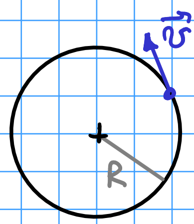
Rotational motion



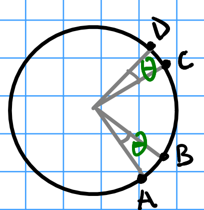
Kinematics : e.g. of a periodic motion

- trajectory : a circle
- period T : time for one complete revolution
- velocity $\vec{v}$: tangential
- speed $v = \frac{2\pi R}{T}$ = const $\leftrightarrow$ uniform circular motion

Describe motion : equations

① $\begin{cases} x(t) \\ y(t) \end{cases}$ – complicated

② $\begin{cases} r(t) \\ \theta(t) \end{cases}$ – easier , namely "circular" $\leftrightarrow$ $r(t) = R$ = const
"uniform" $\leftrightarrow$ angle per time = const



same time to move from A to B
and from C to D

θ = angular displacement

$[\theta] = {}^\circ$ (degrees) = rev (1 rev = 360°) = rad . (2π rad = 1 rev = 360°)

$\theta = 2\pi \leftrightarrow$ full circle ; path length = $2\pi R$

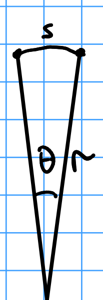
Generalize : angle θ rad $\leftrightarrow$ arc length of θR

$$[\theta] = \frac{[\text{arc length}]}{[R]} = \frac{m}{m} = 1 = \text{dimensionless}$$

Ex. of conversion

$$1^\circ = 1\cancel{^\circ} \times \frac{2\pi \text{ rad}}{360\cancel{^\circ}} = \frac{1 \times 2 \times 3.14..}{360} = \ldots \simeq 0.0175 \text{ rad}$$

Ex two satellites in orbit



$r = 4.23 \times 10^7$ m

$\theta = 2.00^\circ$

θ°	θ, rad	s, m
360	2π	$(2\pi)r$
θ	$\theta\frac{2\pi}{360}$	$(\theta\frac{2\pi}{360})r$

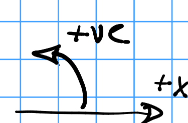
$$\Rightarrow s = \theta^\circ \frac{2\pi \text{rad}}{360^\circ} r = 2.00\cancel{^\circ} \frac{3.14..\text{rad}}{180\cancel{^\circ}} \times 4.23 \times 10^7 m = 1.4765.. \times 10^6 m$$

$$\simeq 1.48 \times 10^6 m = 1.48 \times 10^3 \text{ km} \hat{=} 1500 \text{ km}$$

- angular velocity ω (omega)
  - average $\bar{\omega} = \frac{\theta - \theta_0}{t - t_0} = \frac{\Delta\theta}{\Delta t}$
  - instantaneous $\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$ [or $\dot{\theta}$]
  - units $[\omega] = \frac{rad}{s}$
- angular acceleration α (alpha)
  - average $\bar{\alpha} = \frac{\omega - \omega_0}{t - t_0} = \frac{\Delta\omega}{\Delta t}$
  - intantaneous $\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$ [or $\dot{\omega} = \ddot{\theta}$]
  - units $[\alpha] = \frac{rad}{s^2}$
- summary of kinematics

	recall:
$\omega = \omega_0 + \alpha t$	$v = v_0 + at$
$\bar{\omega} = \frac{1}{2}(\omega + \omega_0)$	$\bar{v} = \frac{1}{2}(v + v_0)$
$\theta = \bar{\omega} t = \frac{1}{2}(\omega + \omega_0) t$	$x = \bar{v} t = \frac{1}{2}(v + v_0) t$
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	$x = v_0 t + \frac{1}{2} a t^2$
for motion with $\alpha = const$	for motion with $a = const$

Ex a watch



a) seconds' hand : $-1\ rev/min$

$$\Rightarrow \omega = \underbrace{-}_{\text{clockwise}} \frac{1 rev}{1 min} = \underbrace{-}_{\text{clockwise}} \frac{2\pi\ rad}{\cancel{1 min} \frac{60 s}{\cancel{1 min}}} = -0.105\ rad/s$$

b) hour hand : $-1\ rev/12 hrs$

$$\Rightarrow \omega = -\frac{2\pi\ rad}{12 \times 3600 s} = -0.000145\ rad/s = 1.45 \times 10^{-4}\ rad/s$$

Ex a fan

$\omega_f = 83.8\ rad/s$

$t = 1.75 s$

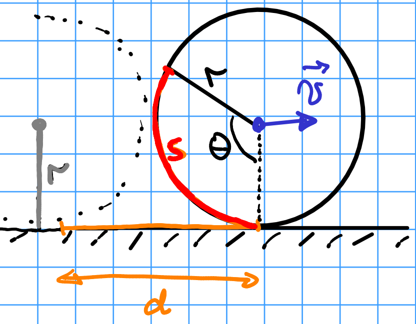
$\alpha = -42.0\ rad/s^2$ (-ve as "deceleration")

$\boxed{\omega = \omega_0 + \alpha t} \Rightarrow \omega_0 = \omega - \alpha t =$

$= 83.8\ rad/s - (-42.0\ rad/\cancel{s^2})\ 1.75\ \cancel{s}$

$\Rightarrow \omega_0 \simeq 157\ rad/s$

Ex a rolling tire



$$\boxed{s = \theta r}$$

$d = s$

(no slipping)

- s = distance along the outer edge, e.g. $s = 2\pi r$ for 1 rev.

  here $\boxed{s = \theta r}$

  linear, angular displacements

- ω = const $\Rightarrow$ $\theta = \omega \Delta t$

- d = distance along ground, $d = v \Delta t$
- no slipping : $s = d \Rightarrow \theta r = \omega \Delta t \, r = v \Delta t \Rightarrow \boxed{v = \omega r}$

  linear, angular velocities

- similarly, $\boxed{a = \alpha r}$

  linear, angular accelerations

Ex "crack-the-whip" – skaters, marching band

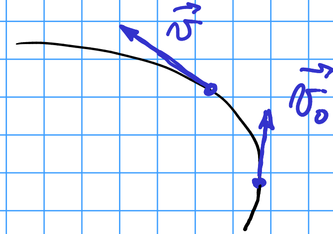
same ω, different r $\rightarrow$ different $v = \omega r$

ω describes the rotation of the whole body

v describes the speed of <u>one point</u> on the wheel, depends on r

$v = v_T$ = tangential speed

$a = a_T$ = tangential acceleration



a_T reflects only changes in <u>speed</u>

$$\overline{a_T} = \frac{v - v_0}{t - t_0} = \frac{\Delta v}{\Delta t}$$

But, $\vec{v}$ is a vector, and its direction is also changing

Even if $\overline{a_T} = \phi$, i.e. $v = v_0$ but $\vec{v} \neq \vec{v}_0$ (i.e. $\vec{a} \neq \phi$)

Ex. the applet : Observe how, for small Δt :

- $\vec{v} \rightarrow \vec{v}_0$
- $\Delta\vec{v} \rightarrow$ tangential to the $\vec{v}$ circle, $\Delta \vec{v} \perp \vec{v}$
- $\Delta v \simeq$ arc length along the circle of radius v, i.e. θv

Put it together :

$$\Delta v \simeq v\theta = v(\omega \Delta t) = v \frac{v}{r} \cdot \Delta t = \frac{v^2}{r} \Delta t$$

$v = \omega r$

$\omega = \frac{v}{r}$

$$a = \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$, with direction $\vec{a} \perp \vec{v}$

Since $\vec{v}$ = tangential to the circular trajectory

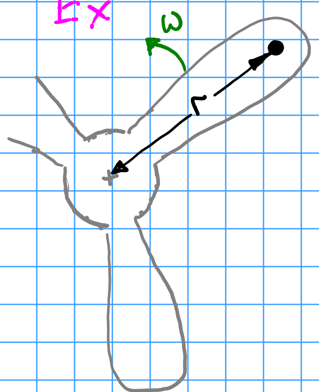
$\Rightarrow$ $\vec{a} \perp \vec{v}$ is toward the center of the circular trajectory

$$\boxed{a_c = \frac{v_T^2}{r}}$$

centripetal acceleration

here, just magnitudes, direction of $\vec{a}_c$ implicit

Ex



choose some point, a distance r from the center of rotation

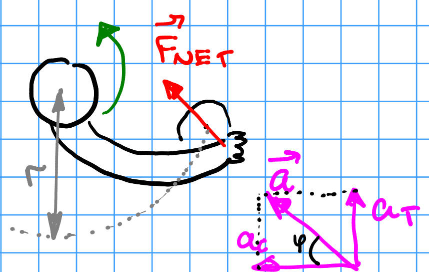
$$a_c = \frac{v_T^2}{r} = \frac{(\omega r)^2}{r} = \omega^2 r$$

$$\frac{a_{c,1}}{a_{c,2}} = \frac{\omega_1^2 \cancel{r}}{\omega_2^2 \cancel{r}} = \left(\frac{\omega_1}{\omega_2}\right)^2 = \left(\frac{440 \cancel{\frac{rev}{min}}}{110 \cancel{\frac{rev}{min}}}\right)^2 = 16$$

Note: a general result $\boxed{a_c = \frac{v_T^2}{r} = \omega^2 r}$

Ex a discus thrower

top view



"from rest"	:	$\omega_0 = 0$
final	:	$\omega = +15 \frac{rad}{s}$
in time	:	$\Delta t = 0.270 s$
arm	:	$r = 0.810 m$

1) tangential : $\alpha = \frac{\omega - \cancel{\omega_0}^{\,0}}{\Delta t} = \frac{\Delta \omega}{\Delta t} = \frac{\omega}{\Delta t}$ $\Rightarrow$ $a_T = \alpha r = \frac{\omega r}{\Delta t}$

2) centripetal : $a_c = \omega^2 r$ (direction: toward the center)

3) total : $a^2 = a_c^2 + a_T^2 = \left(\frac{\omega r}{\Delta t}\right)^2 + (\omega^2 r)^2 = (\omega r)^2 \left[\frac{1}{\Delta t^2} + \omega^2\right]$

$$a = \omega r \sqrt{\frac{1}{\Delta t^2} + \omega^2} = 15.0 \frac{\cancel{rad}}{s} \times 0.810 m \sqrt{\frac{1}{0.270^2 s^2} + \left(15 \frac{\cancel{rad}}{s}\right)^2}$$

$$= 187.72 \ldots \frac{m}{s^2} \simeq 188 \frac{m}{s^2}$$

$$\varphi = \arctan \frac{a_T}{a_c} = \arctan \frac{\cancel{\omega r}/\Delta t}{\omega^{\cancel{2}} \cancel{r}} = \arctan \frac{1}{\omega \Delta t} = \ldots \simeq 13.9°$$

Dynamics of rotational motion

$\exists$ acceleration $\Rightarrow$ $\exists$ a force responsible : centripetal force

$$F_c = m a_c = \frac{m v^2}{r} = m \omega^2 r$$

$s = \theta r$
$v = \omega r$
$a = \alpha r$

$F_c = \emptyset \iff$ motion in a straight line

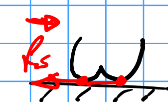
$F_c \neq \emptyset \iff$ motion along a curve
(if F_c = const, along a circle)

$\exists$ = "there is"

F_c can be provided by :

- model airplane : $F_c = T$ = tension in a wire
- satellite, Moon : F_c = gravitational $= G\frac{Mm}{r^2}$
- car turning : $F_c = f_s$, static friction of tires on road
- passenger in a turning car

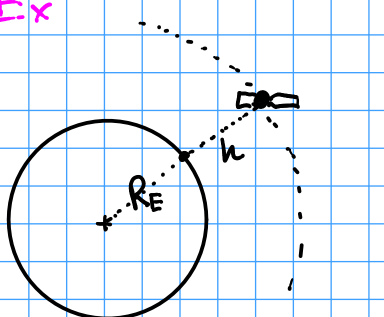
  $F_c = f_s$ = static friction (bum / seat) for small a_c

  $F_c = F_N$ from the door

Ex



$$h = 5.0 \times 10^2 \, \underset{}{\overset{10^3}{k}}m = 5.0 \times 10^5 \, m$$

$$T = 95 \, min$$

$$v = \frac{2\pi (R_E + h)}{T} = \frac{2\pi (6.4 \times 10^6 m + 5.0 \times 10^5 m)}{95 \times 60 s}$$

$$\simeq 7.6 \times \underset{k}{\underset{\shortparallel}{10^3}} m/s = 7.6 \, km/s$$

$$\Rightarrow a_c = \frac{v^2}{(R_E + h)} = \frac{4\pi^2 (R_E + h)^{\cancel{2}}}{T^2 \cancel{(R_E+h)}} = \dots \simeq \underset{g \, @ \, r = R_E + h}{8.4 \frac{m}{s^2}} < \underset{g \, @ \, r = R_E}{9.8 \frac{m}{s^2}}$$

Let us calculate g (acceleration due to gravity, free fall) @ h

$$F = G \frac{M_E}{(R_E + h)^2} m = m g_h \text{ , where } g_h = \frac{G M_E}{(R_E + h)^2}$$

When $h = \emptyset$, $r = R_E$ $\quad g = \frac{G M_E}{R_E^2}$

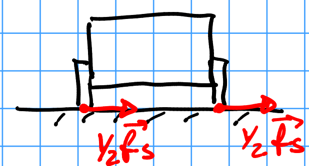
$$g_h = \underbrace{\frac{G M_E}{R_E^2}}_{= g} \times \frac{R_E^2}{(R_E + h)^2} = g \left(\frac{R_E}{R_E + h} \right)^2 = g \left(\frac{6.4 \times 10^6 \cancel{m}}{6.4 \times 10^6 + 5.0 \times 10^5 \cancel{m}} \right)^2$$

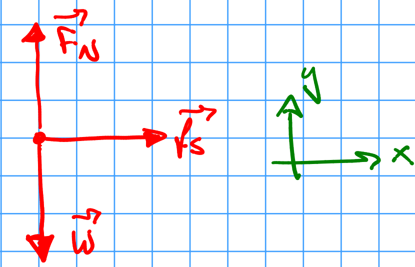
5×10^5
$= 0.5 \times 10^6$

$$g_h = g\left(\frac{6.4}{6.9}\right)^2 = 8.431\ldots \frac{m}{s^2} \simeq \underline{8.4\ \frac{m}{s^2}}$$

! $g_h = 8.4\ \frac{m}{s^2}$ ⇒ the satellite (and the Moon!) is falling!
All the time!

Ex car turning





$$y:\ +F_N - W = \emptyset \quad \Rightarrow \quad F_N = mg$$

$$x:\ +f_s = F_c = ma_c = m\frac{v^2}{r}$$ ← F_c comes from f_s, friction is what turns the car

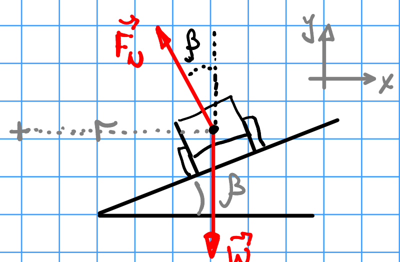
$$\Rightarrow \frac{\cancel{m}v^2}{r} = f_s \leq f_s^{(max)} = \mu_s F_N = \mu_s \cancel{m} g$$

$$\frac{v^2}{r} \leq \mu_s g$$

max safe $v = \sqrt{\mu_s r g}$, <u>independent of m</u> !

⇒ a sandbag in the trunk will not help. SLOW DOWN!

Ex a banked road helps



$$y:\quad +F_N \cos\beta - W = \emptyset$$

$$x:\quad -F_N \sin\beta = -F_c = -m\frac{v^2}{r}$$

$$F_N = \frac{W}{\cos\beta} = \frac{mg}{\cos\beta}$$

$$\Rightarrow +\cancel{m}g\frac{\sin\beta}{\cos\beta} = +\frac{\cancel{m}v^2}{r} = F_c$$

$$v = \sqrt{rg\tan\beta}$$

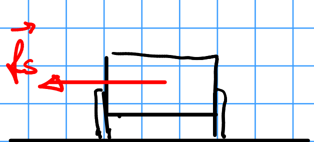
cf: $v \leq \sqrt{\mu_s r g}$

even if $\mu_s = \emptyset$, i.e. all of F_c is from F_N

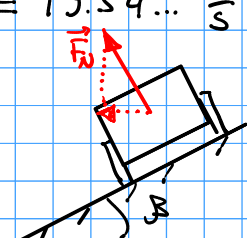
Ex $r = 120\,m \quad \beta = 18^\circ$

$$v_T = \sqrt{120\,m \times 9.80\frac{m}{s^2} \times \tan 18^\circ} = 19.54\ldots \frac{m}{s} \simeq 20\frac{m}{s} \simeq 70\frac{km}{h}$$

Ex friction vs. banking



$\mu_s = 0.80,\ \beta = \emptyset$



$\mu_s = \emptyset,\ \beta \neq \emptyset$

$$\left.\begin{matrix}\text{Same } v_T \\ r\end{matrix}\right\} \Rightarrow \text{same } a_c = \frac{v_T^2}{r} \Rightarrow \text{same } F_c$$

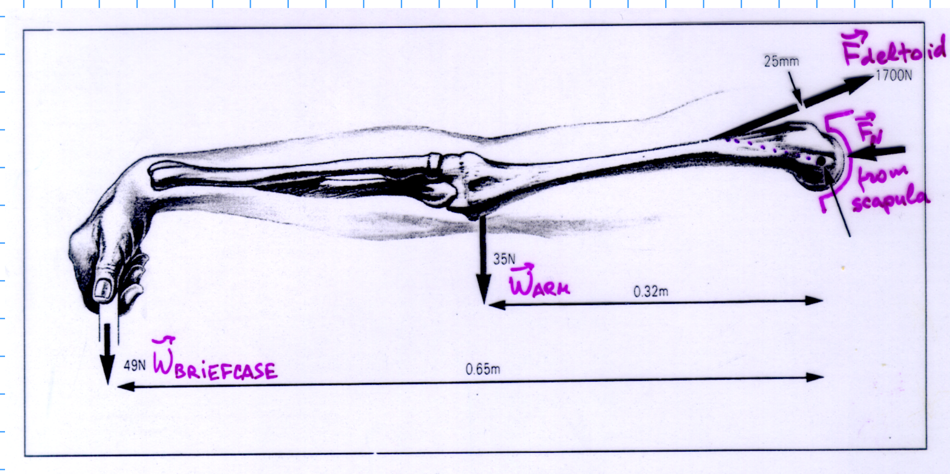
$$\mu_s \cancel{N} = \cancel{N} \tan\beta$$

$$\beta = \arctan \mu_s = \arctan(0.80)$$

Compare $\quad v_T = \sqrt{\mu_s r g} \quad$ vs. $\quad v_T = \sqrt{r g \tan\theta}$

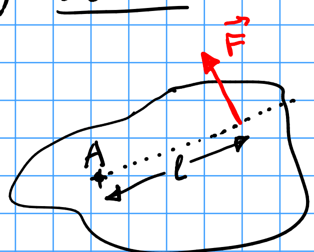
Torque

FIG. 3.1 *The author holding a briefcase (above) and (top) the forces that act on his arm.*



- when $\vec{a} \neq 0 \Rightarrow \exists\ \vec{F} = m\vec{a} \neq 0$
  when $\alpha \neq 0 \Rightarrow \exists\ a_T = \alpha r \neq 0$
  $\Rightarrow \exists$ a torque

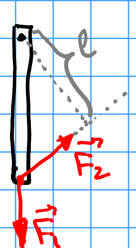
- rotation is always
  1) of a rigid body that has size (cannot "rotate" a point)
  2) about some axis



torque = force × lever arm

$$\tau_A = F\ell$$

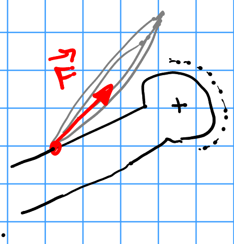
- sign of torque = sense of rotation ; +ve −ve

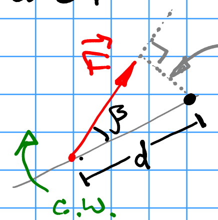
- 

  $F_1 = F_2$
  F_1 does not cause a rotation , $\tau_1 = 0$
  F_2 does $\Rightarrow \tau_2 > 0$

- $\tau_1 = 0$
  $\left.\begin{array}{l}\tau_2 > 0 \\ \tau_3 > 0\end{array}\right\} \tau_3 > \tau_2$

Ex a shoulder



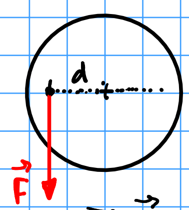


$\ell = d \sin\beta$

$\tau = -Fd\sin\beta$

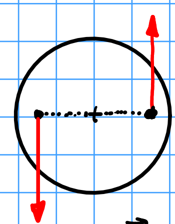
c.w.

Ex a puck on ice + 2 rubber bands



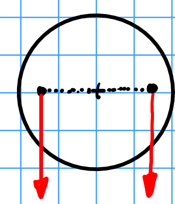
$\sum \vec{F} \neq 0$

$\sum \tau = Fd > 0$



$\sum \vec{F} = 0$

$\sum \tau = 2Fd > 0$



$\sum \vec{F} \neq 0$

$\sum \tau = 0$

- condition of translational eq-m : $\sum \vec{F} = 0$
  —— " —— rotational —— " —— : $\sum \tau = 0$

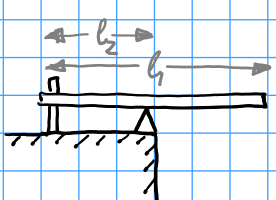
Eq-m, redefined:

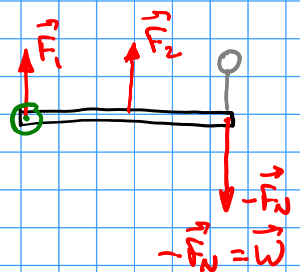
$$\boxed{\begin{array}{l}\sum F_x = 0 \\ \sum F_y = 0 \\ \sum \tau = 0\end{array}}$$

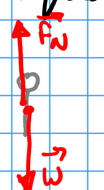
} no translational acceleration

no rotational accelerations

- $\sum \tau = 0$ is a source of equations

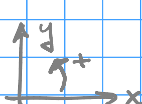






$\vec{F}_N + \vec{W} = 0$

eq-m of diver



$W = 530\,N$

$\ell_1 = 3.90\,m$

$\ell_2 = 1.40\,m$

New: choose the axis of rotation (about which to calculate $\sum \tau = 0$)

Eq-m: $y: \quad +F_1 + F_2 - W = 0 \quad \Rightarrow \quad F_1 = W - F_2 = W - W\dfrac{\ell_1}{\ell_2}$

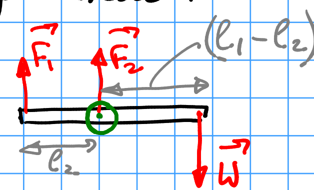
$\tau: \quad +F_2\ell_2 - W\ell_1 = 0 \quad \Rightarrow \quad F_2 = W\dfrac{\ell_1}{\ell_2}$

$$\Rightarrow \quad \underbrace{F_1 = W\left(1 - \frac{l_1}{l_2}\right)} = 530\,N\left(1 - \frac{3.90\,m}{1.40\,m}\right) \simeq \underbrace{-946\,N} \quad -ve!$$

$$F_2 = W\frac{l_1}{l_2} = 530\,N\,\frac{3.90\,m}{1.40\,m} \simeq \underbrace{1.48\times10^3\,N} \quad +ve$$

$\Rightarrow$ F_2 is +ve $\Rightarrow$ true $\vec{F}_2$ is as drawn

F_1 is −ve $\Rightarrow$ true $\vec{F}_1$ is <u>down</u>, opposite of drawn



Alternative solution: τ about fulcrum

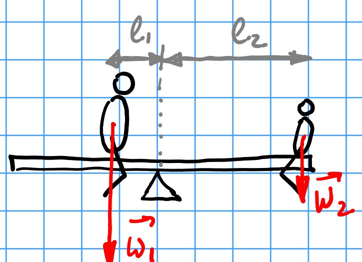
$$y: \quad +F_1 + F_2 - W = \emptyset$$

$$\tau: \quad \underbrace{-F_1 l_2}_{c.w.} \underbrace{- W}_{c.w.}(l_1 - l_2) = \emptyset \Rightarrow F_1 = -W\,\frac{l_1 - l_2}{l_2} = W\,\frac{l_2 - l_1}{l_2}$$

$$\Rightarrow \quad \underbrace{F_1 = W\left(1 - \frac{l_1}{l_2}\right)} \leftarrow \text{same!}$$

EFTS: the tray problem: use the same method, more terms.

- <u>Center of gravity</u>



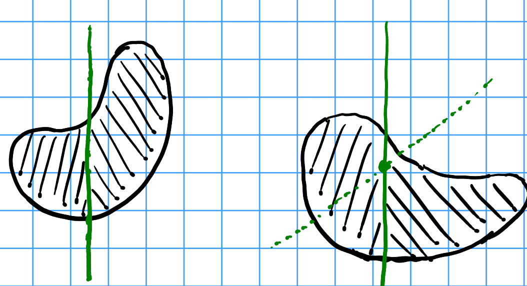
Eq-m: $\sum \tau = \emptyset$ or $\tau_{LHS} + \tau_{RHS} = \emptyset$

$+W_1 l_1 - W_2 l_2 = \emptyset$ or $\tau_{LHS} = -\tau_{RHS}$

$\Rightarrow \quad W_1 l_1 = W_2 l_2$

$$\Rightarrow \quad \boxed{\frac{W_1}{W_2} = \frac{l_2}{l_1}}$$

i.e. a heavier person must sit closer to center

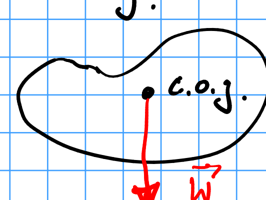


c.o.g. = the point about which $\tau_{RHS} = -\tau_{LHS}$ in <u>every</u> direction

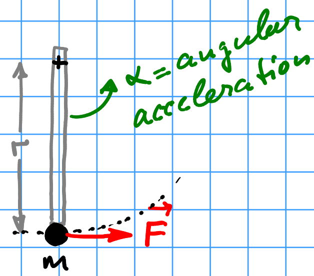
i.e. weight generates no torque about the c.o.g.

In a problem involving $\vec{W}$ do not need to consider its τ_W <u>if</u> all torques are calculated about the c.o.g.

$\Rightarrow$ $\vec{W}$ "acts" on the body at c.o.g.



N2L for rotations



$\vec{F}$ produces tangential $a_T = \frac{F}{m}$

but $a_T = \alpha r = \frac{F}{m}$

$\Rightarrow F = m \alpha r$

Also: $\tau = Fr = m\alpha r^2 = \underbrace{(mr^2)}_{\text{constant for a given body } (m,\, r)}\alpha$

$\Rightarrow$ define $\boxed{I \equiv mr^2}$, a moment of inertia

$\Rightarrow \boxed{\tau = I\alpha}$ N2L for rot.

c.f. $F = ma$: $F \to \tau$, $m \to I$, $a \to \alpha$

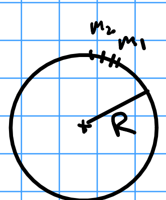
just like F causes object of mass m to have acceleration a, τ causes object of moment of inertia I to have ang. acc. α.

- extended bodies consist of "particles", but α is the same $\forall$

$$\tau_{total} = \tau_1 + \tau_2 + \ldots = (m_1 r_1^2)\alpha + (m_2 r_2^2)\alpha + \ldots$$
$$= [m_1 r_1^2 + m_2 r_2^2 + \ldots]\alpha = I_{body}\,\alpha$$

$$\Rightarrow \boxed{\sum \tau = I_{body}\,\alpha}$$

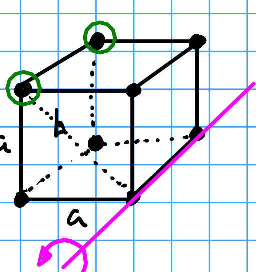
Ex a thin hoop of radius R and mass M



$r_1 = r_2 = \ldots = R$

$I_{hoop} = m_1 r_1^2 + m_2 r_2^2 + \ldots = (m_1 + m_2 + \ldots)R^2 = MR^2$

Ex



$a^2 + a^2 = b^2$

$2a^2 = b^2$

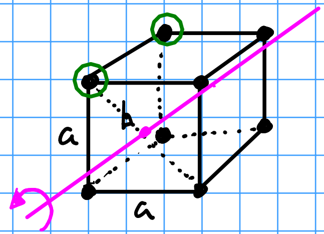
$a = 0.20\,m$

$m = 0.10\,kg$

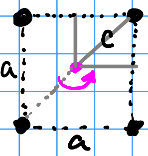
a) axis through one edge of the cube

$$I = \sum_i m_i r_i^2 = 2 \times \emptyset + 4(ma^2) + 2(m\underset{\underset{2a^2}{\parallel}}{b^2})$$

$$\Rightarrow I = 8ma^2 = 8 \times 0.10 \times (0.20)^2$$
$$= 0.0032\ \text{kg m}^2$$



b) axis through the center of the cube



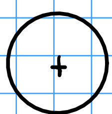
$$c^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 = \frac{a^2}{4} + \frac{a^2}{4} = \frac{a^2}{2}$$ ✓

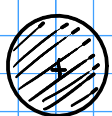
or $c = \frac{1}{2} b \Rightarrow c^2 = \frac{1}{4} b^2 = \frac{1}{4}(2a^2) = \frac{a^2}{2}$ ✓

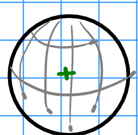
$$I = \sum_i m_i r_i^2 = 8 (m c^2) = 8 m \frac{a^2}{2} = 4 m a^2 = 0.0016 \ kg \cdot m^2$$

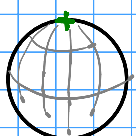
! the moment of inertia depends on where the axis is.

Ex some simple bodies

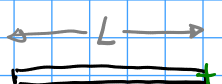

hoop, about the center .... — MR^2


uniform disk, about center .... $\frac{1}{2} MR^2$


uniform sphere, about center .... $\frac{2}{5} MR^2$

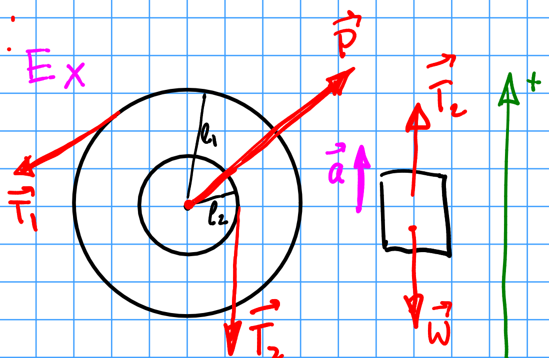

uniform sphere, about a point on surface .... $\frac{7}{5} MR^2$

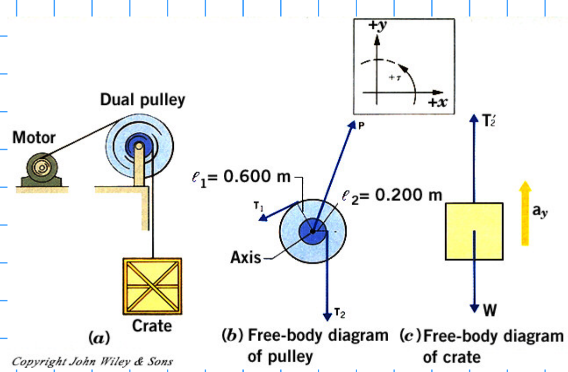

a uniform rod, about midpoint $\frac{1}{12} ML^2$



—— " —— " — endpoint $\frac{1}{3} ML^2$

Ex





(a) (b) Free-body diagram of pulley (c) Free-body diagram of crate

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① $\sum \vec{F} = m\vec{a}$

$$a = \frac{+T_2 - W}{m} \times \frac{g}{g} = \frac{T_2 - W}{W} \cdot g = \left(\frac{T_2}{W} - 1\right) g$$ (I)

② $\sum \vec{F} = \emptyset \Rightarrow$ can find $\vec{P}$

③ $\sum \tau = I\alpha \Rightarrow \alpha = \frac{+T_1 \ell_1 - T_2 \ell_2}{I}$ (II)

2 eq-ns, 3 unknowns

④ "no slipping": $\underset{\sim\sim}{a = \alpha \ell_2}$ ← sub (I) & (II) into this

$$\left(\frac{T_2}{W} - 1\right) g = \frac{T_1 \ell_1 - T_2 \ell_2}{I} \cdot \ell_2 = \frac{T_1 \ell_1 \ell_2}{I} - \frac{T_2 \ell_2^2}{I}$$

$$T_2 \left[\frac{g}{W} + \frac{\ell_2^2}{I}\right] = T_1 \frac{\ell_1 \ell_2}{I} + g$$

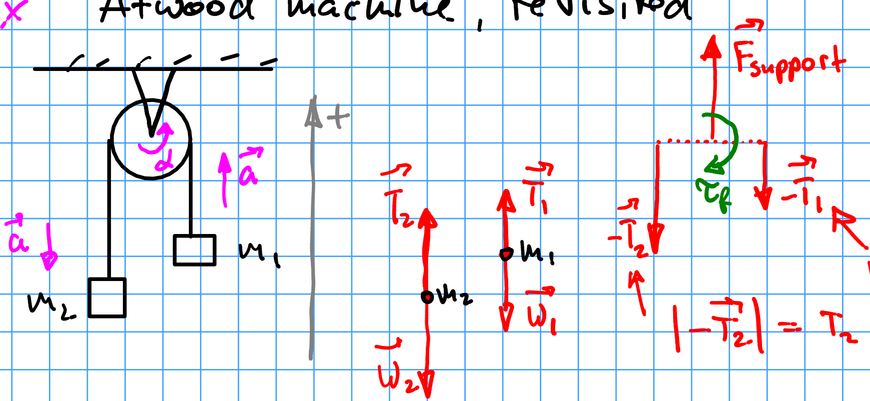
EFTS

$$\underset{\sim\sim}{T_2} = \frac{T_1 \frac{\ell_1 \ell_2}{I} + g}{\frac{g}{W} + \frac{\ell_2^2}{I}} \times \frac{I}{I} = \underset{\sim\sim\sim\sim\sim\sim}{\frac{T_1 \ell_1 \ell_2 + g I}{\frac{g I}{W} + \ell_2^2}} = \ldots \simeq \underset{\sim\sim\sim\sim\sim\sim}{4.96 \times 10^3 \, N}$$

could go to numbers here

$$\underset{\sim\sim}{a} = \left(\frac{T_2}{W} - 1\right) g = \left(\frac{4958.2\ldots}{4420} - 1\right) 9.80 \simeq \underset{\sim\sim\sim\sim}{1.19 \, \frac{m}{s^2}}$$

Ex Atwood machine, revisited



$r = 0.15 m$ (of the pulley)
$m_1 = 0.40 kg$
$m_2 = 0.80 kg$
$m_p = 0.20$
$\tau_f = 0.35 N$

m_1 : $+T_1 - m_1 g = m_1 a$
m_2 : $+T_2 - m_2 g = -m_2 a$
pulley: $\underset{\text{c.c.w.}}{+T_2 r} \underset{\text{c.w}}{- T_1 r} - \tau_f = I\alpha$

3 eq.ns, 3 unknowns

Disk: $I = \frac{1}{2} m_p r^2$

No slipping: $a = \alpha r$, $\alpha = \frac{a}{r}$

physics
algebra

$$(T_2 - m_2 g) - (T_1 - m_1 g) = -m_2 a - m_1 a$$

$$(T_2 - T_1) + (m_1 - m_2) g = -(m_1 + m_2) a$$

$$\underset{\sim\sim\sim\sim}{(T_2 - T_1)} = -(m_1 - m_2) g - (m_1 + m_2) a$$

$$(T_2 - T_1) r - \tau_f = -(m_1 - m_2) g r - (m_1 + m_2) a r - \tau_f = I\alpha = \frac{1}{2} m_p r^{\cancel{2}} \frac{a}{\cancel{r}}$$

$$-(m_1 - m_2) g r - \tau_f = (m_1 + m_2) \underline{a r} + \frac{1}{2} m_p \underline{a r}$$

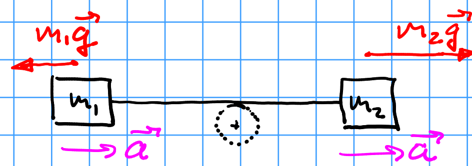
$$-(m_1 - m_2) g r - \tau_f = \left(m_1 + m_2 + \frac{1}{2} m_p\right) a r$$

$$\Rightarrow a = \frac{-(m_1 - m_2) g r - \tau_f}{(m_1 + m_2 + \frac{1}{2} m_p) r} = \frac{-(0.40 - 0.80) \cdot 9.80 \cdot 0.15 - 0.35}{(0.80 + 0.40 + \frac{1}{2} 0.20) \cdot 0.15} = 1.22\ldots \simeq 1.2 \, \frac{m}{s^2}$$

Note: for massless, frictionless pulley: $m_p = \emptyset$, $\tau_f = \emptyset$

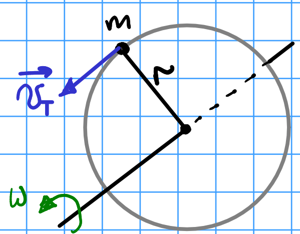
$$\Rightarrow a = \frac{-(m_1 - m_2) g r}{(m_1 + m_2) r} = \frac{m_2 - m_1}{m_2 + m_1} g \quad \checkmark \text{ as before!}$$

which could be seen from:



- rotational kinetic energy

Ex of a particle



$v_T = r\omega$

$$K_R = \frac{1}{2} m v_T^2 = \frac{1}{2} m (r\omega)^2 = \frac{1}{2} \underbrace{m r^2}_{=I} \omega^2$$

$$\boxed{K_R = \frac{1}{2} I \omega^2}$$

again: rotation $\begin{Bmatrix} I & \leftrightarrow & m \\ \omega & \leftrightarrow & v \end{Bmatrix}$ translation

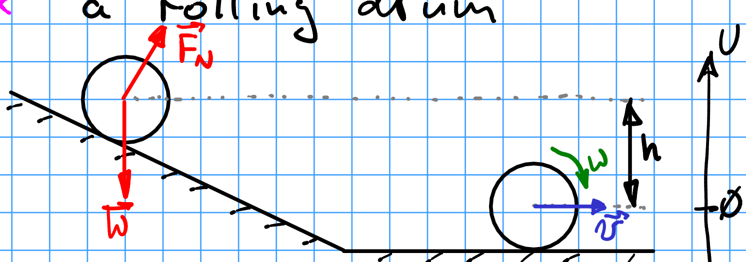
Ex of an extended body:

$$K_R = \sum_i \frac{1}{2} m_i v_i^2 = \sum_i \frac{1}{2} m_i r_i^2 \underbrace{\omega^2}_{\text{same } \forall i} = \frac{1}{2}\left(\sum_i m_i r_i^2\right) \omega^2$$

$K_R = \frac{1}{2} I_{body} \, \omega^2$ where $I_{body} = \sum_i m_i r_i^2$

general result!

Ex a rolling drum



No (rolling) friction, $F_{gravity}$ is conservative

$\Rightarrow E_{total} = \text{const}$

As usual: no slipping: $v = \omega r$

$\Downarrow$

$\omega = \frac{v}{r}$

$$E_{top} = mgh = E_{bottom} = \frac{1}{2} m v^2 + \frac{1}{2} I_{drum} \, \omega^2$$

$$\Rightarrow mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \left(\frac{v}{r}\right)^2 = \frac{1}{2}\left(m + \frac{I}{r^2}\right) v^2$$

$$v^2 = \frac{2mgh}{m + I/r^2} = \frac{2m}{m + I/r^2} gh$$

Empty drum : $I = mr^2 \Rightarrow v = \sqrt{\frac{2m}{m+m} gh} = \sqrt{gh}$

Full drum : $I = \frac{1}{2} mr^2 \Rightarrow v = \sqrt{\frac{2m}{m + \frac{1}{2}m} gh} = \sqrt{\frac{4}{3} gh}$