$\boxed{\text{Dynamics}}$

Aristotle : stop pushing $\Rightarrow$ motion stops

Newton : stop pushing $\Rightarrow$ motion continues with $\vec{v} = \text{const}$

In practice : push to overcome friction, etc.

"push"/"pull" $\Rightarrow$ force, a vector

Newton's laws of motion : a summary of experimental observations, + a point of view

$\boxed{\text{N1L}}$ no force $\Rightarrow$ no change in $\vec{v}$

contained within N1L is the postulate about the existence of inertial frames of reference

- N1L only works in the i.f.of r.
- i.f.of r. are the ones where N1L holds

$\boxed{\text{N2L}}$ $\vec{a} = \dfrac{\vec{F}}{m}$ or $\sum\limits_i \vec{F}_i = m\vec{a}$

sum of $\vec{F}_1 + \vec{F}_2 + \ldots$

net applied force determines $\vec{a} \propto \vec{F}$

Experiment : same $\vec{F}$
- large m $\Rightarrow$ $|\vec{a}|$ is small
- small m $\Rightarrow$ $|\vec{a}|$ is large

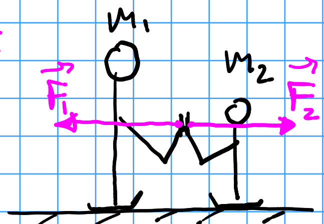
contained within N2L is the postulate that the objects have mass, a measure of inertia of the object

inertia $\equiv$ how an object responds to an applied force

$\boxed{\text{N3L}}$ all forces are due to interactions

$$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A} \qquad \left(\vec{F}_B = -\vec{F}_A \right)$$

Ex father and daughter on skates



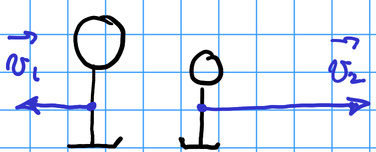
N3L: $\vec{F}_{1 on 2} = -\vec{F}_{2 on 1}$ or $\vec{F}_2 = -\vec{F}_1$

N2L: $\vec{a}_1 = \dfrac{\vec{F}_1}{m_1}$ $\quad \vec{a}_2 = \dfrac{\vec{F}_2}{m_2} = -\dfrac{\vec{F}_1}{m_2}$

$\vec{a}_1 \uparrow\downarrow \vec{a}_2$

$m_1 > m_2 \Rightarrow |\vec{a}_1| < |\vec{a}_2|$ or $a_1 < a_2$

Kinematics (during contact):



$$\begin{cases} \vec{V}_1 = \cancel{\vec{V}_{01}} + \vec{a}_1 t = \vec{a}_1 \cdot t_{contact} \\ \vec{V}_2 = \cancel{\vec{V}_{02}} + \vec{a}_2 t = \vec{a}_2 \cdot t_{contact} \end{cases}$$

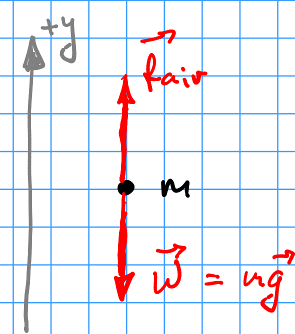
$a_1 < a_2 \Rightarrow v_1 < v_2$ $\qquad v_1 = |\vec{v}_1|$ etc.

Note: for $t > t_{contact}$, $a_1 = a_2 = \emptyset \Rightarrow \begin{cases} v_1 = const \\ v_2 = const \end{cases}$

Ex. a skydiver

$v = const \Rightarrow a = \emptyset \Rightarrow$ net $F = \emptyset \Rightarrow$ Eq.-m.

$\sum_i \vec{F}_i = \emptyset$



Identify: gravity $= -mg$

air resistance $= +f_{air}$

$\Rightarrow +f_{air} - mg \underset{\text{N2L}}{=} ma \underset{\text{eq-m}}{=} \emptyset \Rightarrow f_{air} = mg$

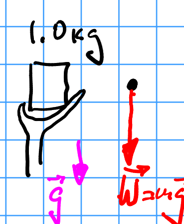
Note: $\vec{W} = m\vec{g}$ (or $W = |\vec{W}| = |m\vec{g}| = mg$) = weight

weight $\neq$ mass

weight = force due to gravity $\qquad -g = -9.80\,\frac{m}{s^2}$

mass = measure of inertia $\qquad -mg = -9.80\,N$

$1N = 1\,\frac{kg\,m}{s^2}$



	fundamental	derived
SI	$[m] = kg$ $[t] = s$ $[l] = m$	$[t], [l] \rightarrow [a] = \frac{m}{s^2}$; $[m], [a] \rightarrow [F] = N = \frac{kg\,m}{s^2}$
BI	$[F] = lb$ $[t] = s$ $[l] = ft$	$[t], [l] \rightarrow [a] = \frac{ft}{s^2}$; $[F], [a] \rightarrow [m] = 1\,\frac{lb\,s^2}{ft} = 1$ slug

Supermarket "conversion":

1 lb "=" 0.45 kg ??

force   mass

Ex "weight-loss" program

200 lb to ∅ lb in 2 mins

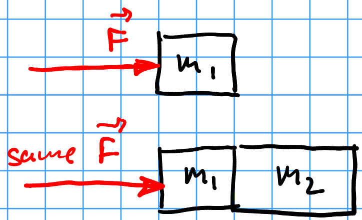
SpaceX launch

vs. 200 lb to 199 lb in 2 weeks

exercise.

(g), (m) $\Rightarrow$ $w = mg$

Ex a problem with no numbers



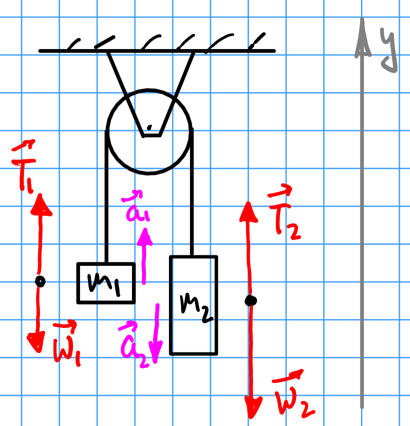
$\vec{a}$ N2L: $\vec{a} = \dfrac{\vec{F}}{m_1}$

same $\vec{F}$: $\frac{1}{3}\vec{a}$: $\frac{1}{3}\vec{a} = \dfrac{\vec{F}}{m_1+m_2}$

$\vec{a} = \dfrac{1}{m_1}\vec{F} = \dfrac{3}{m_1+m_2}\vec{F}$

$$\Rightarrow \frac{1}{m_1} = \frac{3}{m_1+m_2} \Rightarrow \frac{m_1+m_2}{m_1} = 3 \Rightarrow 1 + \frac{m_2}{m_1} = 3 \Rightarrow \frac{m_2}{m_1} = 2$$

Ex Atwood machine



$m_1 = 0.55$ kg

$m_2 = 0.80$ kg

$|\vec{a}_1| = |\vec{a}_2| = a$

(or $\vec{a}_1 = -\vec{a}_2$)

$|\vec{T}_1| = |\vec{T}_2| = T$

$$\boxed{\sum_i \vec{F}_i = m\vec{a}}$$

For m_1: $+T_1 - m_1 g = m_1 a$

m_2: $+T_2 - m_2 g = -m_2 a$

$\Rightarrow$ 2 eq-ns, 2 unknowns

$$\Rightarrow (T - m_1 g) - (T - m_2 g) = m_1 a - (-m_2 a)$$

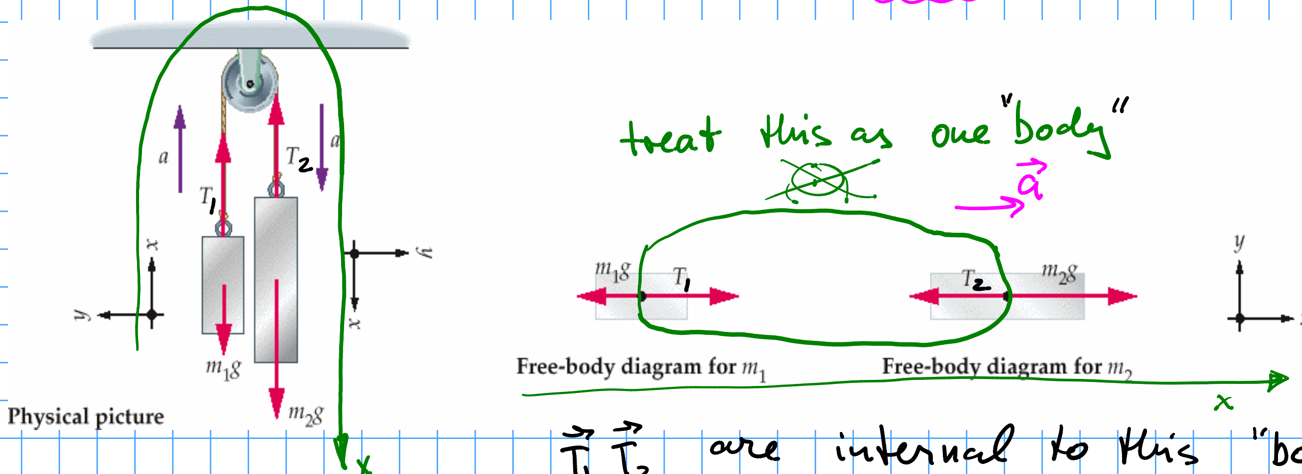
$$(m_2 - m_1) g = (m_2 + m_1) a$$

$$a = \frac{m_2 - m_1}{m_2 + m_1} g$$

$$a = \frac{0.80\cancel{kg} - 0.55\cancel{kg}}{0.80\cancel{kg} + 0.55\cancel{kg}} \, 9.80\frac{m}{s^2} = 1.8148\ldots \frac{m}{s^2} \simeq 1.8\frac{m}{s^2}$$

$$T = m_1 a + m_1 g = m_1(a+g) = m_1\left[\frac{m_2-m_1}{m_2+m_1}g + g\right] = m_1\left[\frac{m_2-m_1}{m_2+m_1} + 1\right]g$$

$$= m_1 \frac{(m_2-m_1) + (m_2+m_1)}{m_2+m_1} g = \frac{2m_1m_2}{m_1+m_2} g = 6.388\ldots \underbrace{\frac{kg\,m}{s^2}}_{=N} \simeq 6.4N$$

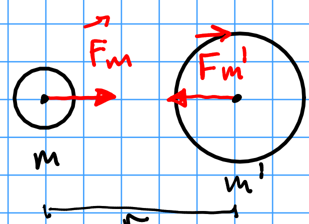


$\vec{T}_1, \vec{T}_2$ are internal to this "body"

$$(m_1+m_2)a = +m_2 g - m_1 g = (m_2-m_1)g$$

$$a = \frac{m_2-m_1}{m_2+m_1} g \qquad \checkmark \text{ same.}$$

- Gravity is a non-contact force



$$\boxed{F = G\frac{m\,m'}{r^2}}$$

NL of Universal Gravitation

NLUG

$$G = 6.67 \times 10^{-11} \frac{N m^2}{kg^2}$$

$$\text{N3L}: \quad \vec{F}_m = -\vec{F}_{m'}$$

Ex. Earth $\quad M_E = 5.98 \times 10^{24}\,kg$

$$R_E = 6.38 \times 10^6\,m = 6380\,km$$

$$F = m \underbrace{\frac{GM_E}{R_E^2}}_{=g} \quad \text{for an object of mass } m \text{ on Earth's surface}$$

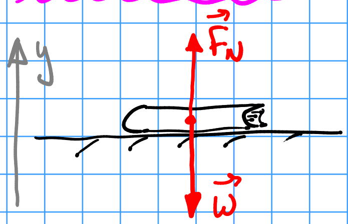
$$g = \frac{GM_E}{R_E^2} = \frac{6.67\times10^{-11}\,N\cancel{m^2} \times 5.98\times10^{24}\,\cancel{kg}}{\cancel{kg^2}\,(6.38\times10^6)^2\,\cancel{m^2}} = 9.80\,\frac{N}{kg} = 9.80\,\frac{m}{s^2}$$

($N = kg\,m/s^2$)

& $F = mg$, as usual

This is an extension of NLUG to large spheres of uniform density. When $r = R_E$ we call this $F = W$ = weight, $\boxed{W = mg}$

- contact forces between two bodies



$\vec{F}_N$ normal force is $\perp$ to the contact surface

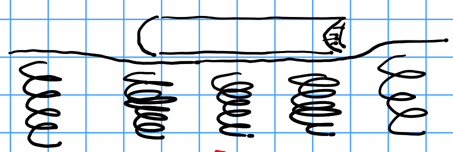
$\vec{F}_N$ is exerted by the table on the book

$\vec{W}$ is (exerted by the Earth) weight of the book

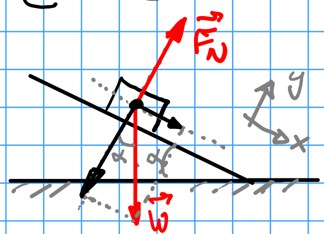
Book is in eq-m: $\vec{a} = \emptyset$

$\Rightarrow \vec{F}_{Total} = \vec{F}_N + \vec{W} = \emptyset \Rightarrow +F_N - W = \emptyset$, $\boxed{F_N = W = mg}$

Physial origin of $\vec{F}_N$: a molecular "mattress" model of atoms in solids:



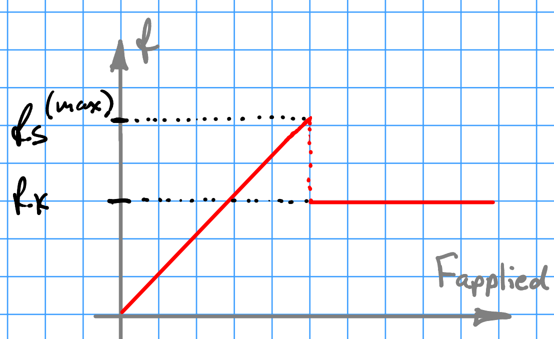
Note: "normal" = $\perp$ surface, not necessarily "up"



y-part of $\vec{W}$ presses block into the surface

$\Rightarrow \boxed{F_N = W\cos\alpha}$

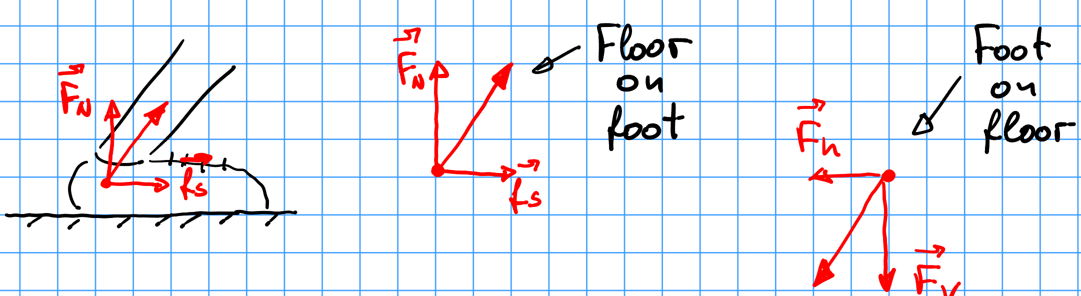
- tangential component of contact force = static friction f_s



$$\boxed{f_s^{(max)} = \mu_s F_N}$$

μ_s = coefficient of static friction

Origin: molecular welds



Increase $\vec{F}_h \rightarrow \vec{f}_s$ increases to match $\Rightarrow$ no acceleration of foot rel. to floor

When $\vec{F}_h$ grows too large $\rightarrow$ foot slips $\Rightarrow$ $a \neq \emptyset$ anymore

Experimental fact : $f_s = \mu_s F_N$ (magnitudes only $\vec{f}_s \perp \vec{F}_N$)

$[\mu_s] = 1$, dimensionless $\qquad \mu_s \lesssim 0.01$ "smooth" ; $\mu_s \gtrsim 1$ "rough"

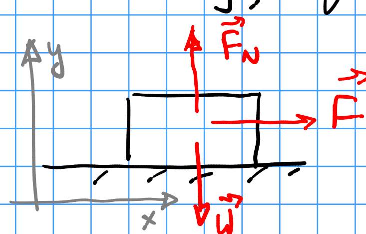
Once slipping starts, friction changes over to kinetic friction

Experimental fact: $f_K = \mu_K F_N$, $\vec{f}_K$ opposes motion

μ_K = coefficient of kinetic friction

- μ_s, μ_K do not depend on contact area
- f_K does not depend on speed (for small speeds)
- normally, $\mu_K \leq \mu_s$, i.e. $f_K \leq f_s^{(max)}$

Ex



$W = 45.0N \qquad \mu_s = 0.650$

$F = 36.0N \qquad \mu_K = 0.420$

① Move? $\quad f_s^{(max)} = \mu_s F_N = \mu_s W = \dots = 29.25N$

$y: +F_N - W = \emptyset \Rightarrow F_N = W$

$f_s^{(max)} < F \Rightarrow F$ overcomes friction, block moves

② How?

N2L

$$\begin{cases} +F_N - W = \emptyset \rightarrow F_N = W \text{ (as before)} \\ +F - f_K = F - \mu_K F_N = m a_x \end{cases} \qquad \boxed{f_K = \mu_K F_N}$$

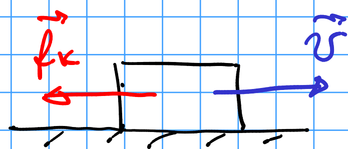
$\Rightarrow W = mg \Rightarrow m = \frac{W}{g}$

$\Rightarrow \frac{W}{g} a_x = F - \mu_K F_N = F - \mu_K W$

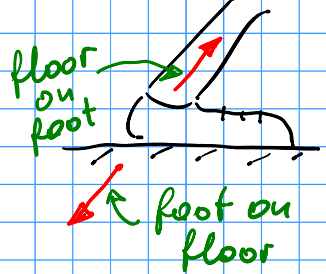
$\Rightarrow a_x = \frac{g}{W}(F - \mu_K W) = g\left(\frac{F}{W} - \mu_K\right)$

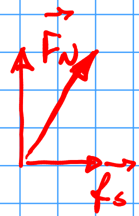
$a_x = 9.80 \frac{m}{s^2}\left(\frac{36.0\cancel{N}}{45.0\cancel{N}} - 0.420\right) = \dots = 3.72 \frac{m}{s^2}$

Note: $\vec{f}_k$ opposes motion



if $\vec{v}$ = "forward"
$\vec{f}_k$ = "back"

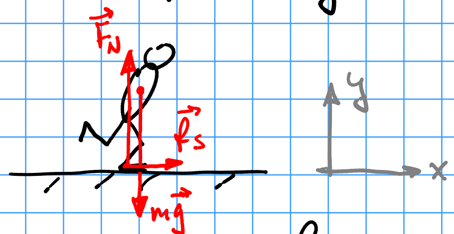




$\Rightarrow$ here $\vec{f}_s$ propels us forward

no friction $\Leftrightarrow$ no ability to move forward (or to stop)

Ex accelerating on ice



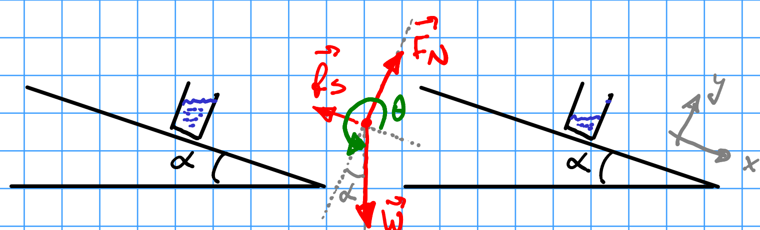
$y: +F_N - mg = ma_y = \emptyset \rightarrow F_N = mg$

$x: +f_s = ma_x$

$$\Rightarrow \quad a_x = \frac{f_s}{m} \le \frac{f_s^{(max)}}{m} = \frac{\mu_s F_N}{m} = \frac{\mu_s \cancel{m} g}{\cancel{m}} = \underline{\mu_s g}$$

$$\underline{a_x} \le a_x^{(max)} = \underline{1.57 \ \tfrac{m}{s^2}}$$

Ex



In standard notation:

$\theta = 270 + \alpha$

x: $W\cos\theta \ldots$

y: $W\sin\theta \ldots$

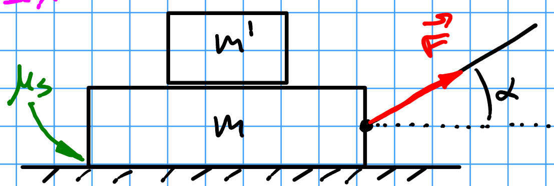
Eq-m: $\left.\begin{aligned} x&: \ W\sin\alpha - f_s = \emptyset \\ y&: \ -W\cos\alpha + F_N = \emptyset \end{aligned}\right\}$ $\emptyset \Leftrightarrow$ eq-m (no acceleration)

$\Rightarrow F_N = W\cos\alpha \quad \Rightarrow \quad \cancel{W}\sin\alpha = f_s \le f_s^{(max)} = \mu_s F_N = \mu_s \cancel{W}\cos\alpha$

$\Rightarrow \quad \dfrac{\sin\alpha}{\cos\alpha} \le \mu_s \qquad \underline{\tan\alpha \le \mu_s}$

! Eq-n does not contain W (or m) $\rightarrow$ both glasses start slipping at the same time!

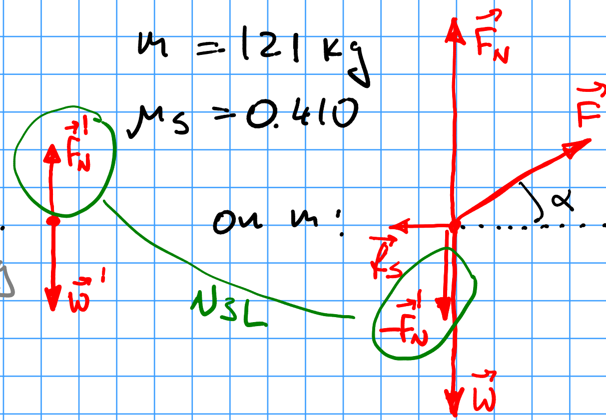
Ex



$F = 661\,N$ $\quad m = 121\,kg$

$\alpha = 20.0°$ $\quad \mu_s = 0.410$

Forces on m': $\quad$ on m:



Eq-m: $\quad$ no motion in y-direction:

m': $\quad +F_N' - m'g = \emptyset \quad \Rightarrow \quad F_N' = m'g$

m: $\quad +F_N + F\sin\alpha - F_N' - mg = \emptyset$

$\Rightarrow \quad F_N + F\sin\alpha - (m+m')g = \emptyset \qquad (I)$

and $\quad$ no motion in x-direction: $\rightarrow +x$

m: $\quad F\cos\alpha - f_s = \emptyset$

$\Rightarrow \quad F\cos\alpha = f_s \le f_s^{(max)} = \mu_s F_N \qquad (II)$

Express F_N from (I), sub. into (II)

$\Rightarrow \quad F\cos\alpha \le \mu_s\left[(m+m')g - F\sin\alpha\right]$

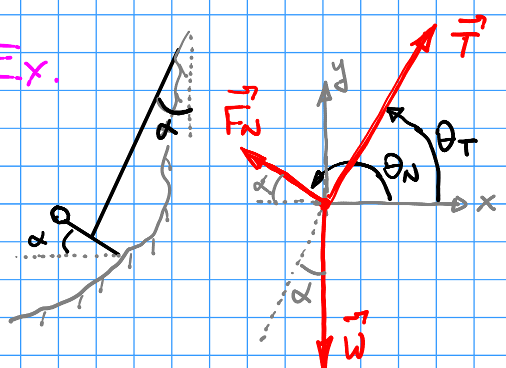
$$\frac{F\cos\alpha}{\mu_s} + F\sin\alpha \le (m+m')g$$

$$\frac{F}{g}\left[\frac{\cos\alpha}{\mu_s} + \sin\alpha\right] - m \le m'$$

$$\min(m') = \frac{661\,N}{9.80\,\frac{m}{s^2}}\left[\frac{\cos 20.0°}{0.410} + \sin 20.0°\right] - 121\,kg = 56.7\,kg$$

$\Rightarrow \quad m' \ge 56.7\,kg$

Ex.



$W = 650\,N$ $\quad \theta_T = 90 - \alpha = 72.0°$

$\alpha = 18.0°$ $\quad \theta_N = 180 - \alpha = 162°$

Eq-m:

x: $\quad F_N\cos\theta_N + T\cos\theta_T = \emptyset \qquad (I)$

y: $\quad F_N\sin\theta_N + T\sin\theta_T - W = \emptyset \qquad (II)$

Sub (I) into (II):

$$F_N = -T\,\frac{\cos\theta_T}{\cos\theta_N}$$

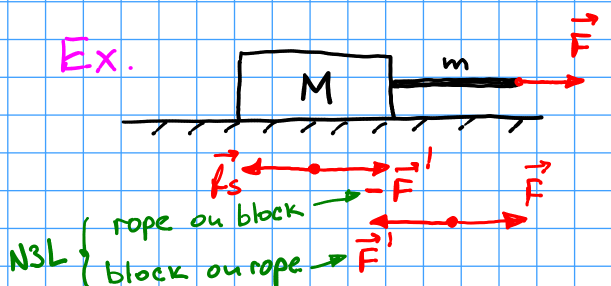
$$\Rightarrow \quad -T\,\frac{\cos\theta_T}{\cos\theta_N}\cdot\overbrace{\sin\theta_N}^{=\tan\theta_N} + T\sin\theta_T = W$$

$$T = \frac{W}{-\cos\theta_T\,\tan\theta_N + \sin\theta_T} = \frac{650\,N}{\sin 72^\circ - \cos 72^\circ\cdot\tan 162^\circ}$$

$$T = 618\ N$$

This is an example of tension, a contact force

Ex.



Eq-m : $a_{block} = \emptyset \Rightarrow a_{rope} = \emptyset$

$$\Rightarrow \quad \vec{F} + \vec{F}' = \emptyset \Rightarrow +F - F' = \emptyset$$

$$\vec{f_s} + (-\vec{F}') = \emptyset \Rightarrow -f_s + (-F') = \emptyset$$

$$F = F' = f_s$$

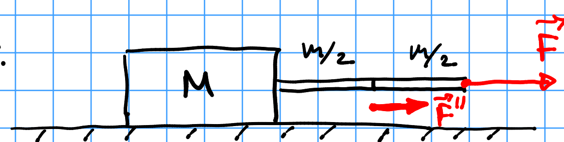
In general, $\vec{a} \neq \emptyset$

$$\Rightarrow \quad \vec{a} = \frac{\vec{F}}{M+m} \leftarrow \text{block + rope together}$$

$$\Rightarrow \quad \vec{F}' = M\vec{a} = M\,\frac{\vec{F}}{M+m} = \frac{M}{M+m}\,\vec{F}$$

and $\vec{F}' = \vec{F}$ only if $m = \emptyset$ (i.e. a massless rope)

Note:

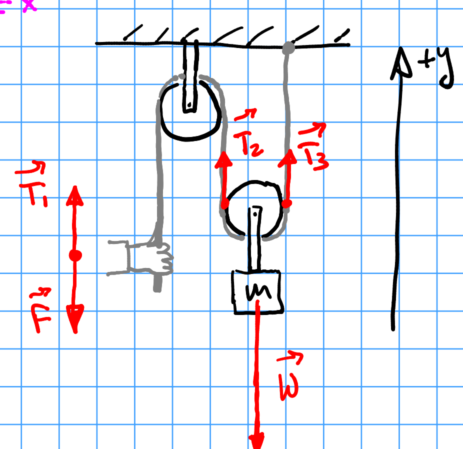


$\vec{F}''$ = force on the left $\frac{1}{2}$ rope

$$\vec{F}'' = \left(M + \frac{m}{2}\right)\vec{a} = \frac{M + m/2}{M+m}\,\vec{F} = \vec{F} \text{ iff } m = \emptyset$$

Generalize: any point along the rope: tension is the same everywhere within a massless rope.

Ex



Hand is motionless:

$$\vec{F} + \vec{T_1} = \emptyset$$

$$\Rightarrow \quad -F + T_1 = \emptyset \Rightarrow \quad F = T_1 = \overbrace{T\,(= T_2 = T_3)}^{\text{for massless rope}}$$

Pulley + mass is motionless:

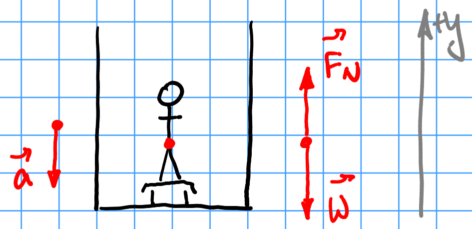
$$\vec{T_2} + \vec{T_3} + \vec{W} = \emptyset$$

$$\Rightarrow \quad +T_2 + T_3 - W = \emptyset \Rightarrow \quad W = 2T$$

$$F = \frac{1}{2}W$$

$\frac{1}{2}$ force of straight lifting

Ex non-equilibrium applications of N2L



$$+F_N - mg = -ma \quad \leftarrow \text{non-zero R.H.S.}$$

$$F_N = m(g-a)$$

e.g. down with $a = \frac{1}{3}g$: $F_N = m\left(g - \frac{1}{3}g\right) = \frac{2}{3}mg$ (33% under weight)

$\vec{a}$ up with $a = \frac{1}{3}g$: $F_N = m\left(g + \frac{1}{3}g\right) = \frac{4}{3}mg$ (33% over weight)