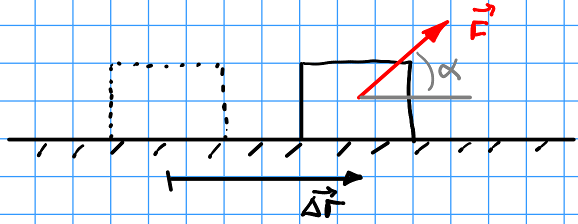
Work and Energy



displacement $\Delta\vec{r}$

force $\vec{F}$, $\angle\alpha$ w.r.to $\Delta\vec{r}$

$$\boxed{\text{Work} \equiv F\,\Delta r\cos\alpha}$$

Ex pushing a cart ($\alpha = \emptyset$) $\Rightarrow$ Work $= F\,\Delta r$

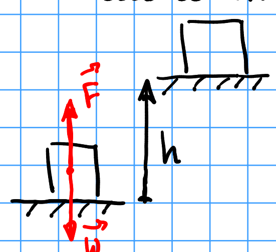
In general, "work done by $\vec{F}$ along $\Delta\vec{r}$"

- no displacement $\Rightarrow$ no work done, $W = \emptyset$ | vs. W = weight
- no force $\Rightarrow$ $W = \emptyset$
- $\alpha = 90°$ ($\vec{F} \perp \Delta\vec{r}$) $\Rightarrow$ $\cos\alpha = \emptyset$ = $W = \emptyset$, no work done
- $\alpha = \emptyset$ $\Rightarrow$ $\cos\alpha = 1 = \max(\cos\alpha)$ i.e. max work done

Work : a scalar $[W] = [F][\Delta r] = 1N \cdot 1m = 1\,\text{Joule} = 1J$

Ex a person lifts a 10.0 kg toolbox from the floor, without acceleration, and puts it on a shelf, 1.50 m high
an approximation

"no acceleration": $\vec{F} + \vec{W} = \emptyset \Rightarrow F = W = mg$



① Work done by person:

$W_F = F\,\Delta r\cos\emptyset° = mgh = 10.0 \times 9.80 \times 1.50 = \underline{147\,J}$

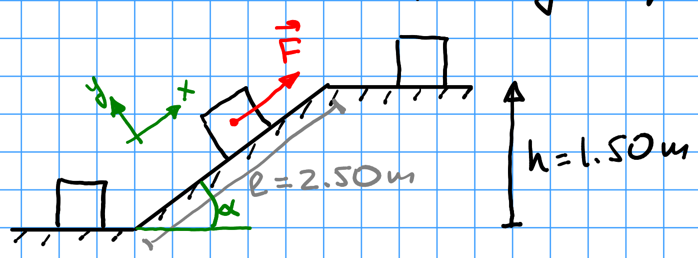
② Work done by gravity

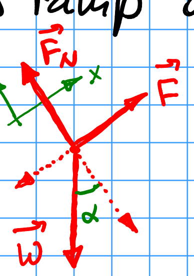
$W_g = W\,\Delta r\cos 180° = -mgh = \underline{-147\,J}$

③ Work done by the net force: $W_{NET} = \emptyset$ since $\vec{F}_{NET} = \emptyset$ (pseudo eq-m)

$W_{NET} = W_F + W_g = \emptyset$

Ex same toolbox, using a frictionless ramp of 2.50 m





$\left.\begin{array}{l} h = 1.50m \\ \ell = 2.50m \end{array}\right\}$ $\sin\alpha = \dfrac{h}{\ell}$

No friction: $+F - W\sin\alpha = \emptyset \Rightarrow F = W\sin\alpha = W\frac{h}{\ell} = mg\frac{h}{\ell}$

Work done: $W = F\cdot\ell = mg\frac{h}{\cancel{\ell}}\cdot\cancel{\ell} = mgh = 147\,J$ same!

Note: Force: $F = 10.0\times 9.80\times\frac{1.50}{2.50} = 58.8\,N < W = mg$

distance: $\ell > h$

work: same!

$\emptyset$, since $\Delta y = \emptyset$

Again, $W_g = W_{g,x} + W_{g,y} = -mg\frac{h}{\ell}\cdot\ell = -mgh = -147\,J$

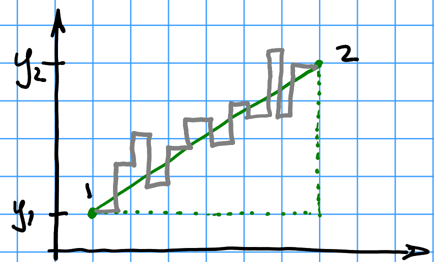
in our y, x

Net force $= \emptyset$ (no acceleration, pseudo eq-m) $\Rightarrow W_{NET} = \emptyset$

$W_F = -W_g$

! || total work done does not depend on path only on net change in elevation.

Generalize:



$W(1\to 2)$ only depends on $h = (y_2 - y_1)$

$$W_g = -mgh = -mg(y_2 - y_1) = -(mgy_2 - mgy_1) = -[U_g(y_2) - U_g(y_1)]$$

where $\boxed{U_g(y) \equiv mgy}$ = gravitational P.E.

U_g is a function of position only, i.e. memory of the path is gone.

Weight is a conservative force. For a c.f. $\vec{F}$:

$$W_{c.f.} = -\Delta U = -[U(\vec{r}) - U(\vec{r}_0)], \text{ with } U(\vec{r}) = U(x, y, z, \ldots)$$

Ex. gravity, electrostatic, elastic, ... are c.f.

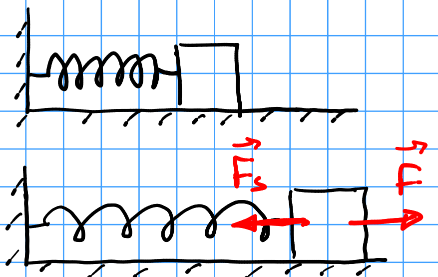
Note: choice of origin is arbitrary

Ex let $U' = mgy + U_0$, where $U_0 = \text{const}$

$$\Rightarrow W_g = -\Delta U' = -[(mgy_2 + U_0) - (mgy_1 + U_0)] = -[mgy_2 - mgy_1]$$

$$= -\Delta U$$ ✓ the same!

Ex elastic force



$$\boxed{F_s = -kx}$$ Hooke's law

"spring"

force resists displacement

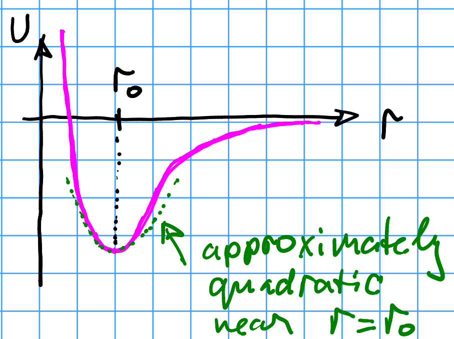
k = spring constant

$$\Rightarrow U_s = \frac{1}{2}kx^2$$ ← a different form of P.E. suitable for a harmonic oscillator

Ex $k = 200\,\frac{N}{m}$ compress to store 20.0 J of PE

$$U_s(x) = \frac{1}{2}kx^2 \Rightarrow x = \sqrt{\frac{2U}{k}} = \sqrt{\frac{2\times 20.0\,J = \cancel{N}\cdot m}{200\,\frac{\cancel{N}}{m}}} = \sqrt{\frac{1}{5}}\,m \simeq 0.447m$$

Ex

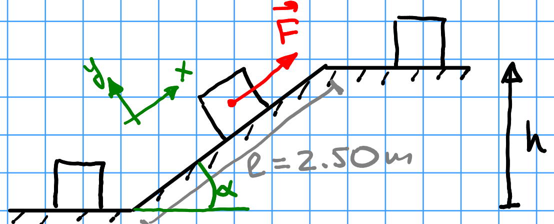


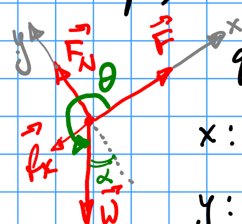
interatomic interaction potential

near eq-m, all potentials look a lot like the spring P.E.

$\Rightarrow$ i.e like a harmonic oscillator.

Ex same toolbox on ramp, add friction





quasi-eq-m (no acceleration)

$$x: \; -W\sin\alpha + \emptyset + F - f_k = ma_x = \emptyset$$

$$y: \; -W\cos\alpha + F_N + \emptyset + \emptyset = ma_y = \emptyset$$

Better: $\theta = 270° - \alpha$

$x: \; \ldots\ldots + W\cos\theta$

$y: \; \ldots\ldots + W\sin\theta$ } would take care of signs & sin/cos

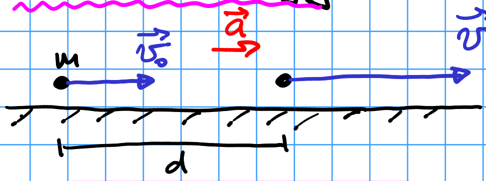
$$\Rightarrow W_{total} = F\ell = [\mu_k W\cos\alpha + W\sin\alpha]\ell$$

$$\sin\alpha = \frac{h}{\ell} \qquad \cos\alpha = \sqrt{1-\left(\frac{h}{\ell}\right)^2} = \sqrt{\frac{\ell^2-h^2}{\ell^2}} = \frac{\sqrt{\ell^2-h^2}}{\ell}$$

$$\Rightarrow W_{total} = \mu_k mg\frac{\sqrt{\ell^2-h^2}}{\cancel{\ell}}\cancel{\ell} + mg\frac{h}{\cancel{\ell}}\cancel{\ell} = \underbrace{\mu_k mg\sqrt{\ell^2-h^2}}_{W_{non\text{-}c.f.}} - \underbrace{\Delta U_g}_{147J,\text{ as before}}$$

$\Rightarrow W_{total} > W_{c.f} = -\Delta U_g = 147\,J$ because of the presence of friction

- kinetic energy



$$\boxed{K \equiv \frac{1}{2} m v^2}$$

kinetic energy

$$v^2 = v_0^2 + 2ad \qquad F = ma$$

$$W = Fd = mad = \frac{1}{2} m \left(v^2 - v_0^2 \right)$$

$$W = \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2$$

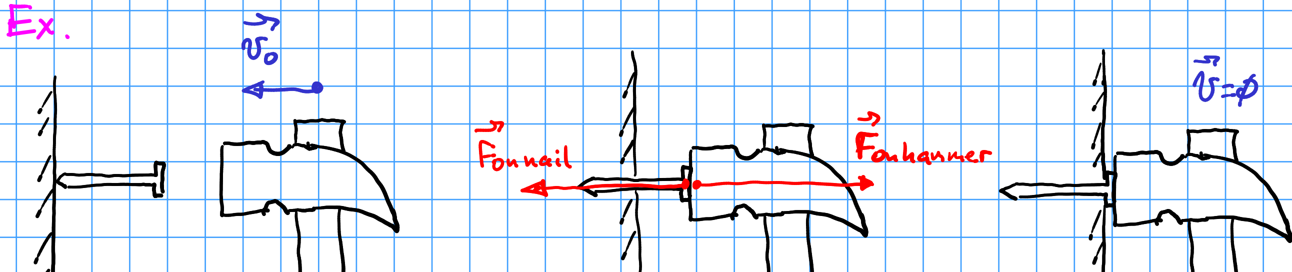
$$W = K - K_0 = \Delta K$$

$U = \text{P.E.} = U(x) =$ energy associated with position

$K = \text{K.E.} =$ energy associated with motion, i.e. the work done to get an object moving; or the ability of the object to do [mechanical] work on its surroundings

- work-energy theorem $\boxed{\Delta K = W}$

Ex.



$K = \frac{1}{2} m v_0^2$ work being done $K = \emptyset$

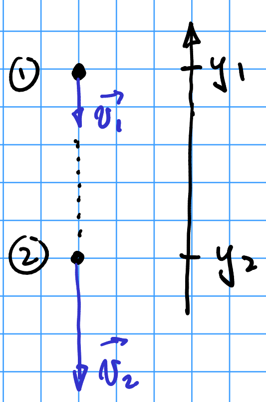
$$W_{\text{on hammer}} = \cancel{\frac{m v_f^2}{2}} - \frac{m v_0^2}{2} = -\frac{m v_0^2}{2}$$

N3L: $\vec{F}_{\text{on nail}} = -\vec{F}_{\text{on hammer}}$

$$\Rightarrow W_{\text{on nail}} = -W_{\text{on hammer}} = -\left(-\frac{m v_0^2}{2} \right) = \frac{m v_0^2}{2}$$

i.e. objects with K.E. ($K \geq \emptyset$, always) can do work on their surroundings.

Ex free fall $F = -mg$



$$W_g = F \Delta y = (-mg)(y_2 - y_1) = -(mgy_2 - mgy_1)$$

② final ① initial

On the other hand:

$$W_g = \Delta K = \underset{\text{final}}{\frac{m v_2^2}{2}} - \underset{\text{initial}}{\frac{m v_1^2}{2}}$$

$$\Rightarrow \quad \frac{m v_2^2}{2} - \frac{m v_1^2}{2} = -\left(mgy_2 - mgy_1\right)$$

$$\Rightarrow \quad \underbrace{\underbrace{\frac{m v_2^2}{2}}_{KE_2} + \underbrace{mgy_2}_{PE_2}}_{\text{final}} = \underbrace{\underbrace{\frac{m v_1^2}{2}}_{KE_1} + \underbrace{mgy_1}_{PE_1}}_{\text{initial}}$$

$\Rightarrow$ the sum of KE and gravitational PE is constant, $\forall y$

$$\boxed{E = KE + PE = \frac{mv^2}{2} + mgy = \text{const}}$$

Ex $v_0 = \emptyset$, $v \neq \emptyset$

$y = \emptyset \leftarrow KE = \emptyset,\ PE = \emptyset \Rightarrow E = \emptyset$

$y = -h \leftarrow KE > \emptyset,\ PE < \emptyset \Rightarrow E = \emptyset$

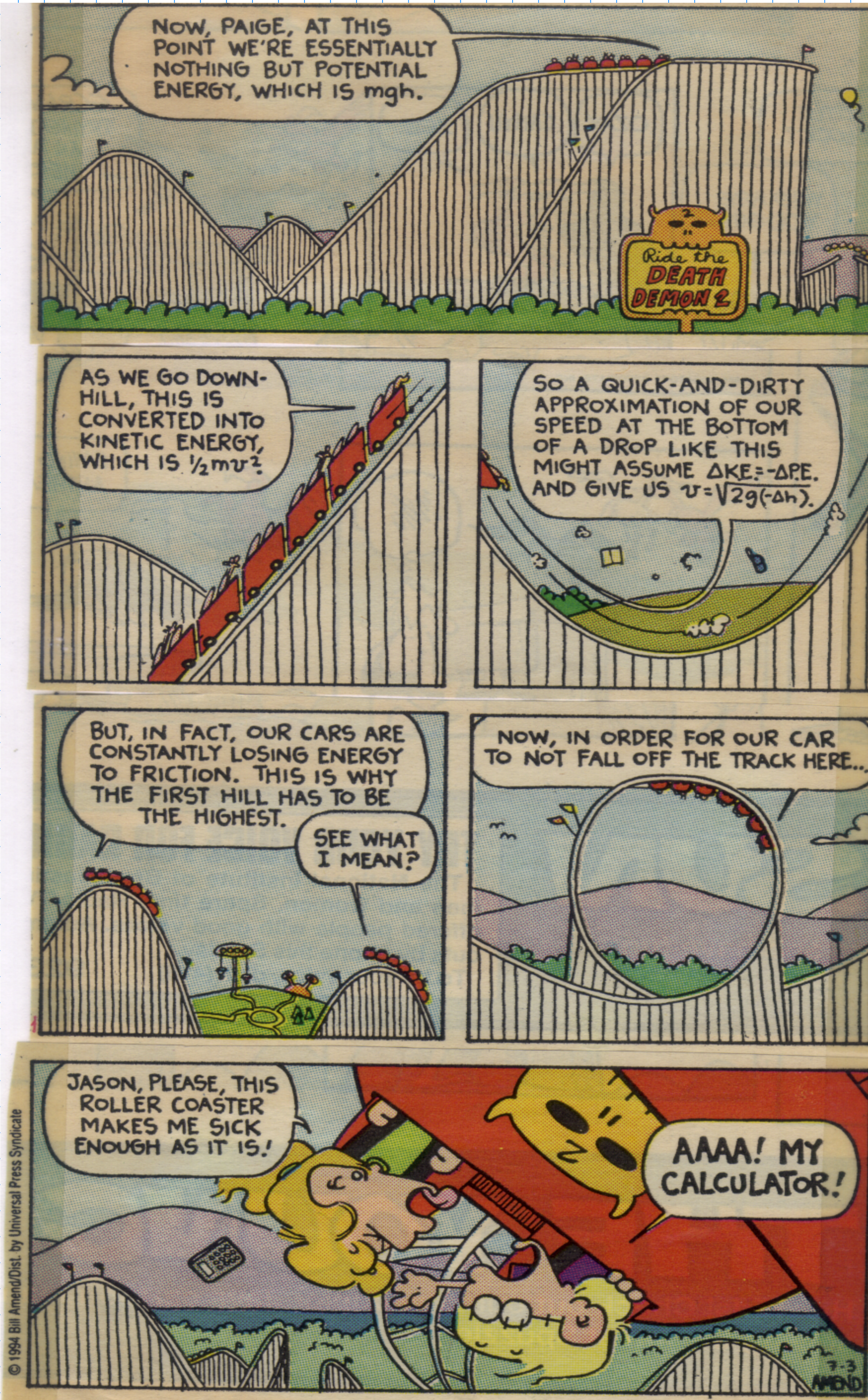
} $E = \text{const.}$

$$\emptyset = \frac{mv^2}{2} + mg(-h) \quad \rightarrow v = \sqrt{2gh}$$

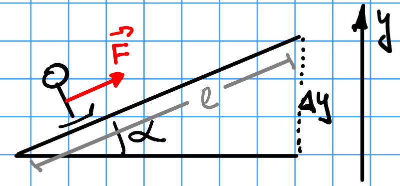
measure of ability to do work on other objects $\rightarrow \dfrac{\cancel{m} v^2}{2} = \cancel{m} g h \leftarrow$ work done by gravity

y: PE	E	KE	PE	E = KE + PE
$\emptyset$	$\emptyset$	0	600 000 J	600 000 J
−200 kJ	$\emptyset$	200 000 J	400 000 J	600 000 J
−400 kJ	$\emptyset$	400 000 J	200 000 J	600 000 J
−600 kJ	$\emptyset$	600 000 J	0	600 000 J

vs.


NOW, PAIGE, AT THIS POINT WE'RE ESSENTIALLY NOTHING BUT POTENTIAL ENERGY, WHICH IS mgh.
Ride the DEATH DEMON 2
AS WE GO DOWNHILL, THIS IS CONVERTED INTO KINETIC ENERGY, WHICH IS $\frac{1}{2}mv^2$.
SO A QUICK-AND-DIRTY APPROXIMATION OF OUR SPEED AT THE BOTTOM OF A DROP LIKE THIS MIGHT ASSUME $\Delta KE = -\Delta PE$. AND GIVE US $v = \sqrt{2g(-\Delta h)}$.
BUT, IN FACT, OUR CARS ARE CONSTANTLY LOSING ENERGY TO FRICTION. THIS IS WHY THE FIRST HILL HAS TO BE THE HIGHEST.
SEE WHAT I MEAN?
NOW, IN ORDER FOR OUR CAR TO NOT FALL OFF THE TRACK HERE...
JASON, PLEASE, THIS ROLLER COASTER MAKES ME SICK ENOUGH AS IT IS!
AAAA! MY CALCULATOR!
7-3
AMEND

Ex. a skier



$\ell = 2830\,m \quad \alpha = 14.6° \quad m = 75.0\,kg$

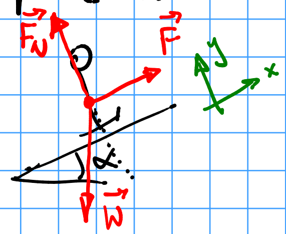
ΔU_g = change in PE $= \Delta(mgy) = mg\,\Delta y$

$= mg\,\ell \sin\alpha$

$\Rightarrow \Delta U_g = \underbrace{\underbrace{75.0\,kg \cdot 9.80\,\frac{m}{s^2}}_{N} \times 2830\,m}_{J} \times \sin 14.6° = \underline{5.24 \times 10^5\,J}$

Note: average force: $W = \bar{F}\ell = \Delta U_g \Rightarrow \bar{F} = \frac{\Delta U_g}{\ell} = mg\sin\alpha$ ✓

Compare this with the "old way":

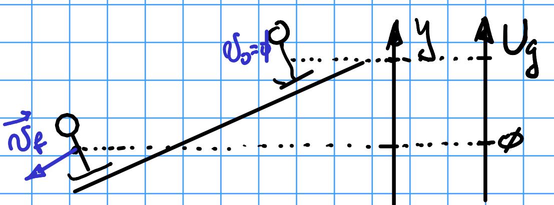


$x: +F - mg\sin\alpha = \emptyset$ (skier @ constant velocity)

$y: +F_N - mg\cos\alpha = \emptyset$

$\Rightarrow F = mg\sin\alpha$ ✓ same.

Ex skier turns around & goes down:



a) no friction: $\Delta(K+U) = \emptyset$

$\Rightarrow \left(\frac{m v_f^2}{2} + \emptyset\right) - \left(\cancel{\frac{m v_0^2}{2}}^{\,\emptyset} + mg\ell\sin\alpha\right) = \emptyset$

$\frac{\cancel{m} v_f^2}{2} = \cancel{m}g\,\ell\sin\alpha$

$v_f = \sqrt{2g\ell\sin\alpha} = \ldots = \underline{118\,\frac{m}{s}}$

b) with friction: $\mu_k = 0.100$

$\Delta K = \underset{-\Delta U_g}{\overset{\shortparallel}{W_g}} + W_{other} \Rightarrow \Delta K + \Delta U_g = \Delta(K + U_g) = W_{other}$

$\Rightarrow \frac{m v_f^2}{2} - mg\ell\sin\alpha = -\mu_k \underset{F_N}{\overset{\shortparallel}{(mg\cos\alpha)}} \cdot \ell$

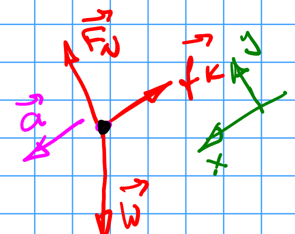
$\vec{f}_K$ opposes the displacement, $\cos(180°) = -1$

$f_K = \mu_K F_N$

$\Rightarrow v_f = \underline{\sqrt{2g\ell(\sin\alpha - \mu_k\cos\alpha)}}$ ✓

$= \ldots = \underline{92.8\,\frac{m}{s}}$

The "old way":



$\cancel{m}g\sin\alpha - \mu_k \cancel{m}g\cos\alpha = \cancel{m}a$

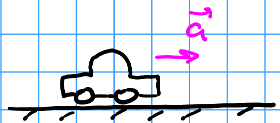
$v_f^2 = \cancel{v_0^2}^{\,\emptyset} + 2ad^{\,=\ell}$

$\Rightarrow v_f = \sqrt{2\ell g(\sin\alpha - \mu_k\cos\alpha)}$ ✓

• power = rate of doing work $\equiv \boxed{\frac{W}{t} = P}$

$[P] = 1\,\text{Watt} = 1\,W = \frac{1\,J}{1\,s}$; $1\,\text{h.p.} = 746\,W$

Ex



$t = 5.00\,s$ $\quad m = 1.10 \times 10^3\,kg$

$a = 4.60\,\frac{m}{s^2}$ $\quad v_0 = \emptyset$ (from rest)

- accelerates : there is a force, $F = ma$ (N2L)
- it acts for time t : $d = \frac{at^2}{2}$
- work done: $W = Fd = \frac{1}{2} m a^2 t^2$

No surprise: $v = at \Rightarrow W = \frac{1}{2} m v^2 = \frac{1}{2} m (at)^2$

- average power : $\bar{P} = \frac{W}{t} = \frac{1}{2} m a^2 t = \ldots = 5.82 \times 10^4\,W$

$= 58.2 \times \underbrace{10^3}_{= k} W = 58.2\,kW = 78\,\text{hp.}$

another way :

$$\bar{P} = \frac{W}{t} = \frac{Fd}{t} = F\frac{d}{t} = F\bar{v}$$

$$\bar{P} = F\,\frac{1}{2}(v + \cancel{v_0}) = \frac{1}{2} F v = \frac{1}{2}(ma)(at) = \text{same.}$$