Chapter 2, Problem 38: Along a straight road through town, there are three speed-limit signs. They occur in the following order: 55, 35, and 25 mi/h, with the 35-mi/h sign located midway between the other two. Obeying these speed limits, the smallest possible time $t_A$ that a driver can spend on this part of the road is to travel between the first and second signs at 55 mi/h and between the second and third signs at 35 mi/h. More realistically, a driver could slow down from 55 to 35 mi/h with a constant deceleration and then do the same thing from 35 to 25 mi/h. This alternative requires a time $t_B$. Determine the ratio $t_B/t_A$.

Solution: Write expressions for both times, then divide them, noticing that the unknown distance $D$ cancels.

$$t_A = \frac{D}{55} + \frac{D}{35} = D \left( \frac{1}{55} + \frac{1}{35} \right)$$

$$t_B = \frac{D}{v_1} + \frac{D}{v_2}$$

$$t_B = \frac{D}{\frac{55 + 35}{2}} + \frac{D}{\frac{35 + 25}{2}}$$

$$t_B = \frac{D}{1} + \frac{D}{1} = D \left( \frac{1}{1} + \frac{1}{1} \right)$$
Chapter 2, Problem 39  You are driving your car, and the traffic light ahead turns red. You apply the brakes for 3.00 s, and the speed of the car decreases to 4.50 m/s. The car's deceleration has a magnitude of 2.70 m/s$^2$ during this time. Determine the car's displacement.

Solution: Using kinematics equations without drawing a position-time graph is an effective method for solving this problem, but making use of a velocity-time diagram is also effective. Even if you don't like velocity-time diagrams, if you get stuck use one as a last resort instead of giving up.

The strategy is to first determine the initial velocity (using the acceleration and final velocity), and then to use it to determine the displacement, which is equal to the area under the graph.

\[
\frac{t_B}{t_A} = \frac{D \left( \frac{1}{45} + \frac{1}{30} \right)}{D \left( \frac{1}{55} + \frac{1}{35} \right)} = 1.19
\]

Thus, the time for the second journey is greater than the time for the first journey, as expected.
Chapter 3, Problem 20  A golfer hits a shot to a green that is elevated 3.0 m above the point where the ball is struck. The ball leaves the club at a speed of 14.0 m/s at an angle of 40.0° above the horizontal. It rises to its maximum height and then falls down to the green. Ignoring air resistance, determine the speed of the ball just before it lands.

\[
\text{Slope} = \frac{\text{rise}}{\text{run}}
\]

\[-2.7 = \frac{4.5 - v_0}{3 - 0}\]

\[v_0 = 4.5 - (-2.7)(3)\]

\[v_0 = 12.6 \text{ m/s}\]

\[d = \text{area under graph} \]
\[d = \text{area of } \triangle + \text{area of rectangle}\]
\[d = \frac{1}{2}(12.6 - 4.5)(3) + 4.5(3)\]
\[d = 25.65 \text{ m}\]
Solution:

Method 1: The x-component of velocity is constant, and we can calculate it because we know the initial velocity. Our main task is therefore to determine the y-component of the final velocity; if we can do this, then we can determine the final speed by applying Pythagoras's theorem to the components of the final velocity.

The y-component of the final velocity can be calculated using kinematics equations, as follows.

\[ v_y^2 - v_{0y}^2 = 2a_y(y) \]

\[ v_y^2 = v_{0y}^2 + 2a_y(y) \]

\[ v_y^2 = (14 \sin 40°)^2 + 2(-9.8)(3) \]

\[ v_y = -4.71 \text{ m/s} \]
Final speed:

\[ v^2 = (4 \cos 40^\circ)^2 + (4 \sin 40^\circ)^2 \]

\[ v = 11.7 \text{ m/s} \]

Method 2: Use the principle of conservation of mechanical energy.

\[
\frac{1}{2} m v_0^2 + m g (0) = \frac{1}{2} m v^2 + m g (3)
\]

\[
\frac{1}{2} v_0^2 = \frac{1}{2} v^2 + 3 g
\]

\[
v_0^2 = v^2 + 6 g
\]

\[
v^2 = v_0^2 - 6 g
\]

\[
v = \sqrt{14^2 - 6 (9.8)}
\]

\[ v = 11.7 \text{ m/s} \]

Note that both methods produce the same result, as they must, but the solution using energy methods is simpler. The moral is to be on the lookout for possibilities to use energy methods, as they are often easier.
Chapter 3, Problem 26  In the absence of air resistance, a projectile is launched from and returns to ground level. It follows a trajectory similar to that shown in Figure 3.10 and has a range of 30 m. Suppose the launch speed is doubled, and the projectile is fired at the same angle above the ground. What is the new range?

Solution: Consider the first motion, where we’ll call the initial speed $v_1$ and the initial velocity’s angle $\theta$. The horizontal displacement (also called the range) of the projectile is $x_1$, where

$$x_1 = (v_1 \cos \theta)t_1 \quad (1)$$

To determine the time $t_1$ at which the projectile reaches the ground again, ask yourself what is the condition that describes the projectile reaching the ground again? The answer: When the projectile reaches the ground again, the $y$-component of the displacement is zero. Thus,

$$\Delta y = v_{0y} t_1 + \frac{1}{2} a_y t_1^2$$

$$0 = v_{0y} t_1 - \frac{1}{2} g t_1^2$$

$$0 = t_1(v_{0y} - \frac{1}{2} g t_1)$$

$$t_1 = 0 \quad \text{or} \quad v_{0y} - \frac{1}{2} g t_1 = 0$$

Reject $\frac{1}{2} g t_1 = v_{0y}$

$$g t_1 = 2 v_{0y}$$

$$t_1 = \frac{2 v_1 \sin \theta}{g} \quad (2)$$

The value $t_1 = 0$ is rejected because it applies when the projectile leaves the ground, and we are interested in the time when the projectile returns to the ground, which is the other value for $t_1$. 

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Substituting this value of $t_1$ into equation (1), we obtain

$$X_1 = (v_1 \cos \Theta) t_1,$$

$$X_1 = (v_1 \cos \Theta) \frac{2v_1 \sin \Theta}{g},$$

$$X_1 = \frac{2v_1^2 \sin \Theta \cos \Theta}{g}.$$

Exactly the same reasoning can be applied to the second motion, which is projected at the same angle but with a different initial speed. The result is, therefore,

$$X_2 = \frac{2v_2^2 \sin \Theta \cos \Theta}{g}.$$

Thus,

$$\frac{X_2}{X_1} = \frac{\left(\frac{2v_2^2 \sin \Theta \cos \Theta}{g}\right)}{\left(\frac{2v_1^2 \sin \Theta \cos \Theta}{g}\right)}$$

$$\frac{X_2}{X_1} = \frac{v_2^2}{v_1^2}$$

$$X_2 = \left(\frac{v_2}{v_1}\right)^2.$$
Note that the horizontal displacement for the second projectile is 4 times as large as for the first projectile. Does this make sense? Note from equation (2) that doubling the initial speed (keeping the initial angle the same) doubles the flight time; then from equation (1) we can see that doubling the initial speed and the flight time quadruples the range.

Chapter 3, Problem 60    The captain of a plane wishes to proceed due west. The cruising speed of the plane is 245 m/s relative to the air. A weather report indicates that a 38.0-m/s wind is blowing from the south to the north. In what direction, measured with respect to due west, should the pilot head the plane?

Solution:
Chapter 4, Problem 34  A neutron star has a mass of $2.0 \times 10^{30}$ kg (about the mass of our sun) and a radius of $5.0 \times 10^{3}$ m (about the height of a good-sized mountain). Suppose an object falls from rest near the surface of such a star. How fast would this object be moving after it had fallen a distance of 0.010 m? (Assume that the gravitational force is constant over the distance of the fall and that the star is not rotating.)

Solution: Use Newton's second law to determine the acceleration of the object. Then use kinematics to determine the final speed of the object.

$$F = ma$$

$$\frac{GMm}{r^2} = ma$$

$$a = \frac{GM}{r^2}$$
Chapter 4, Problem 48  Traveling at a speed of 16.1 m/s, the driver of an automobile suddenly locks the wheels by slamming on the brakes. The coefficient of kinetic friction between the tires and the road is 0.720. What is the speed of the automobile after 1.30 s have elapsed? Ignore the effects of air resistance.

Solution: Start by drawing a free-body diagram of the car. Suppose that the car is initially moving to the right. Then use the free-body diagram
to write Newton's second law in both the vertical and horizontal directions. Doing this will help us determine the car's acceleration; once we have the car's acceleration, we can use kinematics equations to determine its final speed.

\[ y: \quad N - mg = m(0) \quad (1) \]
\[ x: \quad -f = ma \quad (2) \]
\[ f = \mu N \quad (3) \]

\[ (1) \rightarrow N = mg \]
\[ (1) + (3) \rightarrow f = \mu mg \]
\[ (2) \rightarrow a = -\frac{f}{m} \]

\[ a = -\frac{\mu mg}{m} \]
\[ a = -\mu g \]

Now use a kinematics equation to determine the final speed:

\[ \Delta v = at \]
\[ v_f - v_i = at \]
\[ v_f = v_i + at \]
\[ v_f = v_i - \mu g t \]
\[ v_f = 16.1 - 0.720(9.8)(1.2) \]
Chapter 4, Problem 82
To hoist himself into a tree, a 72.0-kg man ties one end of a nylon rope around his waist and throws the other end over a branch of the tree. He then pulls downward on the free end of the rope with a force of 358 N. Neglect any friction between the rope and the branch, and determine the man's upward acceleration.

Solution:

\[ 2T - mg = ma \]

\[ a = \frac{2T - mg}{m} \]

\[ a = \frac{2(358) - 72(9.8)}{72} \]

\[ a = 0.14 \text{ m/s}^2 \]
rope are not the same. Find the tensions in the rope to the left and to the right of the mountain climber.

Solution: Draw a free-body diagram for the person. Because the person is in static equilibrium, both components of acceleration are zero.

\[\text{x:}\]
\[T_2 \sin 80^\circ - T_1 \sin 65^\circ = m \cdot 0\]

\[\text{y:}\]
\[T_1 \cos 65^\circ + T_2 \cos 80^\circ - mg = m \cdot 0\]

\[\implies T_2 \sin 80^\circ = T_1 \sin 65^\circ\]
\[T_2 = T_1 \frac{\sin 65^\circ}{\sin 80^\circ}\]

\[\implies T_1 \cos 65^\circ + \left(T_1 \frac{\sin 65^\circ}{\sin 80^\circ}\right) \cos 80^\circ = mg\]
A pulley problem with friction present: Problem 5 from the December 2015 final exam.

Two objects (with masses 45.0 kg and 21.0 kg) are connected by a massless string that passes over a massless, frictionless pulley. The pulley hangs from the ceiling. Air resistance results in a 5-N force on the 45.0-kg block and a 10-N force on the 21.0-kg block.

(a) Determine the acceleration of the system.
(b) Determine the tension in the string.

Solution: The pulley is massless, so the tension in each side of the string is the same, and we can therefore use the same symbol $T$.
to represent the tension in each side of the string. Draw a free-body diagram for each block, and choose downwards to be the positive direction for the heavy block, and upwards to be the positive direction for the light block. In this way, we can use the same symbol $a$ for the acceleration of each block.

Note that the frictional forces oppose the motions of each block, so the friction force acts upward on the heavy block (which moves downward) and downward on the light block (which moves upward).

Now write Newton's second law for each block, and solve the resulting equations.

\[ 45g - T - 5 = 45a \quad ① \]
\[ T - 21g - 10 = 21a \quad ② \]

\[ ① + ② : 45g - 21g - 15 = 66a \]

\[ a = \frac{24g - 15}{15} \]
Chapter 5, Problem 30  The drawing shows a baggage carousel at an airport. Your suitcase has not slid all the way down the slope and is going around at a constant speed on a circle (r = 11.0 m) as the carousel turns. The coefficient of static friction between the suitcase and the carousel is 0.760, and the angle $\theta$ in the drawing is 36.0°. How much time is required for your suitcase to go around once?

Solution:
\[ \begin{align*}
x : & \quad f \cos 36^\circ - N \sin 36^\circ = ma = \frac{mv^2}{r} \quad (1) \\
y : & \quad N \cos 36^\circ + f \sin 36^\circ - mg = m(0) \quad (2) \\
\quad f = \mu N \quad (3) \\
(3) + (1) \implies & \quad \mu N \cos 36^\circ - N \sin 36^\circ = \frac{mv^2}{r} \quad (4) \\
(3) + (2) \implies & \quad N \cos 36^\circ + \mu N \sin 36^\circ = mg \quad (5)
\end{align*} \]

\[
N \left( \mu \cos 36^\circ - \sin 36^\circ \right) = \frac{mv^2}{r} \\
N \left( \cos 36^\circ + \mu \sin 36^\circ \right) = mg
\]

\[
\begin{align*}
v &= \sqrt{gr \left( \frac{\mu \cos 36^\circ - \sin 36^\circ}{\cos 36^\circ + \mu \sin 36^\circ} \right)} \\
v &= 1.52 \text{ m/s}
\end{align*}
\]
Chapter 5, Problem 36  A satellite circles the earth in an orbit whose radius is twice the Earth's radius. The Earth's mass is 5.98 x 10^{24} kg, and its radius is 6.38 x 10^6 m. What is the period of the satellite?

Solution: Begin by applying Newton's law of motion to the satellite; then use Newton's law of gravity for the "F" side of the equation:

\[
F = ma \\
\frac{GMm}{r^2} = ma \\
\frac{GM}{r^2} = a \\
\frac{GM}{r} = \frac{v^2}{r} \\
\frac{GM}{r} = \left(\frac{2\pi r}{T}\right)^2
\]

\[
t = \frac{2\pi r}{v} = \frac{2\pi (11)}{1.52} = 45.35 \text{ s}
\]
Chapter 5, Problem 38  A satellite has a mass of 5850 kg and is in a circular orbit $4.1 \times 10^5$ m above the surface of a planet. The period of the orbit is 2.00 hours. The radius of the planet is $4.15 \times 10^6$ m. What would be the true weight of the satellite if it were at rest on the planet’s surface?

Solution: The strategy is always the same for these types of problems; write Newton's second law of motion for the orbiting object, combine it with Newton's law of gravity, and then use $2\pi r = \nu T$ if necessary.
The specific strategy in this problem is to first determine the mass of the planet using the general strategy given above, using the radius of the satellite's orbit. Then we'll use Newton's law of gravity to determine the satellite's weight (i.e., the gravitational force that the planet exerts on the satellite) when the satellite is resting on the planet's surface.

\[ F = ma \]

\[ \frac{GMm}{r^2} = m \frac{v^2}{r} \]

\[ \frac{GM}{r} = \frac{v^2}{r} \]

\[ \frac{GM}{r} = \left( \frac{2\pi r}{T} \right)^2 \]

\[ \frac{GM}{r} = \frac{4\pi^2 r^2}{T^2} \]

\[ M = \frac{4\pi^2 r^3}{6T^2} \]

Note that in the previous equation \( r \) is the radius of the satellite's orbit, which is

\[ r = 4.15 \times 10^6 \text{ m} + 4.1 \times 10^5 \text{ m} \]

\[ r = 4.15 \times 10^6 \text{ m} + 0.41 \times 10^6 \text{ m} \]
\[ r = 4.56 \times 10^6 \text{ m} \]

Also note that the period of the satellite is 2 h, which must be converted to SI units (i.e., seconds) before being inserted into the equation. Thus, \( T = 2 \text{ h} = 7200 \text{ s} \).

Continuing with the calculation of the planet's mass, we have

\[
M = \frac{4\pi^2 r^3}{GT^2}
\]

\[
M = \frac{4\pi^2 (4.56 \times 10^6)^3}{(6.67 \times 10^{-11})(7200)^2}
\]

\[
M = 1.083 \times 10^{24} \text{ kg}
\]

The weight of the satellite when it rests on the planet's surface is

\[
F = \frac{GMm}{R^2}
\]

where \( R \) is the radius of the planet.

\[
F = \frac{(6.67 \times 10^{-11})(1.083 \times 10^{24})(5.850)}{(4.15 \times 10^6)^2}
\]

\[
F = 24,500 \text{ N}
\]

Chapter 5, Problem 46  A stone is tied to a string (length = 1.10 m) and whirled in a circle at the same constant speed in two different ways. First, the circle is horizontal and the string is nearly parallel to the ground. Next, the circle is vertical. In the vertical case the
maximum tension in the string is 15.0% larger than the tension that exists when the circle is horizontal. Determine the speed of the stone.

Solution: When the stone is swinging in a vertical circle, at which point of the motion is the tension in the string the greatest? To answer this question, draw free body diagrams for the stone at various points of its motion. Also note the key point that the speed of the stone is the same at all points of the motion.

Recall that the centripetal acceleration of the stone is $\frac{v^2}{r}$, and if the speed is constant, then so is the centripetal acceleration. How can it be that the centripetal acceleration, which points towards the centre of the circle, is constant even though the direction of the weight force, which is downwards, sometimes points towards the centre of the circle (left diagram), sometimes points directly away from the centre of the circle (right diagram), and sometimes elsewhere (middle diagram)? Evidently the tension force adjusts itself, both in magnitude and direction, so that the speed remains constant. Study the diagrams carefully and give this some thought, and see if you can arrive at the conclusion that the tension force is greatest when the stone is at the bottom of the vertical circle.

On the other hand, the tension is constant when the stone is whirled in a horizontal circle. Let $F$ represent the tension in the string when the stone moves in a horizontal circle, so that
When the string is at the bottom of the vertical circle,

\[ T - mg = \frac{m v^2}{r} \]  

Substituting the expression for \( F \) from equation (1) into equation (2), and solving for the speed \( v \), we obtain

\[ 1.15 \frac{m v^2}{r} - mg = \frac{m v^2}{r} \]

\[ 1.15 \frac{v^2}{r} - g = \frac{v^2}{r} \]

\[ 1.15 \frac{v^2}{r} - \frac{v^2}{r} = g \]

\[ (1.15 - 1) \frac{v^2}{r} = g \]

\[ 0.15 \frac{v^2}{r} = gr \]

\[ v^2 = \frac{gr}{0.15} \]

\[ v = \sqrt{\frac{(0.8)(1.1)}{0.15}} \]
Problem 8 from Test 5: The radius of Planet X is $4.73 \times 10^7$ m, its mass is $5.91 \times 10^{25}$ kg, and its rotational period is 26.5 h. The radius of the orbit of Planet X about a star is $3.27 \times 10^{11}$ m, and the orbital period is 1.34 Earth years.

(a) Determine the surface gravity on Planet X.
(b) Determine the orbital speed (in m/s) of Planet X.
(c) Determine the acceleration (in m/s$^2$) of the centre of Planet X.
(d) Determine the force that the star exerts on Planet X.
(e) Determine the mass of the star.

Solution: Let $R$ represent the radius of Planet X and let $r$ represent the radius of the orbit of Planet X around its star.

(a) Suppose a small object of mass $m$ is at the surface of Planet X, whose mass is $M$. The force that Planet X exerts on the small object is

$F = mg$

$$g = \frac{GM}{R^2}$$

$$g = \frac{(6.67 \times 10^{-11})(5.91 \times 10^{25})}{(4.73 \times 10^7)^2}$$

$$g = 1.76 \text{ m/s}^2$$
(b) \[ \nu = \frac{2 \pi r}{T} \]

\[ \nu = \frac{2 \pi \left( 3.27 \times 10^{11} \right)}{(1.34)(365.25)(24)(3600)} \]

\[ \nu = 4.86 \times 10^4 \text{ m/s} \]

(c) \[ a = \frac{\nu^2}{r} \]

\[ a = \frac{(4.86 \times 10^4)^2}{3.27 \times 10^{11}} \]

\[ a = 7.22 \times 10^{-3} \text{ m/s}^2 \]

(d) Let \( m \) represent the mass of Planet X, and let \( M \) represent the mass of the star. The force exerted by the star on Planet X can be obtained from the result of Part (c.) and Newton's second law of motion:

\[ F = ma \]

\[ F = (5.91 \times 10^{25}) \left( 7.22 \times 10^{-3} \right) \]

\[ F = 4.27 \times 10^{23} \text{ N} \]

(e.) Using the result of Part (d), the force that the star exerts on Planet X is

\[ F = \frac{GMm}{r} \]
Chapter 6, Problem 24  A skier slides horizontally along the snow for a distance of 21.0 m before coming to rest. The coefficient of kinetic friction between the skier and the snow is 0.050. Initially, how fast was the skier going?

Solution: The work done by friction on the skier reduces the skier's kinetic energy. (Because the skier moves horizontally, there is no change in the skier's gravitational potential energy, and so we don't need to consider gravitational potential energy.)

The work done by friction is equal to the frictional force times the displacement; the frictional force can be determined by drawing a free-body diagram, as follows:

\[
\begin{align*}
G \frac{M m}{r^2} &= F \\
M &= \frac{Fr^2}{Gm} \\
M &= \frac{(4.27 \times 10^{23})(3.027 \times 10^{11})^2}{(6.67 \times 10^{-11})(5.91 \times 10^{25})} \\
M &= 1.16 \times 10^{31} \text{ kg}
\end{align*}
\]
\[ f = \mu mg \]

According to the work-energy principle, the work done by friction on the skier is equal to the skier's change in kinetic energy. The frictional force is in the opposite direction to the skier's displacement, so the work done by friction on the skier is negative.

\[ W = \Delta K \]
\[ -f \cdot d = \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2 \]
\[ -\mu mg \cdot d = \frac{1}{2} m (0)^2 - \frac{1}{2} m v_0^2 \]

\[ \mu g d = \frac{1}{2} v_0^2 \]

\[ v_0 = \sqrt{2 \mu g d} \]
\[ v_0 = \sqrt{2 \left(0.05\right)(9.8)(21)} \]

\[ v_0 = 4.54 \text{ m/s} \]

Chapter 6, Problem 46  A pendulum consists of a small object hanging from the ceiling at the end of a string of negligible mass. The string has a length of 0.75 m. With the string hanging vertically, the object is given an initial velocity of 2.0 m/s parallel to the ground and swings upward in a circular arc. Eventually, the object comes to a momentary halt at a point where the string makes an angle \( \theta \) with its initial vertical orientation and then swings back downward. Find the angle \( \theta \).
Solution: Use the principle of conservation of mechanical work. Identify two interesting points on the motion, and observe that the total mechanical energy is the same at each point. The two points of interest are the point at the lowest point of the motion (which I'll call position 1), and the highest point of the motion (which I'll call position 2).

Is it clear that mechanical energy is conserved? Doesn't the tension force do work on the small object hanging from the string? No, the tension force does no work on the object, because the tension force is always perpendicular to the motion of the small object. Thus, mechanical energy is conserved. (Of course, we ignore friction, including air resistance.)

\[
\frac{1}{2} m v_1^2 + m g y_1 = \frac{1}{2} m v_2^2 + m g y_2
\]

\[
\frac{1}{2} m v_1^2 + m g (0) = \frac{1}{2} m (0)^2 + m g h
\]

\[
\frac{1}{2} m v_1^2 = m g h
\]

\[
\frac{1}{2} v_1^2 = g h
\]

\[
h = \frac{v_1^2}{2g}
\]

Note from the diagram that
Chapter 6, Problem 54  A student, starting from rest, slides down a water slide. On the way down, a kinetic frictional force (a non-conservative force) acts on her. The student has a mass of 83.0 kg, and the height of the water slide is 11.8 m. If the kinetic frictional force does $-6.50 \times 10^3$ J of work, how fast is the student going at the bottom of the slide?

Solution: The strategy again is to use the work-energy principle, only this time both kinetic energy and gravitational potential energy are involved.

$$W_{nc} = \Delta K + \Delta U$$
Chapter 7, Problem 24  A 0.015-kg bullet is fired straight up at a falling wooden block that has a mass of 1.8 kg. The bullet has a speed of 810 m/s when it strikes the block. The block originally was dropped from rest from the top of a building and had been falling for a time $t$ when the collision with the bullet occurs. As a result of the collision, the block (with the bullet in it) reverses direction, rises, and comes to a momentary halt at the top of the building. Find the time $t$.

Solution: As usual, we'll choose upwards to be the positive direction. Because the block fell from rest, its velocity just before it collides with the bullet is $-gt$. By the principle of conservation of momentum, immediately after the collision the block + bullet has velocity $v$, where

$$W_{nc} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 + m g y_f - m g y_i$$

$$W_{nc} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 + m g y_f^0 - m g y_i$$

$$\frac{1}{2} m v_f^2 = W_{nc} + m g y_i$$

$$m v_f^2 = 2W_{nc} + 2mg y_i$$

$$v_f^2 = \frac{2W_{nc}}{m} + 2gy_i$$

$$v_f^2 = \frac{2(-6500)}{83} + 2(9.8)(11.8)$$

$$v_f = 8.64 \text{ m/s}$$
Because the block + bullet system just reaches the original height from which the block was dropped with zero speed, it follows that \( v = gt \). Thus,

\[
1.815gt = (0.015)(810) - 1.8gt
\]

\[
3.615gt = (0.015)(810)
\]

\[
t = (0.015)(810)/(3.615g)
\]

\[
t = 0.343 \text{ s}
\]

Chapter 7, Problem 38  A ball is attached to one end of a wire, the other end being fastened to the ceiling. The wire is held horizontal, and the ball is released from rest (see the drawing). It swings downward and strikes a block initially at rest on a horizontal frictionless surface. Air resistance is negligible, and the collision is elastic. The masses of the ball and block are, respectively, 1.60 kg and 2.40 kg, and the length of the wire is 1.20 m. Find the velocity (magnitude and direction) of the ball (a) just before the collision, and (b) just after the collision.
Solution: (a) By the principle of conservation of mechanical energy,

\[ \frac{1}{2} m_1 v_{1B}^2 = m_1 g h \]

\[ v_{1B}^2 = 2gh \]

\[ v_{1B} = \sqrt{2gh} \]

\[ v_{1B} = \sqrt{2(0.8)(1.2)} \]

\[ v_{1B} = 4.85 \text{ m/s} \]

(b) By the principle of conservation of momentum during the collision,

\[ m_1 v_{1B} + m_2 (0) = m_1 v_{1A} + m_2 v_{2A} \]

\[ v_{1B} = v_{1A} + \frac{m_2}{m_1} v_{2A} \]

\[ v_{1B} = v_{1A} + 1.5 v_{2A} \]  

\[ 1 \]

Because the collision is elastic, kinetic energy is also conserved in the collision, and therefore

\[ \frac{1}{2} m_1 v_{1B}^2 + \frac{1}{2} m_2 (0)^2 = \frac{1}{2} m_1 v_{1A}^2 + \frac{1}{2} m_2 v_{2A}^2 \]

\[ m_1 v_{1B}^2 = m_1 v_{1A}^2 + m_2 v_{2A}^2 \]

\[ v_{1B}^2 = v_{1A}^2 + \frac{m_2}{m_1} v_{2A}^2 \]
\[ V_{IB} = \sqrt{2} + 1.5 \sqrt{2} \]  

1. \[ V_{2A} = \frac{V_{IB} - V_{IA}}{1.5} \]  

2. \[ V_{IB} = V_{IA}^2 + 1.5 \left( \frac{V_{IB} - 2V_{IB}V_{IA} + V_{IA}^2}{1.5} \right) \]  

\[ 1.5V_{IB} = 1.5V_{IA}^2 + V_{IB}^2 - 2V_{IB}V_{IA} + V_{IA}^2 \]  

\[ 0 = 2.5V_{IA}^2 - 2V_{IB}V_{IA} - 0.5V_{IB}^2 \]  

\[ 0 = 5V_{IA}^2 - 4V_{IB}V_{IA} - V_{IB}^2 \]  

\[ V_{IA} = \frac{-(-4V_{IB}) \pm \sqrt{16V_{IB}^2 - 4(5)(-V_{IB}^2)}}{2(5)} \]  

\[ V_{IA} = \frac{4V_{IB} \pm \sqrt{36V_{IB}^2}}{10} \]  

\[ V_{IA} = \frac{4V_{IB} \pm 6V_{IB}}{10} \]  

\[ V_{IA} = V_{IB} \quad \text{OR} \quad V_{IA} = -\frac{V_{IB}}{5} \]  

\( \sqrt{\text{reject}} \)  

\[ V_{IA} = -0.97 \text{ m/s} \]
Chapter 7, Problem 40  A mine car (mass = 440 kg) rolls at a speed of 0.50 m/s on a horizontal track, as the drawing shows. A 150-kg chunk of coal has a speed of 0.80 m/s when it leaves the chute. Determine the speed of the car–coal system after the coal has come to rest in the car.

Solution: The $y$-component of momentum of the system is not conserved; the $y$-component of momentum of the falling coal is reduced to zero by a normal force exerted on the coal by the car.

The $x$-component of momentum of the system is conserved. Therefore, the speed $v$ of the car + coal system after the collision satisfies

\[
(440 + 150)v = 440(0.5) + 150(0.8)\cos 25^\circ
\]
The figure shows a graph of the angular velocity of a rotating wheel as a function of time. Although not shown in the graph, the angular velocity continues to increase at the same rate until $t = 8.0 \text{ s}$. What is the angular displacement of the wheel from 0 to 8.0 s?

Solution: The slope of the graph is the angular acceleration. Thus,

$$\alpha = \frac{6 - (-9)}{5 \text{ s}} \frac{\text{rad}}{\text{s}^2} = 3 \frac{\text{rad}}{\text{s}^2}$$

$$\Delta \theta = \omega_0 t + \frac{1}{2} \alpha t^2$$
Chapter 8, Problem 38  A fisherman is reeling in a fish. In 9.5 s the fisherman winds 2.6 m of fishing line onto a reel whose radius is 3.0 cm (assumed to be constant as an approximation). The line is being reeled in at a constant speed. Determine the angular speed of the reel.

Solution:

\[ \Delta \theta = (-9)(8) + \frac{1}{2}(3)(8)^2 \]
\[ \Delta \theta = 24 \text{ rad} \]

Chapter 8, Problem 58  A dragster starts from rest and accelerates down a track. Each tire has a radius of 0.320 m and rolls without slipping. At a distance of 384 m, the angular speed of the wheels is 288 rad/s. Determine (a) the linear speed of the dragster and (b) the magnitude of the angular acceleration of its wheels.

Solution:
Chapter 9, Problem 24  The figure shows a bicycle wheel resting against a small step whose height is \( h = 0.120 \text{ m} \). The weight of the wheel is \( W = 25.0 \text{ N} \) and its radius is \( r = 0.340 \text{ m} \). A horizontal force is applied to the axle of the wheel. As the magnitude of the applied force increases, there comes a time when the wheel just begins to rise up and loses contact with the ground. What is the magnitude of the force when this happens?

\[(a)\] \( v = r \omega \)
\[v = (0.32 \text{ m})(288 \text{ rad/s}) = 92.2 \text{ m/s} \]

\[(b)\] \( \alpha = \frac{v^2}{2r} \)
\[r \alpha = \frac{v^2}{2r} = \frac{92.2^2}{2(0.32)(384)} \approx 34.6 \text{ rad/s}^2 \]
Solution: The wheel contacts the step and the ground, so there are two normal forces acting on the wheel, one vertically upward from the ground, and one from the corner of the step. We are asked to determine the minimum applied force that causes the wheel to lift off the ground, which will reduce the normal force from the ground to zero; thus, we can omit this force from our free-body diagram.

We'll calculate torques about an axis perpendicular to the page at the corner of the step; the torque of the normal force exerted by the corner of the step on the wheel will therefore be zero, and so it also is omitted from the diagram. (We might also have had to write down the balance of forces on the wheel, but it turns out that we can solve the problem using torques only.)
Chapter 9, Problem 28  A man drags a 72-kg crate across the floor

Just before the wheel starts to lift off the ground, the net torque is still zero. Therefore,

\[ -F(r-h) + mg\ell = 0 \]

\[ F = \frac{mg\ell}{r-h} \]

\[ F = \frac{(25)(0.2592)}{0.22} \]

\[ F = 29.5 \text{ N} \]

Provided that the applied force is less than or equal to this value, the net torque exerted on the wheel about an axis perpendicular to the page and through the corner of the step will be zero, and so the wheel will not lift off the ground. However, any force that is even slightly greater than this value will cause the wheel to lift off the ground.
at a constant velocity by pulling on a strap attached to the bottom
of the crate. The crate is tilted 25° above the horizontal, and the
strap is inclined 61° above the horizontal. The centre of gravity of
the crate coincides with its geometrical centre, as indicated in the
figure. Find the magnitude of the tension in the strap.

Solution: The crate has a constant velocity, and does not rotate, so
it is in equilibrium. (This is a dynamic equilibrium, not a static
equilibrium, but it's still equilibrium.) This means that both the net
force and the net torque acting on the box is zero.

First draw a free-body diagram:

The net force on the box in the x-direction is zero:
The net force on the box in the \( y \)-direction is also zero:

\[
T \cos 61^\circ - f = 0
\]

The net torque acting on the box is a little more complicated to calculate, so we'll take it step-by-step. First notice that it would be wise to choose the bottom-left corner of the box as a reference point for torques; more specifically, we'll calculate torques about an axis coming out of the page at the bottom-left corner of the box. By doing this, we ensure that the normal force and friction force do not appear in the torque equation, as their torques will be zero.

Let's calculate the torques of the other two forces in detail. First, we'll calculate the torque of the weight force:

\[
r^2 = 0.45^2 + 0.2^2
\]

\[
r = 0.4924\text{ m}
\]

\[
\tan \theta = \frac{0.4}{0.9}
\]

\[
\theta = 23.96^\circ
\]
Now let's calculate the torque of the tension force:

\[ \tau_1 = - (mg) r \cos (\theta + 25^\circ) \]

\[ \tau_1 = - (72)(9.8)(0.4924) \cos (48.96) \]

\[ \tau_1 = - 228.13 \text{ N} \cdot \text{m} \]

Now let's calculate the torque of the tension force:

\[ \tau_2 = (T \sin 36^\circ)(0.9) \]

The box has zero angular acceleration, and so the net torque is zero. Therefore,

\[ \tau_2 + \tau_1 = 0 \]

\[ (T \sin 36^\circ)(0.9) - 228.13 = 0 \]
Chapter 9, Problem 44  The drawing shows the top view of two doors. The doors are uniform and identical. Door A rotates about an axis through its left edge, and door B rotates about an axis through its center. The same force is applied perpendicular to each door at its right edge, and the force remains perpendicular as the door turns. No other force affects the rotation of either door. Starting from rest, door A rotates through a certain angle in 3.00 s. How long does it take door B (also starting from rest) to rotate through the same angle?

Solution: Using the table of moments of inertia found in the textbook, it follows that the moment of inertia of Door A about its
left end is four times as large as the moment of inertia of Door B about its centre:

$$I_A = 4 I_B$$

The torque of the given force on Door A about axis A is twice as large as the torque of the given force on Door B about axis B, because the magnitudes of the forces are the same, they are both perpendicular to the doors, and the lever arm is twice as large for Door A:

$$\tau_A = 2 \tau_B$$

Applying the rotational analogue of Newton's second law to each door,

$$\tau_A = I_A \alpha_A \quad \text{and} \quad \tau_B = I_B \alpha_B$$

it follows that

$$\frac{\tau_A}{\tau_B} = \frac{I_A \alpha_A}{I_B \alpha_B}$$

$$\frac{2 \tau_B}{\tau_B} = \frac{4 I_B \alpha_A}{I_B \alpha_B}$$

$$2 = \frac{4 \alpha_A}{\alpha_B}$$

$$\alpha_B = 2 \alpha_A$$
Because each door begins at rest, it follows that

$$\theta_A = \frac{1}{2} \alpha_A t_A^2$$

and

$$\theta_B = \frac{1}{2} \alpha_B t_B^2$$

Because the two angular displacements are equal, it follows that

$$\frac{1}{2} \alpha_B t_B^2 = \frac{1}{2} \alpha_A t_A^2$$

$$\alpha_B t_B = \alpha_A t_A$$

$$(2 \alpha_A) t_B = \alpha_A t_A$$

$$t_B = \frac{1}{2} t_A$$

$$t_B^2 = \frac{1}{2} (3s)^2$$

$$t_B = 2 \cdot 12 \text{ s}$$