PHYS 1P21/1P91 Introductory Physics I

- www.physics.brocku.ca → Courses → 1P21 or 1P91
- course outline
- PHYS 1P91 (only): - Sakai
  - ioLab portable data acquistion device
  - "ioLab Online" software

  more details in your 1st labs, next week

- **Problem-solving strategy**
  1. read and re-read; identify what is known/unknown
  2. make a drawing
  3. use the laws of Physics to establish formal relationships, $\vec{F} = m\vec{a}$
  4. perform algebraic manipulations to get the final result
  5. substitute numerical values; watch significant figures
  6. does it make sense?

  Marking: 1-4 worth 80-85%, 5-6 worth 15-20%.

- treat units as algebraic quantities

Ex. $1\,\text{mi} = 1.61\,\text{km} \Rightarrow \frac{1.61\,\text{km}}{1\,\text{mi}} = 1.$

$\Rightarrow$ = "therefore"

$$1\,\frac{\text{mi}}{\text{gal}} = \frac{1\,\cancel{\text{mi}}}{1\,\text{gal}} \times \frac{1.61\,\text{km}}{1\,\cancel{\text{mi}}} = \frac{1.61\,\text{km}}{1\,\text{gal}} = 1.61\,\frac{\text{km}}{\text{gal}}$$

! Units are carried along with numbers in calculations

Ex. that's insane!

distance $= 2.6\times10^3\,\text{cm}$

fuel amt. $= 1.3\,\ell$

fuel economy $= \frac{\text{distance}}{\text{amt. of fuel}} = \frac{2.6\times10^3\,\text{cm}}{1.3\,\ell}$ (10^{-2})

$$= \frac{2.6\times10^3\times10^{-2}\,\text{m}}{1.3\,\ell}$$

$1\,\text{cm} = 0.01\,\text{m} = 10^{-2}\,\text{m}$

$1\,\text{m} = 0.001\,\text{km} = 10^{-3}\,\text{km}$

$1\,\text{cm} = 10^{-2}\,\text{m} = 10^{-2}\cdot10^{-3}\,\text{km} = 10^{-5}\,\text{km}$

$$= \frac{2.6\times10^3\times10^{-2}\times10^{-3}\,\text{km}}{1.3\,\ell}$$

$$= 2.0\times10^{-2}\,\frac{\text{km}}{\ell} = 0.020\,\frac{\text{km}}{\ell}$$

typical car: $10\text{-}15\,\frac{\text{km}}{\ell}$

$\Rightarrow$ this answer fails the sanity check... for a car. OK for space shuttle.

Ex. dimensional analysis $\quad v = v_0 + at \quad (*)$

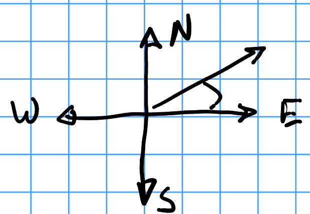
$$[v] = [v_0] = \frac{m}{s} \qquad [a] = \frac{m}{s^2} \qquad [t] = s$$

$$\Rightarrow \frac{m}{s} = \frac{m}{s} + \frac{m}{s^{\cancel{2}}} \cdot \cancel{s} = \frac{m}{s} + \frac{m}{s} \quad \checkmark$$

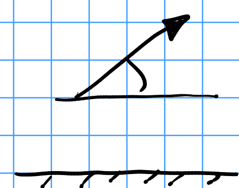
$\Rightarrow$ Eq. $(*)$ is dimensionally correct

Vectors

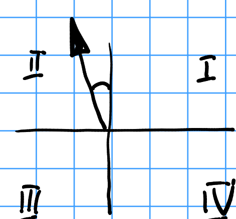
- there are many ways to specify a direction



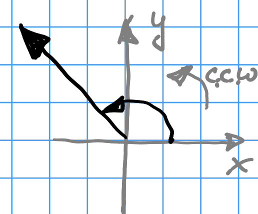
"25° North of East"



"45° up from horizontal"



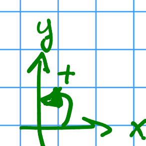
"30° in the second quadrant"



"$\theta = 135°$"

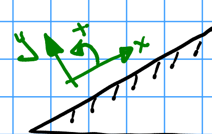
"$\theta = 135°$" means $135°$ c.c.w from the +ve x-axis

In the corner of your work, always indicate the standard angle convention, to remind yourself (and graders)



Often it is convenient to tilt the reference frame

e.g. on an inclined plane:



Ex. displacement is a vector

$\underbrace{\text{displacement}}_{\text{a vector}} = \underbrace{\text{distance}}_{\text{a scalar}}$ traveled in a particular direction

$\vec{v}$ $\quad$ $\vec{a}$ $\quad$ $\vec{F}$

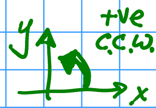
also: velocity, acceleration, force...
- i.e. whenever a direction needs to be specified

Notation:

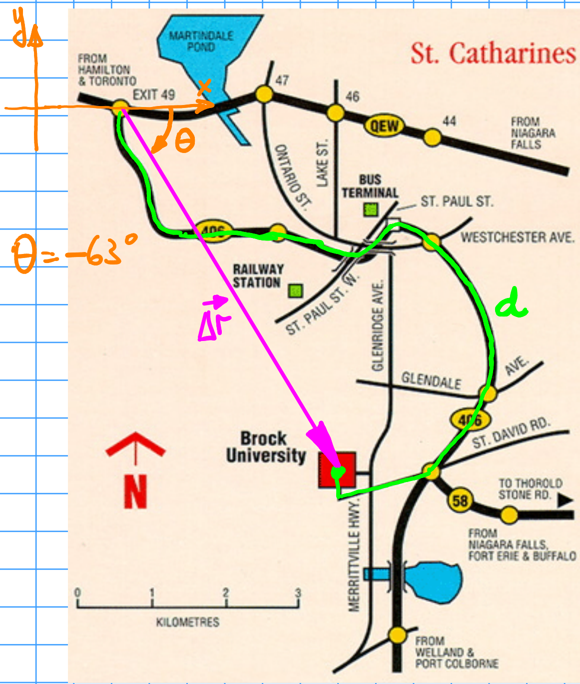
r = scalar

~~bold **r** = vectors in books~~

$\vec{r}$ = vector



Ex. vectors and scalars on a map



average speed $= \dfrac{d}{\Delta t}$ a scalar

$$= \frac{11\,\text{km}}{9\,\text{min}} = \frac{11\cdot 10^3\,\text{m}}{9\,\cancel{\text{min}}} \times \frac{1\,\cancel{\text{min}}}{60\,\text{s}} = \ldots \rightarrow \text{m/s}$$

$$= \frac{11\,\text{km}}{9\,\cancel{\text{min}}} \times \frac{60\,\cancel{\text{min}}}{1\,\text{hr}} \simeq 73\,\text{km/hr}$$

average velocity $= \dfrac{\Delta\vec{r}}{\Delta t}$ a vector

$$= \frac{6\,\text{km}\angle -63^\circ}{9\,\cancel{\text{min}}} \times \frac{60\,\cancel{\text{min}}}{1\,\text{hr}}$$

$$= \frac{6\times 60\,\text{km}\ \angle -63^\circ}{9\,\text{hr}} \simeq 40\,\frac{\text{km}}{\text{hr}}\angle -63^\circ$$

or $40\,\text{km/hr}$, 63° S of E

Ex. the sum of two vectors gives no displacement at all

$\underbrace{\text{displacement}}_{\text{vector}}$ = $\underbrace{\text{distance traveled}}_{\text{scalar}}$ in a particular direction

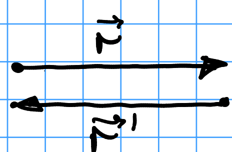
$\vec{r}$

r = scalar

bold **r** = ~~vector in books~~

$\vec{r}$ = vector

$\vec{r} + \vec{r}\,' = \phi$



say "$\vec{r}\,'$" is the opposite of $\vec{r}$", or $\vec{r}\,' = -\vec{r}$

$$\Rightarrow \quad \vec{r} + \vec{r}\,' = \vec{r} + (-\vec{r}) = \vec{r} - \vec{r} = \phi$$

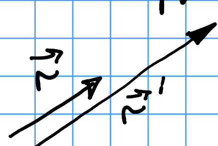
Another point of view:

$$\vec{r} = r\angle\theta \quad \Leftrightarrow \quad \vec{r}\,' = -\vec{r} = r\angle\theta + \pi, \text{ rad}$$
$$r\angle\theta + 180^\circ$$

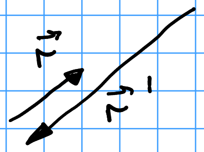
$1\,\text{rad} = \dfrac{180^\circ}{\pi}$

$2\pi\,\text{rad} = 360^\circ$

Ex different magnitudes, same direction



$$\left.\begin{array}{l} r' = 2.1 r \\ \vec{r}\,' \parallel \vec{r} \end{array}\right\} \quad \vec{r}\,' = 2.1\,\vec{r}$$



$$\left.\begin{array}{l} r' = 2.1 r \\ \vec{r}\,' \uparrow\downarrow \vec{r} \end{array}\right\} \vec{r}\,' = -2.1\vec{r}$$

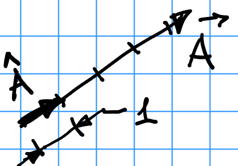
The above are examples of vector subtraction, and of vector multiplication by a scalar: $\vec{r} \equiv r\angle\theta \rightarrow c\vec{r} = \overset{\text{a scalar}}{c}r\angle\theta$ (the same)

i.e. the direction is the same, magnitude changes.

These rules define objects called vectors

Ex. displacements, velocities, forces, etc.

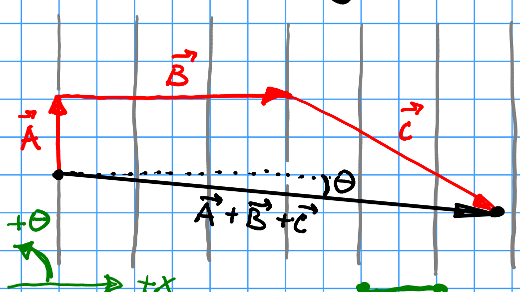
- unit vectors $\underbrace{\forall}_{\text{for All}} \vec{A}$, $\underbrace{\exists}_{\text{there is}}$ $\hat{A}$ s.t. $\vec{A} = \underbrace{A}_{\text{scalar}}\hat{A}$, where $A = |\vec{A}|$ and $|\hat{A}| = 1$, $\hat{A} \uparrow\uparrow \vec{A}$

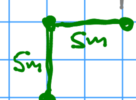


in a sense $\hat{A}$ represents a pure direction of $\vec{A}$

A —— // —— length of $\vec{A}$

- motion along $\underline{\hat{x}}$ is independent of motion along $\underline{\hat{y}}$

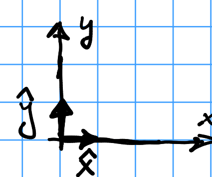




$A = 5.00$ m $\quad \theta_A = +90.0°$

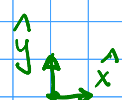
$B = 15.0$ m $\quad \theta_B = 0°$

$C = 18.0$ m $\quad \theta_C = -35.0°$

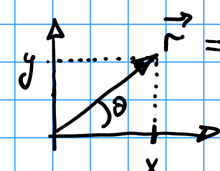


reading

solving

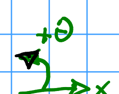


- $\vec{A}$: $A_x = 0$, $A_y = +5.0$ m
- $\vec{B}$: $B_x = +15.0$ m , $B_y = 0$
- $\vec{C}$: $C_x = C\cos\theta = 18.0 \cdot \cos(-35°) = +14.7447\ldots$ m
  $C_y = C\sin\theta = 18.0 \cdot \sin(-35°) = -10.3243\ldots$ m



$\vec{r} = \underbrace{(x, y)}_{\text{coordinates}}$

$$\boxed{\begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \end{array}}$$



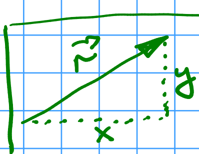
- net displacement , $\vec{A}+\vec{B}+\vec{C}$

along x: $A_x + B_x + C_x = 0 + 15.0 + 14.7447\ldots = 29.7447\ldots$ m

y: $A_y + B_y + C_y = 5.0 + 0 + (-10.3243\ldots) = -5.3243\ldots$ m

or magnitude: $|\vec{A}+\vec{B}+\vec{C}| = \sqrt{(A_x+B_x+C_x)^2 + (A_y+B_y+C_y)^2}$

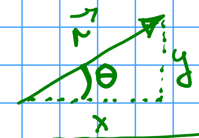
EFTS $= \ldots = 30.2175\ldots$ m $\simeq 30.2$ m



$|\vec{r}|^2 = x^2 + y^2$

direction: $\tan\theta = \frac{(\dots)_y}{(\dots)_x} = \frac{-5.3243\dots \cancel{m}}{29.7447\dots \cancel{m}}$

$\theta = \arctan(\dots) = -10.148\dots^\circ \simeq -10.1^\circ$



$\boxed{\tan\theta = \frac{y}{x}}$

Note: rounding off too early:

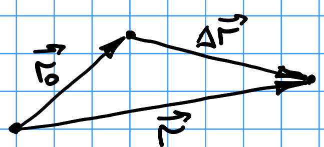
$C_x = 14.7447\dots m \simeq 14.7m \qquad C_y \simeq -10.3m$

Net: $\emptyset m + 15m + 14.7m = 29.7m$

$5.00m + \emptyset m - 10.3m \simeq -5.3m$ lost 1 sig. fig!

⇒ carry extra digit(s), round off only at the end.

- Time dependence: velocity and acceleration



$\Delta\vec{r} \equiv \vec{r} - \vec{r}_0$ = total displacement vector

$\Delta t \equiv t - t_0$ = total elapsed time

Average velocity: $\boxed{\overline{\vec{v}} \equiv \frac{\Delta\vec{r}}{\Delta t}}$ (a vector)

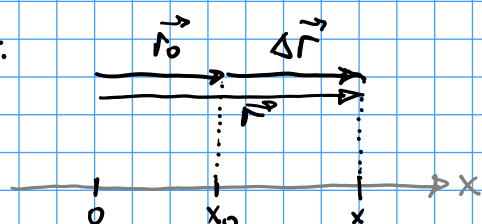
Δ = Delta = "a change in"

$\equiv$ = "defined as"

overbar = "average"

$\overline{A}$ = average of A

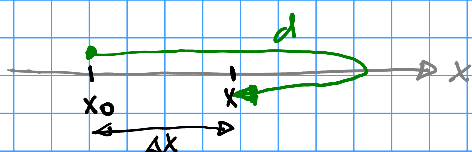
In 1D:



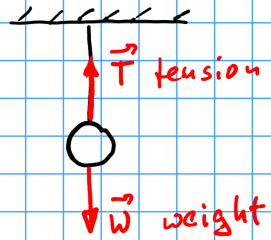
$\Delta x = x - x_0$

c.f. average speed $= \overline{v} \equiv \frac{d}{\Delta t}$ (a scalar)

In general, distance traveled $d \neq \Delta x \quad (d \neq |\Delta\vec{r}| = \Delta r)$



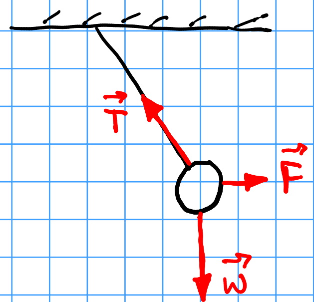
- forces are vectors, too: satisfy the same algebra rules as displacements, but:
  - units of force are N (Newtons)
  - drawing to scale means now: ~~10m = 1cm~~ 100N = 1cm



$\vec{T}$ = tension = force <u>on</u> the ball <u>by</u> the string

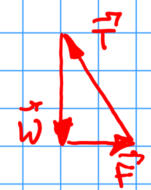
$\vec{W}$ = weight = force <u>on</u> the ball $\underbrace{\text{by the Earth}}_{\text{an example of non-contact force}}$

Ex apply an additional force, $\vec{F}$

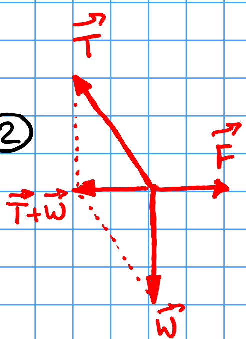


later will learn that eq-m is when (equilibrium)

$$\vec{W} + \vec{T} + \vec{F} = \emptyset$$

① 

← tail-to-tip adding the three forces must add up to zero, i.e. form a closed triangle.

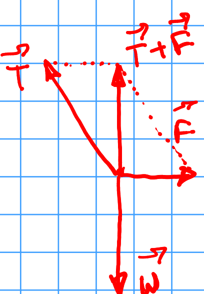
② 

Another way:

$$\vec{W} + \vec{T} + \vec{F} = (\vec{W} + \vec{T}) + \vec{F} = \emptyset$$

$$\vec{W} + \vec{T} = -\vec{F}$$

i.e. (vector) $\vec{W} + \vec{T}$ is the opposite of (vector) $\vec{F}$

③ 

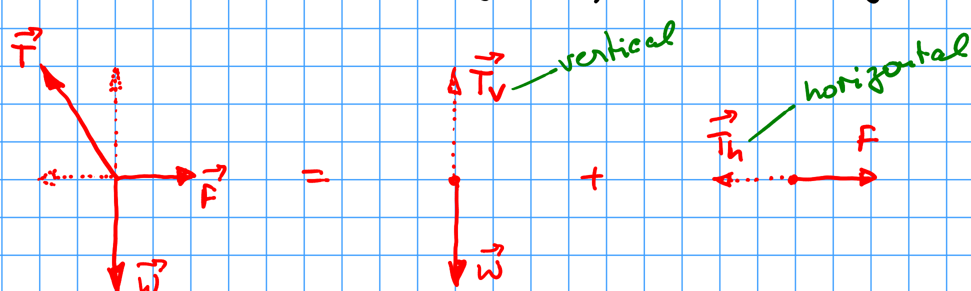
Another way: $(\vec{T} + \vec{F}) + \vec{W} = \emptyset$

$$\vec{T} + \vec{F} = -\vec{W}$$

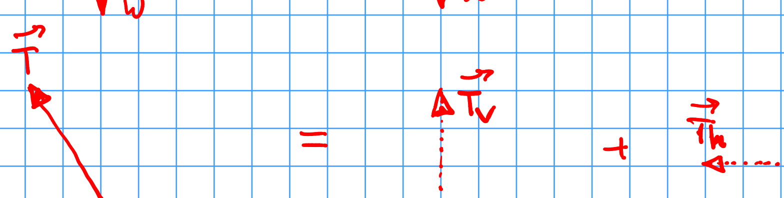
$\Rightarrow$ can add vectors in any order !!

Above, $\vec{T}$ contributes along
- vertical, counteracting $\vec{W}$
- horizontal, counteracting $\vec{F}$

Graphically:



or:



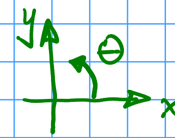
Algebraically: $\vec{T}+\vec{w}+\vec{F} = (\vec{T}_h+\vec{T}_v)+\vec{w}+\vec{F}$

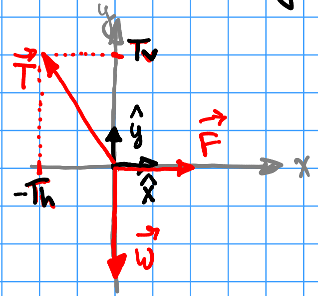
$= (\vec{T}_h+\vec{F})+(\vec{T}_v+\vec{w}) = \emptyset$

vertical : $\vec{T}_v+\vec{w}=\emptyset \Rightarrow \vec{T}_v = -\vec{w}$

horizontal : $\vec{T}_h+\vec{F}=\emptyset \Rightarrow \vec{T}_h = -\vec{F}$

④ Another way, using a reference frame :





$|\hat{x}| = |\hat{y}| = 1$

scalars

$\vec{T} = \vec{T}_h + \vec{T}_v = \underbrace{-T_h\hat{x}}_{\text{vector}} + \underbrace{T_v\hat{y}}_{\text{vecto}}$

Note: $T_h = |\vec{T}_h|$, $T_h > \emptyset$

$(-T_h)$ is -ve

(T_v) is +ve

T_h, T_v are the scalar components along $\hat{x}, \hat{y}$

or (T_x, T_y)

In this way, a single vector equation becomes two scalar eq-ns, one for each of the directions defined by $\hat{x}, \hat{y}$

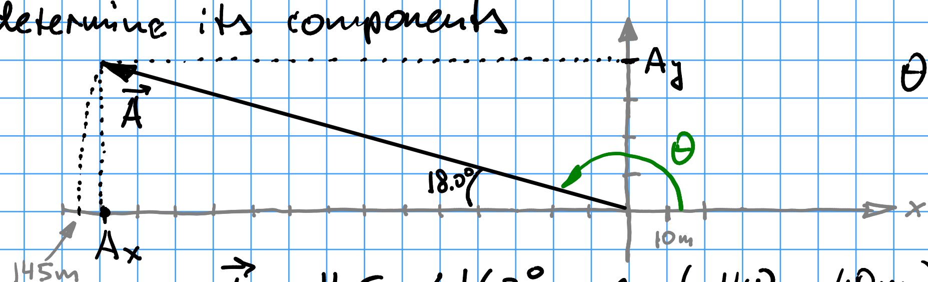
$$\vec{T}+\vec{w}+\vec{F}=\emptyset \longrightarrow \begin{cases} -T_x + F = \emptyset & \text{along } \hat{x} \\ +T_y - W = \emptyset & \text{along } \hat{y} \end{cases}$$

! an algebraic way of adding vectors.

Ex vector of magnitude 145 m, 18.0° "above the -ve x-axis"

determine its components

①



$\theta = 180° - 18.0° = 162°$

$\vec{A} = 145\text{m}\angle 162° \approx (-140\text{m}, 40\text{m})$ graphical, approximate

② algebraically (a refinement)

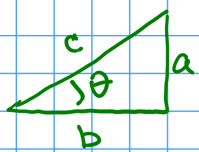
$A_x = 145\text{m} \cdot \cos(162°) = -138\text{m}$

$A_y = 145\text{m} \cdot \sin(162°) = 44.8\text{m}$

$$\boxed{\begin{aligned} A_x &= A\cos\theta \\ A_y &= A\sin\theta \end{aligned}}$$

$\Rightarrow \vec{A} = \underbrace{145\text{m}\angle 162°}_{\text{polar form}} = \underbrace{-138\text{m}\,\hat{x} + 44.8\text{m}\,\hat{y}}_{\text{Cartesian form}}$

A math detour



$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$a^2 + b^2 = c^2$

$\frac{a^2}{c^2} + \frac{b^2}{c^2} = 1$

$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$

$$\boxed{\begin{aligned} &\cos^2\theta + \sin^2\theta = 1 \\ &\cos^2\theta - \sin^2\theta = \cos 2\theta \\ &2\cos\theta\sin\theta = \sin 2\theta \end{aligned}}$$

memorize!

$A_x = A\cos\theta$

$A_y = A\sin\theta$

$A^2 = A_x^2 + A_y^2$

$\tan\theta = \frac{A_y}{A_x}$

Ex two runners

$$\boxed{\bar{v} = \frac{d}{\Delta t}} \Rightarrow \Delta t = \frac{d}{\bar{v}}$$

d ← distance traveled

$\bar{v}$ ← average speed

time difference $= \Delta t_2 - \Delta t_1 = \frac{d}{\bar{v}_2} - \frac{d}{\bar{v}_1} = d\left(\frac{1}{\bar{v}_2} - \frac{1}{\bar{v}_1}\right)$

$= 10\,\cancel{\text{km}}^{10^3} \left(\frac{1}{4.27\,\frac{\cancel{m}}{s}}\,s - \frac{1}{4.38\,\frac{\cancel{m}}{s}}\,s\right) = 58.81\ldots\ \text{s} \simeq 58.8\,\text{s}$

$\vec{v}$ = velocity vector

$\bar{v}$ = average speed

$\vec{\bar{v}}$ = average velocity (a vector)

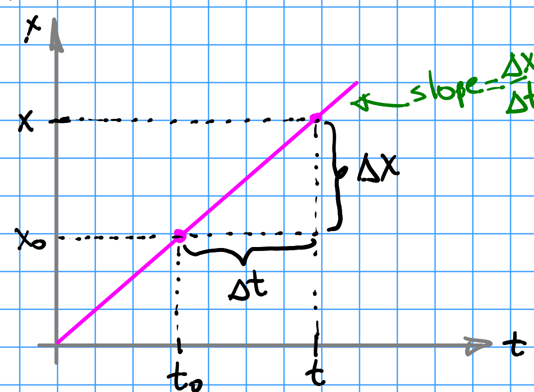
Graphically, v = slope

$v = \frac{\Delta x}{\Delta t} \Rightarrow \Delta x = v\Delta t$

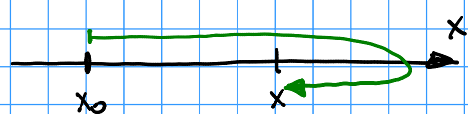
and since often $x_0 = 0$ @ $t = 0$

$\Rightarrow \quad x = vt$

v ← slope



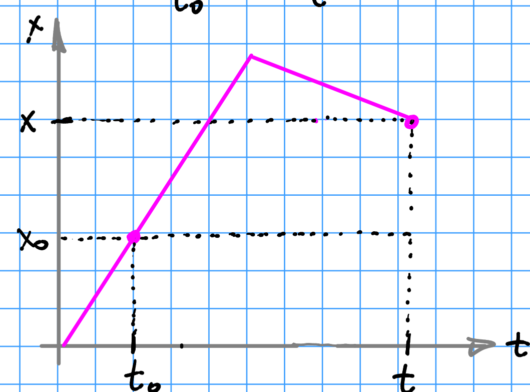
Ex



piecewise-uniform motion

+ve slope : toward larger x

−ve slope : —— " —— smaller x



- instantaneous speed
- instantaneous velocity

$$\boxed{\begin{aligned} v &= \lim_{\Delta t \to 0} \frac{d}{\Delta t} \\ \vec{v} &= \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} \end{aligned}}$$

lim = "limit"

- acceleration = rate of change of velocity

Ex 0 to 100 km/h in 7.00 s

$v_0 = 0$ (starting from standing still)

$\bar{a} = \frac{\text{change in speed}}{\Delta t} = \frac{\Delta v}{\Delta t} = \frac{v - \cancel{v_0}}{\Delta t} = \frac{100\,\text{km}/\cancel{h}}{7.00\,\text{s}} \times \frac{1\cancel{h}}{3600\,\text{s}}$

$$a = \frac{100 \times 10^3\ m}{7.00 \times 3600\ s^2} \simeq 3.97\ \frac{m}{s^2}$$

$$\boxed{\vec{\bar{a}} \equiv \frac{\Delta \vec{v}}{\Delta t}}$$

For vectors:

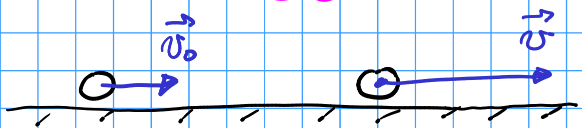
$$\bar{\vec{a}} = \frac{\text{change in velocity}}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v} - \vec{v}_0}{\Delta t}$$

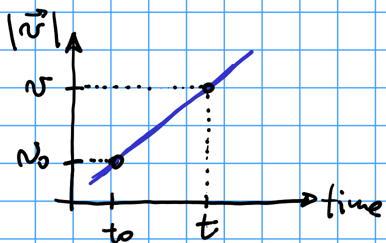
$$[a] = \frac{m}{s^2}$$

Note: $v = v_0$ does <u>not</u> mean $\vec{v} = \vec{v}_0$ and, therefore $\vec{a} = \emptyset$
(e.g. direction may have changed, even though $|\vec{v}| = |\vec{v}_0|$)

Instantaneous acceleration $\quad \vec{a} = \lim_{\Delta t \to \emptyset} \frac{\Delta \vec{v}}{\Delta t}$

Ex.





average $\bar{a} = \frac{v - v_0}{t - t_0}$ = slope of $v(t)$ = const.

re-write as: $\quad v - v_0 = a(t - t_0)$

$$v = v_0 + a(t - t_0)$$

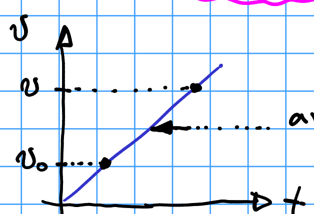
Usually, set $t_0 = \emptyset$

$$\boxed{v = v_0 + at}$$

Note: acceleration vs. deceleration is determined by the <u>relative</u> directions of $\vec{v}$ and $\vec{a}$, <u>not</u> the direction of $\vec{a}$ in some reference frame (+ve or −ve)

Ex spaceship turnaround (qualitatively first)
the same $\vec{a}$ is at first a <u>deceleration</u>, and then an <u>acceleration</u> (in the −ve direction)

• average speed and distance when $a = \text{const}$



average $\bar{v} = \frac{1}{2}(v + v_0)$, but $v = v_0 + at$
i.e. half-way

$$\Rightarrow \bar{v} = \frac{1}{2}\left[(v_0 + at) + v_0\right] = \frac{1}{2}\left[2v_0 + at\right]$$

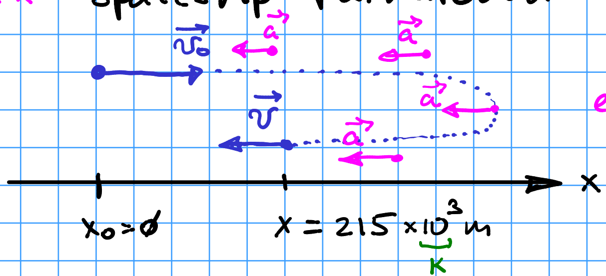
On the other hand $\bar{v} = \frac{d}{\Delta t} = \frac{x - x_0}{t - t_0} = v_0 + \frac{1}{2}at$

As usual, set $x_0 = \emptyset$ at $t_0 = \emptyset$ for convenience

$$\Rightarrow \quad v + \frac{1}{2}at = \frac{x}{t} \quad \Rightarrow \quad \boxed{x = v_0 t + \frac{1}{2}at^2}$$

Full result: $x = x_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2$

Ex spaceship turnaround



$\vec{a}$ everywhere!

Here: $|\vec{v}_0| = 3250 \frac{m}{s}$, $|\vec{a}| = 10.0 \, m/s^2$

$$v = v_0 + at$$

$$t = \frac{v - v_0}{a}$$

$$\Rightarrow \quad x = \bar{v}t = \frac{1}{2}(v + v_0)\frac{v - v_0}{a}$$

recall: $(A+B)(A-B) = A^2 - B^2$

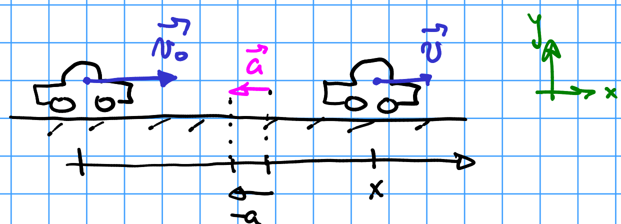
$$\Rightarrow \quad x = \frac{1}{2}(v^2 - v_0^2)\frac{1}{a} \quad \Rightarrow \quad \boxed{v^2 = v_0^2 + 2ax}$$

$$v^2 = v_0^2 + 2ax = \left(3250 \frac{m}{s}\right)^2 - 2\left(-10.0 \frac{m}{s^2}\right) \times 215 \times 10^3 \, m$$

$$= 6,262,500 \frac{m^2}{s^2}$$

$$v = \sqrt{v^2} = \pm 2,502.49 \ldots \frac{m}{s} \simeq \pm 2.50 \frac{km}{s}$$

Ex



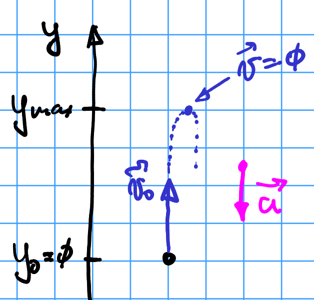
x: $v^2 = v_0^2 + 2(-a)x$

Stop: $v = \emptyset$

$$\Rightarrow \quad \emptyset = v_0^2 - 2ax \quad \Rightarrow \quad x = \frac{v_0^2}{2a}$$

Note: $\frac{x}{x'} = \frac{v_0^2/2a}{(v_0')^2/2a} = \left(\frac{v_0}{v_0'}\right)^2 \Rightarrow$ double the speed $\leftrightarrow$ 4x stopping distance.

Ex free fall $\boxed{\vec{a} = g, \text{ down} = g\angle -\frac{\pi}{2}}$, $g = 9.80 \frac{m}{s^2}$



① $y_{max} = \frac{v_0^2}{2g}$, as before

② time to get to the top

again: $v^2 = v_0^2 + 2ax$

here: $v^2 = \emptyset = v_0^2 + 2(-g)y$

$$\boxed{v = v_0 + at}$$

here: $v = \emptyset$ at the top, $a = -g \Rightarrow \emptyset = v_0 - gt \Rightarrow t_{top} = \frac{v_0}{g}$

③ time to fall back down (to start)

$$\boxed{y = v_0 t + \frac{1}{2} a t^2}$$

"return": $y = y_0 = \emptyset = \left(v_0 + \frac{at}{2}\right) t$ → $v_0 + \frac{at}{2} = \emptyset$ or $t = \emptyset$

for $a = -g$: $v_0 - \frac{gt}{2} = \emptyset$

$t_{return} = \frac{2v_0}{g}$ or $t = \emptyset$? reproduces the initial condition $y = y_0$ @ $t = \emptyset$

④ v upon return

$$\boxed{v^2 = v_0^2 + 2ay}, \quad a = -g$$

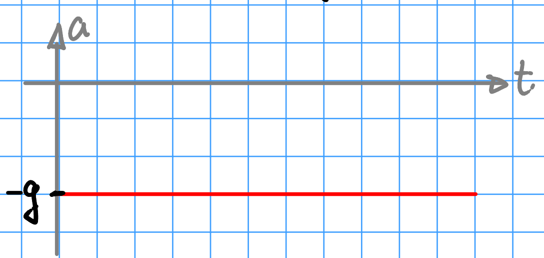
"return": $y = y_0 = \emptyset$

$\Rightarrow v^2 = v_0^2 + 2(-g)\emptyset = v_0^2 \Rightarrow v = \pm v_0$

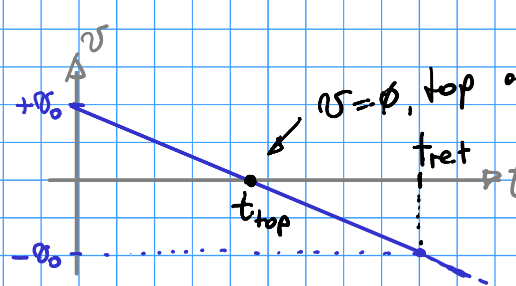
again: $\begin{cases} v = +v_0 & \text{reproduces the initial condition} \\ v = -v_0 & \text{upon return} \end{cases}$ ← select on physical grounds

another way: $t_{ret} = \frac{2v_0}{g} \Rightarrow v = v_0 + at = v_0 + (-g)\cdot\frac{2v_0}{g}$

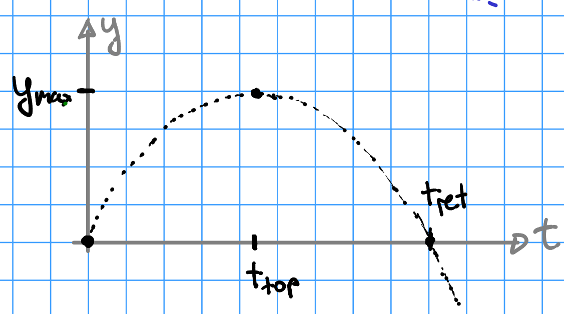
$\Rightarrow v = v_0 - 2v_0 = -v_0$



$a = -g = \text{const}$



$v = v_0 + at = v_0 - gt$



$y = v_0 t + \frac{1}{2} a t^2 = v_0 t - \frac{1}{2} g t^2$

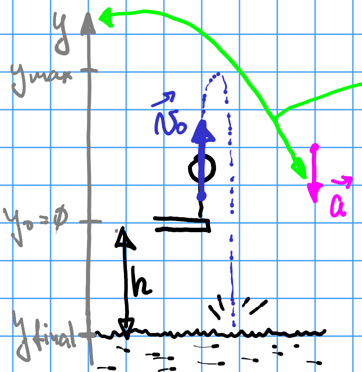
Note: $y = -\frac{g}{2}\left(-\underbrace{\frac{2v_0}{g}}_{= t_{top}} t + t^2\right) = -\frac{g}{2}\left(t^2 - 2t_{top}t \underbrace{+ t_{top}^2 - t_{top}^2}_{\text{added now}}\right)$

looking like $A^2 - 2AB + B^2 = (A-B)^2$

$$\Rightarrow \quad y = \underset{\substack{\text{pointing}\\\text{downward}}}{-}\frac{g}{2}\underbrace{\left(t - t_{top}\right)^2}_{\substack{\text{peak}\\@\, t = t_{top}}} + \underbrace{\frac{g t_{top}^2}{2}}_{\substack{\text{+ve const,}\\\text{a shift up}}}$$

parabola

Ex a diver



$v_0 = 1.80 \frac{m}{s}$

$a = -g = -9.80 \frac{m}{s^2}$

$h = -3.0 m$
(@ splashdown)

Time is not known

$\Rightarrow \boxed{v^2 = v_0^2 + 2ay}$

time = ?

① at $y = y_{max}$, $v = \emptyset \Rightarrow \emptyset = v_0^2 + 2(-g)\, y_{max}$

$\Rightarrow y_{max} = \frac{v_0^2}{2g} = \frac{(1.8 \, m/s)^2}{2 \cdot 9.80 \, m/s^2} \simeq 0.16 \, m$ above the diving board

② hit water when $y = -h = -3.0 m$

$\Rightarrow v^2 = v_0^2 + 2(-g)(-h)$

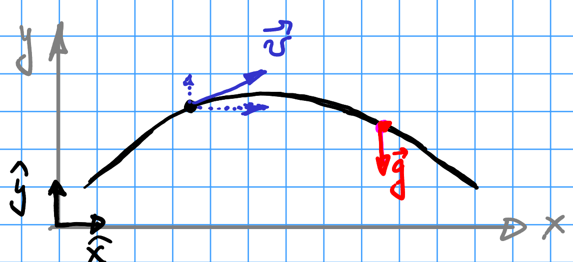
$= \left(1.80 \frac{m}{s}\right)^2 + 2 \cdot 9.80 \frac{m}{s^2} \times 3.0 m \simeq 62.04\ldots \frac{m^2}{s^2}$

$v = \pm 7.876\ldots \frac{m}{s} \simeq \pm 7.9 \frac{m}{s}$

On physical grounds, select $v_{splashdown} = -7.9 \frac{m}{s}$ (i.e. downward)

③ elevation is $y_{max} - (-h) = 3.16 m$ above water

Kinematics in 2D



$\vec{v} = \begin{cases} v_x \\ v_y \end{cases}$ $\qquad \vec{v} = v_x \hat{x} + v_y \hat{y}$

$\vec{a} = \begin{cases} \emptyset \\ -g \end{cases}$ $\qquad \vec{a} = -g\hat{y}$

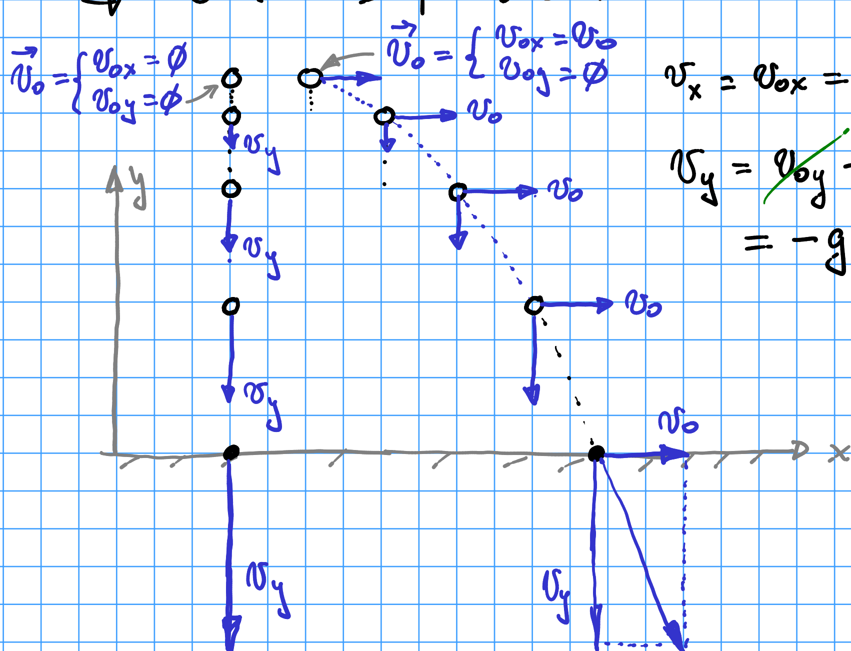
$$\Rightarrow \begin{cases} v_x = v_{0x} + \cancelto{\emptyset}{a_x t} = v_{0x} = \text{const} \\ v_y = v_{0y} + a_y t = v_{0y} - gt \end{cases}$$

- principle of superposition: x-motion is independent of y-motion

	x-motion	y-motion
displacement	x, x_0	y, y_0
velocity	v_x, v_{0x}	v_y, v_{0y}
acceleration	$a_x = \emptyset$	$a_y = -g$
time	t	t
kinematic equations	$v_x = v_{0x} + \cancel{a_x} t$	$v_y = v_{0y} + a_y t$
	$x = \overline{v_x} t$	$y = \overline{v_y} t$
	$x = x_0 + v_{0x} t + \cancel{\frac{1}{2} a_x t^2}$	$y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2 = y_0 + v_{0y} t - \frac{1}{2} g t^2$
	$v_x^2 = v_{0x}^2 + \cancel{2 a_x x}$	$v_y^2 = v_{0y}^2 + 2 a_y y = v_{0y}^2 - 2 g y$
	1D problem	1D problem

2D problem (of projectile motion)

$\Rightarrow$ one 2D problem $\Leftrightarrow$ two 1D problems

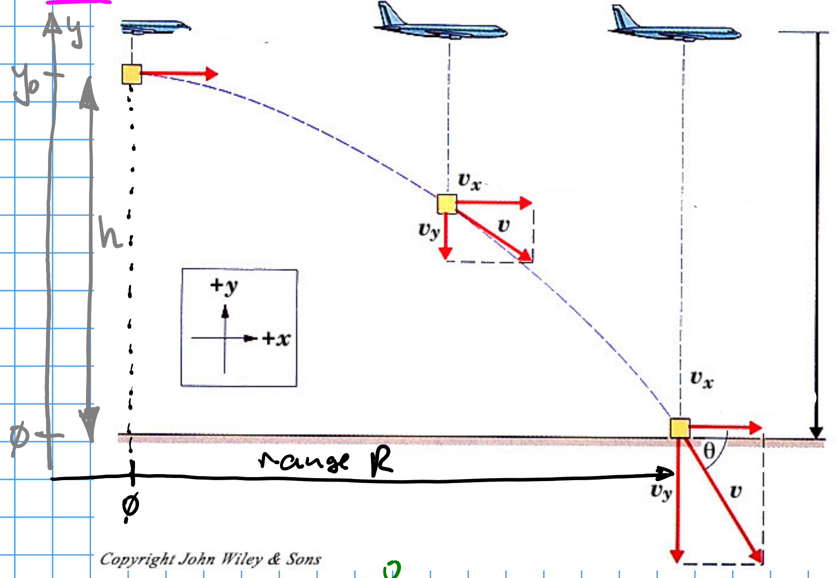


$v_x = v_{0x} = \text{const} = v_0$

$v_y = \cancel{v_{0y}}^{\,\emptyset} - gt \neq \text{const}$
$= -gt$

the two balls hit the ground at the same time, with the same vertical component of the velocity

Ex



delivering a lifeboat

$a = -g$

$g = 9.80 \frac{m}{s^2}$

$\vec{a} = (0, -g)$

at $t = 0$:

$$\begin{cases} v_x = v_{0x} = v_0 \\ v_y = v_{0y} = 0 \end{cases}$$

$\forall t$, $\begin{cases} a_x = 0 \\ a_y = -g \end{cases}$

At $t > t_0 = 0$: (t_0 is the moment of release)

$$\begin{cases} v_x = v_{0x} + \overset{0}{\cancel{a_x}} t = v_{0x} = v_0 = \text{const} \\ v_y = v_{0y} + \underset{-g}{\cancel{a_y}} t = -gt \end{cases}$$

← x-motion is with const. velocity

← y-motion is with const acceleration, $-g\hat{j}$

Q1 : time of flight Q2 : range Q3 : velocity @ impact

① $y = \overset{0}{\cancel{v_{0y}}} t + \frac{1}{2} \underset{-g}{\cancel{a}} t^2 = -\frac{1}{2} g t^2$

At impact : $y = y_0 - h$ or if $y_0 = 0$, $y = -h$

$\Rightarrow -h = -\frac{1}{2} g t^2$ or $t^2 = \frac{2h}{g}$ $\Rightarrow t_{flight} = \sqrt{2h/g}$

Note 1: ignored $-\sqrt{\dots}$ on physical grounds, since the free fall only began @ $t_0 = 0$, something else at $t < t_0 = 0$

Note 2 : could have used $y_0 = h$ and $y = 0$:

$y = y_0 + \overset{0}{\cancel{v_{0y} t}} + \frac{1}{2} \underset{-g}{\cancel{a_y}} t^2 \rightarrow 0 = h + 0 - \frac{1}{2} g t^2 \rightarrow$ same result

② $x = v_{0x} t + \frac{1}{2} \overset{0}{\cancel{a_x}} t^2 = v_0 t$

At impact , $x = R$ $\Rightarrow$ range $R = v_0 t = v_0 \sqrt{\frac{2h}{g}}$

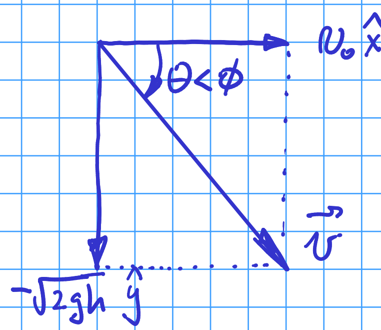
③ components of $\vec{v}$ @ impact

$$\begin{cases} v_x = v_{0x} + \cancel{a_x}^{\,0} t = v_0 \\ v_y = v_{0y} + \underbrace{a_y}_{=-g} t = -gt = -g\sqrt{\frac{2h}{g}} = -\sqrt{g^2\frac{2h}{g}} = -\sqrt{2gh} \end{cases}$$

In polar form:

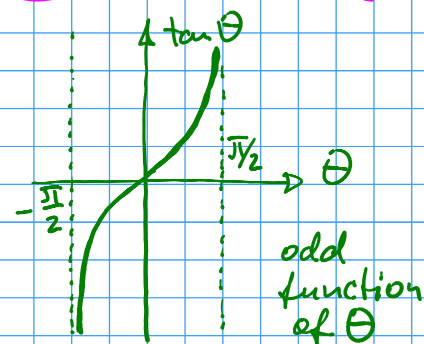
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 + 2gh} \quad \leftarrow \text{magnitude of } \vec{v}$$

$$\theta = \arctan\left(\frac{v_y}{v_x}\right) = \arctan\left(\frac{-\sqrt{2gh}}{v_0}\right) = -\arctan\frac{\sqrt{2gh}}{v_0} \quad \leftarrow \text{direction of } \vec{v}$$



$$\theta = -\arctan\frac{\sqrt{2gh}}{v_0} < 0$$

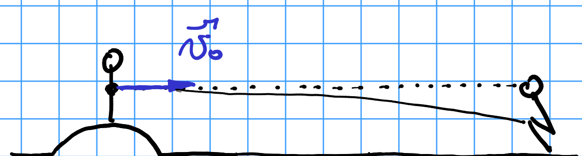
i.e. clockwise from $\hat{x}$



odd : $f(-x) = -f(x), \forall x$

even : $f(-x) = f(x), \forall x$

Ex a pitcher's mound



$v_0 = 40.0\,\frac{m}{s}$

$R = 18.4\,m$

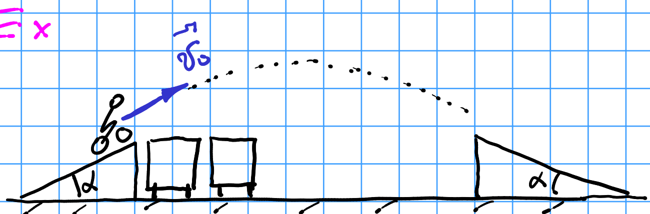
As before:

$$R = v_0\sqrt{\frac{2h}{g}}$$

$$\Rightarrow R^2 = v_0^2\,\frac{2h}{g} \Rightarrow h = \frac{g}{2}\,\frac{R^2}{v_0^2} = \frac{9.80\frac{m}{s^2}\,(18.4\,m)^2}{2\,(40.0\,\frac{m}{s})^2} = 1.0368\ldots\,m \approx 1.04\,m$$

Actual pitcher's mound is only about $\frac{1}{3}$ of that, a partial compensation.

Ex a daredevil jumping over buses



$v_0 = 33.5\,\frac{m}{s}$ $\alpha = 18.0°$

$$x: \quad v_x = v_{0x} + \cancel{a_x}^{\,0} t = v_{0x} = v_0\cos\alpha$$

$$y: \quad v_y = v_{0y} + \underbrace{a_y}_{-g} t = v_{0y} - gt = v_0\sin\alpha - gt$$

x: $x = v_{0x}t + \frac{1}{2}\cancel{a_x}t^2 = v_{0x}t = v_0\cos\alpha\, t$

y: $y = v_{0y}t + \frac{1}{2}a_y t^2 = v_0\sin\alpha\, t - \frac{1}{2}gt^2$

At landing:

① $y = \emptyset$ (landing ramp is the same height)

$$v_0\sin\alpha\, t - \frac{1}{2}gt^2 = \emptyset$$

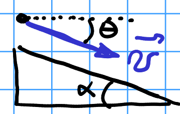
$$t\left[v_0\sin\alpha - \frac{1}{2}gt\right] = \emptyset$$

$t = \emptyset$ (take-off)

$v_0\sin\alpha = \frac{1}{2}gt$

$t = \frac{2v_0}{g}\sin\alpha$ (landing)

② $\vec{v}$ is along the ramp:



$\theta = -\alpha$

$$\tan(-\alpha) = \frac{v_y}{v_x}$$

$\tan = \frac{\sin}{\cos}$

$$-\frac{\sin\alpha}{\cancel{\cos\alpha}} = \frac{v_0\sin\alpha - gt}{v_0\cancel{\cos\alpha}}$$

$$-\sin\alpha = \sin\alpha - \frac{gt}{v_0}$$

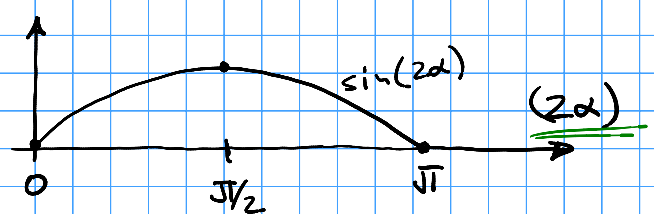
$$+2\sin\alpha = +\frac{gt}{v_0}$$

$$t = \frac{2v_0}{g}\sin\alpha$$

$$x = v_0\cos\alpha\, t = v_0\cos\alpha\,\frac{2v_0}{g}\sin\alpha$$

$$x = \frac{v_0^2}{g}\sin(2\alpha) = \ldots = 67.2\,\text{m}$$

$$\#\text{buses} = \frac{x}{2.74\,\text{m}} = 24.5\ldots = 24 \text{ buses}$$



$\max[\sin(2\alpha)] = 1$ when $(2\alpha) = \frac{\pi}{2}$

or $\alpha = \frac{\pi}{4} = 45°$

⇒ maximum range for ramp @ $\alpha = 45°$

• vary α, max range?

$$R = v_{0x}\,t_{flight} = v_0\cos\alpha\left(\frac{2v_0}{g}\sin\alpha\right) = \frac{v_0^2}{g}(2\sin\alpha\cos\alpha)$$

$$= \frac{v_0^2}{g}\sin(2\alpha) = \max \text{ @ } \alpha = 45°$$

Note: $R = \emptyset$ when $\alpha = \emptyset$ (i.e. never leave ground)

or $2\alpha = 180°$

$\alpha = 90°$ (i.e. straight up)