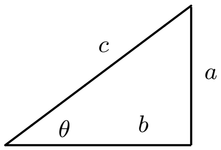
Mathematics

quadratic equation, $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

trigonometry:

$$a^2 + b^2 = c^2$$



$$\begin{aligned} \sin\theta &= a/c \\ \cos\theta &= b/c \\ \tan\theta &= a/b \\ \tan\theta &= \frac{\sin\theta}{\cos\theta} \end{aligned}$$

circular arc (θ in rad):

$$s = \theta r = 2\pi r \text{ for } \theta = 2\pi \text{ (a full circle)}$$

Fundamental Constants

$g =$ 9.80 m s^{-2}
$c =$ speed of light in vacuum $= 3.00 \times 10^8$ ms^{-1}
$e =$ charge of electron $= 1.60 \times 10^{-19}$ C
$m_e =$ mass of electron $= 9.11 \times 10^{-31}$ kg
$m_n =$ mass of neutron or proton $= 1.67 \times 10^{-27}$ kg
$G =$ gravit. constant $= 6.673 \times 10^{-11}$ N m^{2}kg^{-2}
$R_E =$ radius of Earth $= 6.37 \times 10^6$ m
$M_E =$ mass of Earth $= 5.98 \times 10^{24}$ kg
$M_L =$ mass of Moon $= 7.35 \times 10^{22}$ kg
$M_S =$ mass of Sun $= 1.99 \times 10^{30}$ kg
$k_B =$ Boltzmann's constant $= 1.38 \times 10^{-23}$ J/K
$\mu_0 =$ $4\pi \times 10^{-7}$ Wb A^{-1}m^{-1}
$\varepsilon_0 =$ 8.85×10^{-12} C^{2}N^{-1}m^{-2}

Mechanics

uniform linear acceleration $a =$ const:

$$\begin{aligned} v &= v_0 + at \\ \bar{v} &= \frac{1}{2}(v_0 + v) = v_0 + \frac{1}{2}at \\ d &= \bar{v}t = \frac{1}{2}(v_0 + v)t = v_0 t + \frac{1}{2}at^2 \\ v^2 &= v_0^2 + 2ad \end{aligned}$$

Newton's Laws:

N2L: $\vec{a} = \frac{1}{m}\vec{F}_{\text{total}}$

N3L: $\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$

Gravity: $F = G\,\frac{m_1\, m_2}{r^2}$

friction:

$$\begin{aligned} f_s &\leq f_s^{(max)} = \mu_s F_N \\ f_k &= \text{const} = \mu_k F_N, \qquad \mu_k < \mu_s \end{aligned}$$

kinematics of uniform angular acceleration:
(angular displacement θ, velocity ω, acceleration α)

$$\begin{aligned} \omega &= \omega_0 + \alpha t \\ \overline{\omega} &= \frac{1}{2}(\omega + \omega_0) \\ \theta &= \overline{\omega}t = \frac{1}{2}(\omega + \omega_0)t \\ \theta &= \omega_0 t + \frac{1}{2}\alpha t^2 \end{aligned}$$

uniform rolling ($\alpha = 0$) without slipping:

$$\begin{aligned} v_T &= \omega r \\ a_T &= \alpha r \end{aligned}$$

centripetal force and acceleration ($\alpha = 0$):

$$F_c = m a_c = m\,\frac{v_T^2}{r} = m\omega^2 r$$

torque $\tau =$ force $\times$ lever arm $= Fl = Fr\sin\phi$
moment of inertia:

$$I = mr^2 \quad \text{or} \quad I_{\text{body}} = \textstyle\sum_i m_i r_i^2$$

about an axis through the geometrical centre:

$$I_{\text{hoop}} = mr^2 \quad I_{\text{disk}} = \frac{1}{2}mr^2 \quad I_{\text{sphere}} = \frac{2}{5}mr^2$$

N2L for rotations

$$\tau = I\alpha \quad \text{or} \quad \textstyle\sum \tau = I_{\text{body}}\alpha$$

simple harmonic oscillator, $F = -kx$, $U = \frac{1}{2}kx^2$

$$f = 1/T \quad \omega = 2\pi f = \sqrt{k/m}$$

$$\begin{cases} x = A\cos\theta = A\cos\omega t \\ v = -A\omega\sin\omega t \\ a = -A\omega^2\cos\omega t \end{cases}$$

Conservation Laws

work ($= Fd$) done by a conservative force

$$\mathcal{W}_{\text{c.f.}}(\vec{r} \to \vec{r}\,') = -\left[U(\vec{r}\,') - U(\vec{r})\right] = -\Delta U(\vec{r})$$

gravitational potential energy

$$F_{\text{gravity}} = -mg \quad \rightsquigarrow \quad U_g = mgy$$

kinetic energy of translation with velocity $\vec{V}$

$$K_t = \frac{1}{2}\,mV^2$$

kinetic energy of rotation with angular velocity ω

$$K_r = \frac{1}{2}\,I\omega^2$$

conservation of total mechanical energy

$$\Delta\left(U + K_t + K_r\right) = \mathcal{W}_{\text{non-c.f.}}$$

power = rate of doing work $= \mathcal{W}/\Delta t = FV$
1 Watt = 1 J/1 s and 1 h.p. = 746 W
N2L in terms of linear momentum $\vec{p} = m\vec{V}$

$$\text{Impulse} = \overline{\vec{F}}\,\Delta t = \Delta\vec{p} \quad \text{or} \quad \vec{F} = \lim_{\Delta t \to 0}\frac{\Delta\vec{p}}{\Delta t}$$

conservation of momentum for a system as a whole

$$\sum \vec{F}_{\text{ext}} = \lim_{\Delta t \to 0}\frac{\Delta\vec{p}_{\text{total}}}{\Delta t} \qquad \text{(elastic collision, } \Delta E = 0\text{)}$$

Waves and Sound

travelling wave $y = A\cos\left(\frac{2\pi}{\lambda}x \pm \frac{2\pi}{T}t\right)$ with $v = \lambda/T = \lambda f$

$$v_{\text{string}} = \sqrt{\frac{F}{m/L}}$$

sound intensity

$$\beta = 10\,\log\frac{I}{I_0},\ \text{dB} \qquad I_0 = 1 \times 10^{-12}\ \text{W/m}^2$$

point source

$$I = \frac{P}{A} = \frac{P}{4\pi r^2}$$

Doppler (upper sign = approach, lower = recede)

$$f_o = f_s\left(\frac{v \pm v_o}{v \mp v_s}\right) \qquad \text{s=source, o=observer}$$