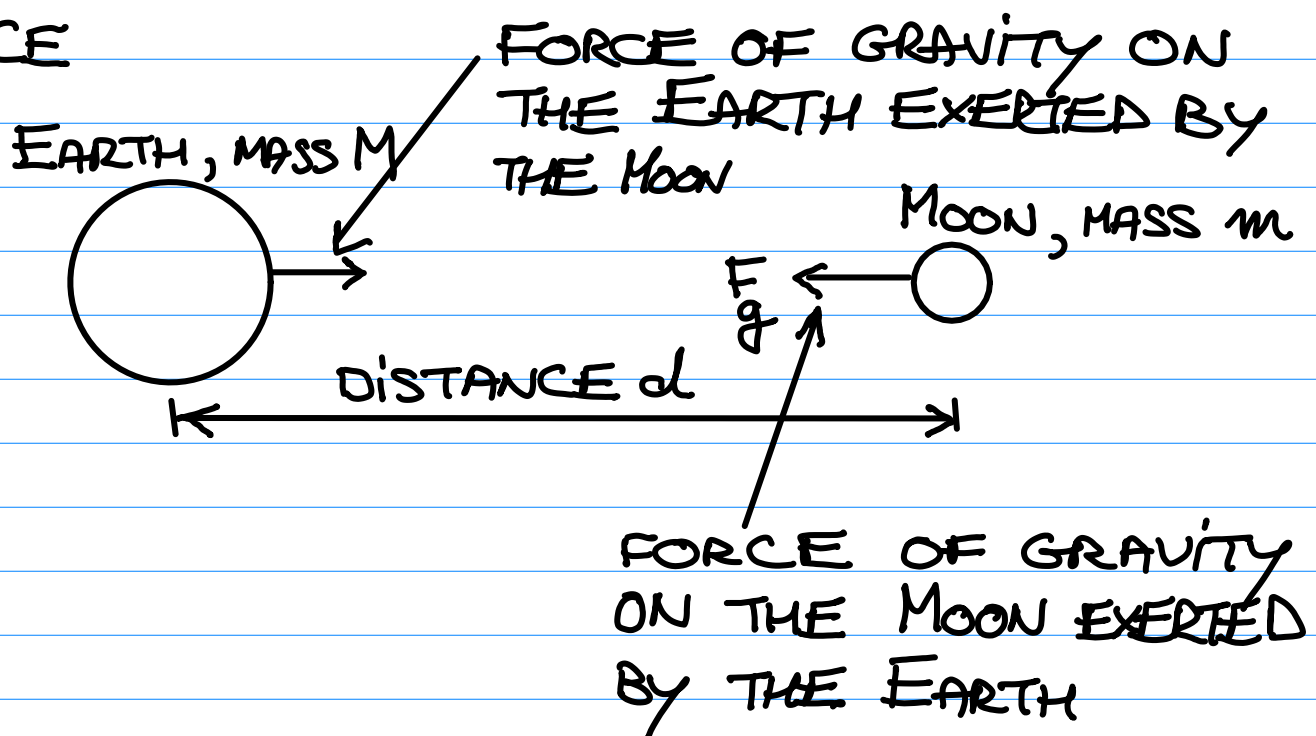
NEWTON CONCLUDED THAT THE EARTH'S GRAVITATIONAL PULL EXTENDS ALL THE WAY TO THE MOON AND BEYOND, AND THAT THE MOON IS FALLING ALL THE TIME UNDER THE INFLUENCE OF EARTH'S GRAVITY. THE MOON HAS JUST THE RIGHT ORBITAL SPEED TO REMAIN IN CIRCULAR ORBIT.

NEWTON'S THIRD LAW OF MOTION:



INTERACTING BODIES EXERT FORCES ON EACH OTHER THAT ARE EQUAL IN MAGNITUDE BUT OPPOSITE IN DIRECTION.

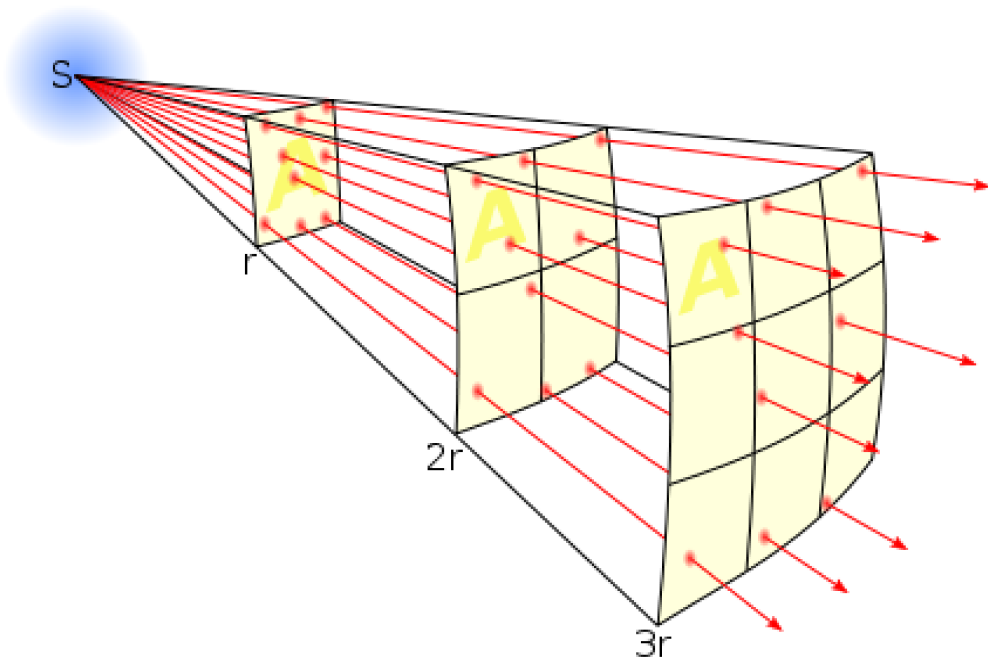
HENCE



WE HAD $F_g \propto m$ AND BY SYMMETRY

$$F_g \propto mM.$$

THERE WERE SOME HEURISTIC ARGUMENTS (BULLIALDUS, HOOKE, WREN, HALLEY, NEWTON) THAT THE FORCE OF GRAVITY DECREASES WITH DISTANCE d AS $1/d^2$ (INVERSE SQUARE LAW):

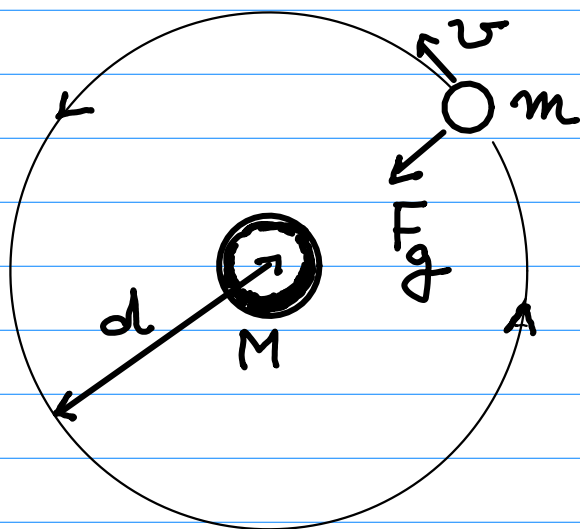


NEWTON'S LAW OF GRAVITY:

$$F_g = G \frac{Mm}{d^2}$$

GRAVITATIONAL CONSTANT
(ITS VALUE WAS DETERMINED
EXPERIMENTALLY MORE THAN
100 YEARS AFTER THE LAW
WAS FORMULATED)

NEWTON WAS ABLE TO EXPLAIN KEPLER'S
LAWS OF PLANETARY MOTIONS BY COMBINING
HIS GENERAL LAWS OF MOTION OF ANY
OBJECT WITH THE LAW OF GRAVITY:



$$m \frac{v^2}{d} = F_g \text{ (SECOND LAW)}$$

$$F_g = G \frac{Mm}{d^2} \text{ (LAW OF GRAVITY)}$$

THEREFORE

$$m \frac{v^2}{d} = G \frac{Mm}{d^2},$$

AND THE ORBITAL SPEED v IS GIVEN BY

$$\underline{v^2 = G \frac{M}{d}} \quad \text{OR} \quad \underline{v = \frac{\sqrt{GM}}{\sqrt{d}}}$$

BUT THE ORBITAL SPEED v IS EQUAL TO

$$v = \frac{\overbrace{2\pi d}^{\text{CIRCUMFERENCE}}}{\underset{\substack{\uparrow \\ \text{ORBITAL PERIOD}}}{P}}.$$

THUS

$$\left(\frac{2\pi d}{P} \right)^2 = G \frac{M}{d}$$

OR

$$\underline{\frac{d^3}{P^2} = \frac{GM}{4\pi^2}} \quad \leftarrow \text{MASS OF THE SUN}$$

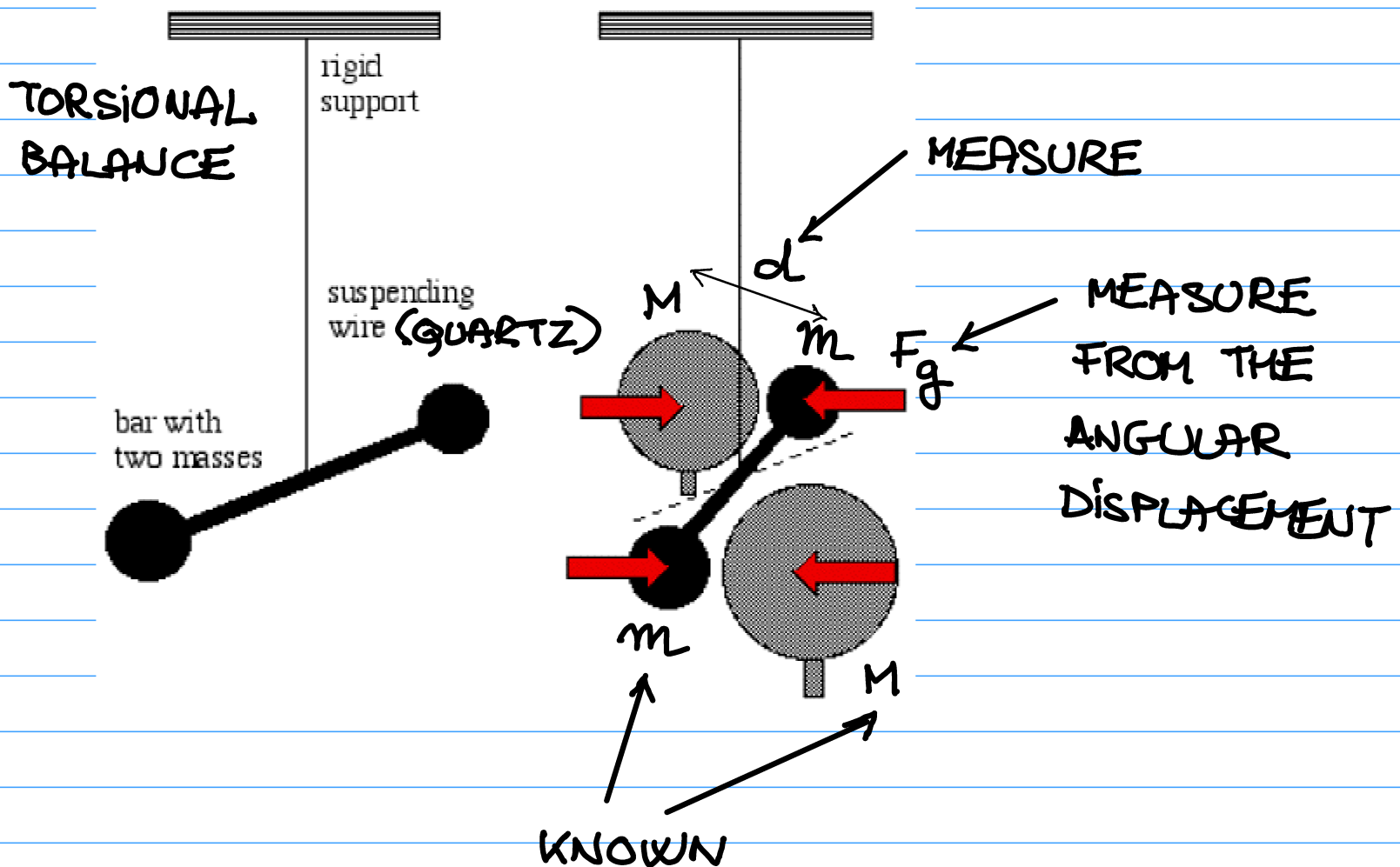
FOR ANY PLANET. THIS IS THE THIRD

KEPLER'S LAW AS FORMULATED BY NEWTON.

NOTE THAT THE MASS M OF THE CENTRAL OBJECT CAN BE DETERMINED FROM THE ORBITAL DATA (DISTANCE d AND ORBITAL PERIOD P) OF THE REVOLVING BODY (m) IF THE VALUE OF THE GRAVITATIONAL CONSTANT G IS KNOWN.

WEIGHING THE EARTH EXPERIMENT:

H. CAVENDISH (1731-1810) IN 1798



DEDUCE G FROM $F_g = G \frac{Mm}{d^2}$

