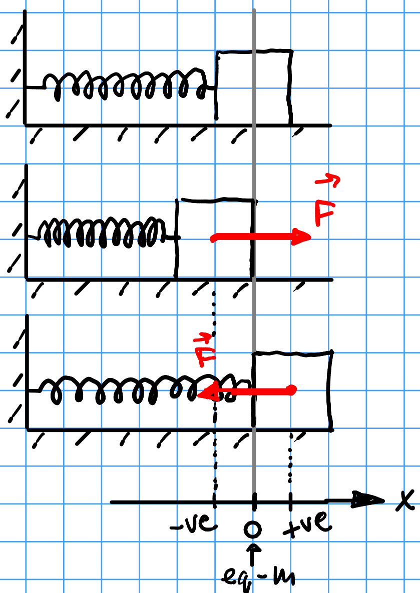
Oscillations and Waves

Simple harmonic oscillator (SHO)

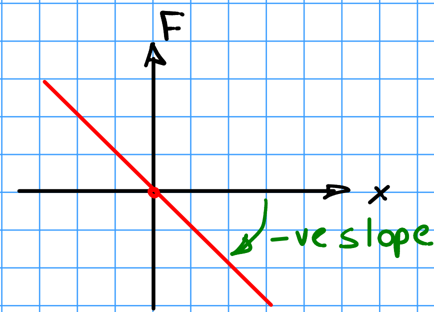
- an ideal spring



$x = \emptyset$, $F = \emptyset$

$x < \emptyset$, $F > \emptyset$

$x > \emptyset$, $F < \emptyset$

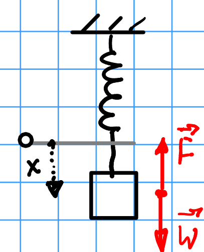


Hooke's law :

$$\boxed{F = -kx}$$

k = spring constant

Ex a spring stretches 0.030 m when an 8.0-kg object is suspended from it. By how much will the spring stretch when it is used to suspend 4.0 kg ?



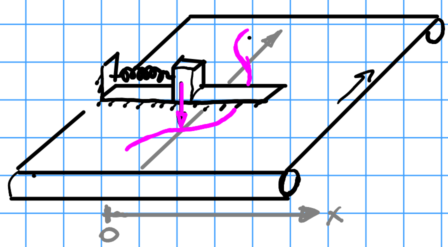
$$\left.\begin{array}{l} F = -kx \\ W = -mg \end{array}\right\} \quad \vec{F} + \vec{W} = \emptyset \Rightarrow -kx - mg = \emptyset \Rightarrow kx = -mg$$

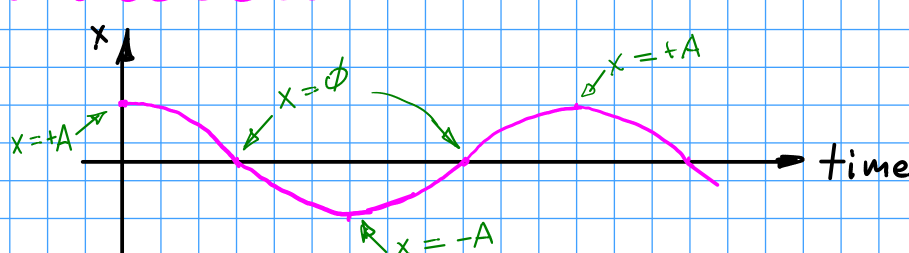
$$\Rightarrow k = -\frac{m_1}{x_1} g = \ldots \quad \text{(could do)}$$

$$\Rightarrow x_2 = -\frac{m_2}{k} g = \frac{m_2 g}{\left(\frac{m_1 g}{x_1}\right)} = x_1 \frac{m_2}{m_1} = -0.030\,\text{m} \times \frac{4.0\,\cancel{\text{kg}}}{8.0\,\cancel{\text{kg}}} = -0.015\,\text{m}$$

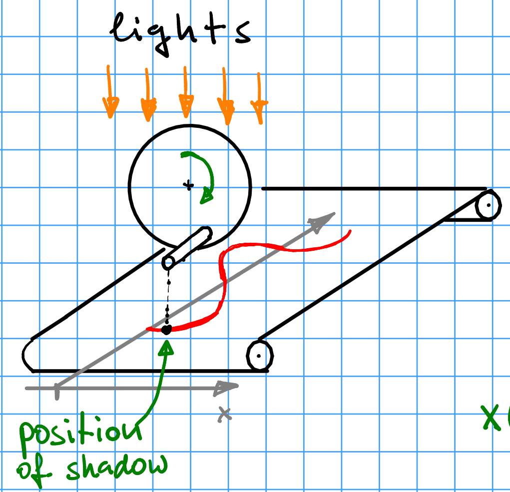
! Could have guessed, since $x \propto F \propto W \propto m$

- for an ideal spring ($F = -kx$) the resulting motion is oscillatory : a simple harmonic oscillator (SHO)

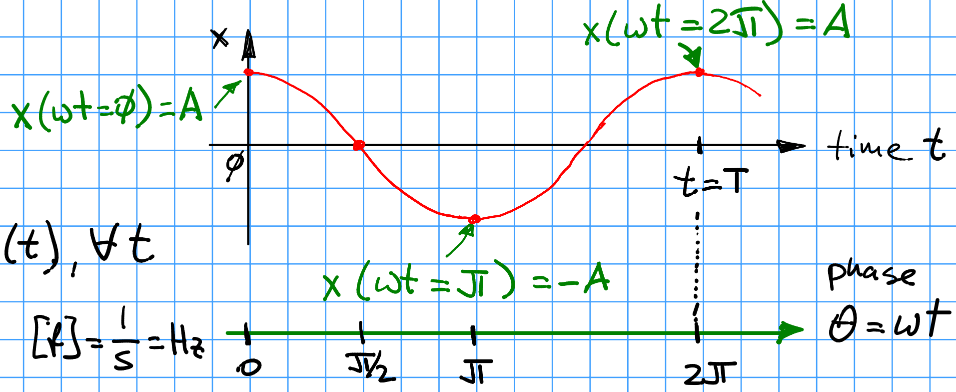




cf



Peg's shadow yields a similar $x(t)$ graph $\Rightarrow$ the same motion as the [x-component of] uniform circular motion:



period T: $x(t+T)=x(t),\ \forall t$

frequency f: $f\equiv\frac{1}{T}\quad [f]=\frac{1}{s}=Hz$

phase [angle] $\theta=\omega t$

Here $\theta=\omega t\Big|_{t=T}=\omega T=2\pi$

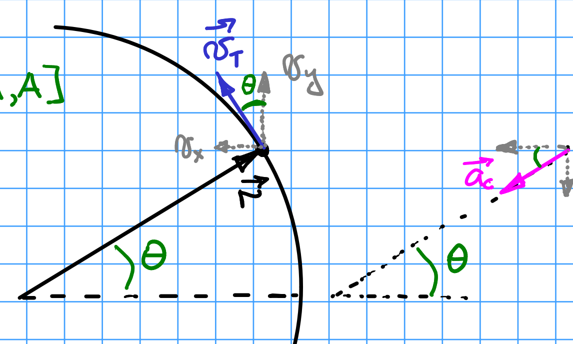
angular frequency: $\boxed{\omega=\frac{2\pi}{T}=2\pi f}$ $\qquad [\omega]=\frac{rad}{s}$

$x\in[-A,A]$

tangential velocity: $v_T=r\omega=A\omega$

$v_x=-v_T\sin\theta$

$=-A\omega\sin\omega t$



centripetal acceleration: $a_c=\frac{v_T^2}{A}=A\omega^2$

$a_x=-a_c\cos\theta=-A\omega^2\cos\omega t$

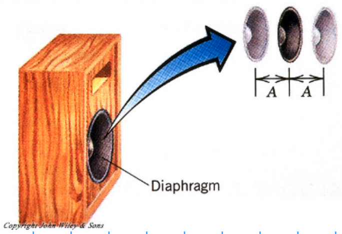
- equations of motion (for both Uniform Circular Motion <u>and</u> SHO):

$$\boxed{\begin{aligned} x&=A\cos(\omega t)\\ v&=-A\omega\sin(\omega t)\\ a&=-A\omega^2\cos(\omega t)\end{aligned}}$$

definitely not! $a=$ const $\not\Rightarrow$ $\begin{cases} a\neq \text{const}\\ v\neq v_0+at\\ x\neq x_0+v_0t+\frac{1}{2}at^2\end{cases}$

- origin in calculus
- analogous to the x-component in uniform circular motion
- periodic in t (period $=T$), <u>or</u> in $\theta=\omega t$ (period $=2\pi$)

Ex. a speaker diaphragm



The diaphragm of a loudspeaker moves back and forth in simple harmonic motion to create sound, as shown in the drawing. The frequency of the motion $f = 1.0$ kHz, and the amplitude $A = 2.0 \times 10^{-4}$m.

Find the maximum acceleration of the diaphragm.

$f = 1.0\,\text{kHz} = 1\times10^3\,\text{Hz}$

$A = 2.0\times10^{-4}\,\text{m} \quad (0.2\,\text{mm})$

$$\max\{a\} = \max\{-A\omega^2\cos(\omega t)\} = A\omega^2 = A(2\pi f)^2$$
$$= 2.0\times10^{-4}\,\text{m} \times 4\pi^2 \times \left(1.0\times10^3\,\tfrac{1}{s}\right)^2 = \ldots \simeq 7.9\times10^3\,\text{m/s}^2$$

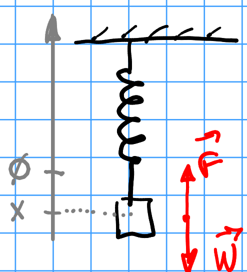
- Force $F = ma = -kx$ (for SHO)

$$m(-\cancel{A}\omega^2\cancel{\cos(\omega t)}) = -k(\cancel{A}\cancel{\cos(\omega t)})$$

$$\omega^2 = \frac{k}{m} \qquad \boxed{\omega = \sqrt{k/m}}$$

spring constant & mass $\Updownarrow$ frequency of oscillations

Ex.



Suppose an object on a vertical spring oscillates up and down at a frequency 5.00 Hz. By how much would this object, hanging at rest, stretch the spring?

$f = 5.00\,\text{Hz} \qquad \omega = 2\pi f = \sqrt{k/m}$

At rest: $-kx - mg = \emptyset \Rightarrow x = -\frac{mg}{k} = -\frac{g}{(k/m)}$

$$\Rightarrow x = -\underbrace{\frac{g}{(2\pi f)^2}} = -\frac{9.80\,\text{m}/\cancel{s^2}}{4\pi^2 \times 5.00^2\,\frac{1}{\cancel{s^2}}} = \ldots = -9.93\times10^{-3}\,\text{m}$$
$$= \underbrace{-9.93\,\text{mm}}$$

Note the sign: $x < \emptyset$

- Energy

potential energy $U = \begin{cases} \emptyset & @\, x = \emptyset \\ W(x) & @\, x \end{cases}$ where $W(x)$ is the work done in moving from $\emptyset$ to x.

But: $W = F\Delta x$ and $F = F(x) \rightarrow$ need average value $\overline{F}$

In fact, $\vec{F} \uparrow\downarrow \vec{\Delta x}$ (recall, $F = -kx$) $\Rightarrow W = F\Delta x\cos(180) = -F\Delta x$

$$\Rightarrow W(x) = -\overline{F}\Delta x = -\tfrac{1}{2}[\cancel{F(\emptyset)}^{\emptyset} + F(x)]x = -\tfrac{1}{2}(-kx)x = \tfrac{1}{2}kx^2$$

$$\Rightarrow \left.\begin{aligned} U &= \tfrac{1}{2}kx^2 \\ K &= \tfrac{1}{2}mv^2 \end{aligned}\right\} \quad E = K + U = \tfrac{1}{2}kx^2 + \tfrac{1}{2}mv^2 = \text{const}$$

verify: $E = \tfrac{1}{2}k(A\cos(\omega t))^2 + \tfrac{1}{2}m(-A\omega\sin(\omega t))^2$

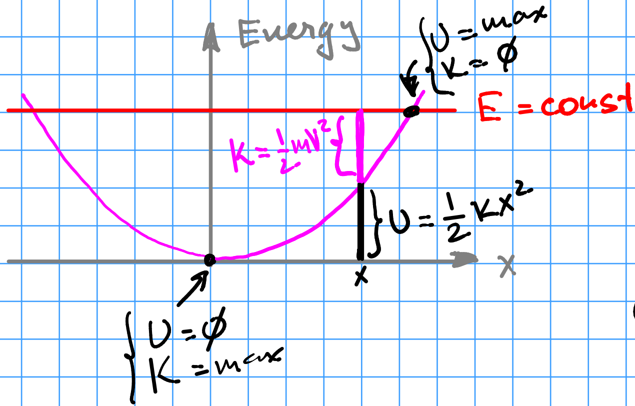
$$= \frac{1}{2} kA^2 \cos^2(\omega t) + \frac{1}{2} m A^2 \omega^2 \sin^2(\omega t)$$

$$\omega^2 = k/m$$

$$= \frac{1}{2} kA^2 \cos^2(\omega t) + \frac{1}{2} A^2 k \sin^2(\omega t) = \frac{1}{2} kA^2 = \text{const}$$

Ex A spring is compressed 0.0800 m and is used to launch an object horizontally with a speed of 2.40 m/s. If the object were attached to the spring, at what angular frequency (in rad/s) would it oscillate?

where is $v = v_{max}$?



@ $x = x_{max}$	@ $x = \phi$
$U_{max} = \frac{1}{2} k x_{max}^2$	$U = \phi$
$K = \phi$	$K = \frac{1}{2} m v_{max}^2$

Conservation of energy:

$$\frac{1}{2} k x_{max}^2 = \frac{1}{2} m v_{max}^2$$

$$\frac{k}{m} = \frac{v_{max}^2}{x_{max}^2} \quad \text{but} \quad \frac{k}{m} = \omega^2$$

$$\Rightarrow \omega = \frac{v_{max}}{x_{max}} = \frac{2.40\ m/s}{0.0800\ m} = 30.0\ rad/s \quad \text{, or}$$

$$f = \frac{\omega}{2\pi} = 4.77\ \frac{1}{s} = 4.77\ Hz$$

Waves in elastic media

Oscillations : stationary periodic displacements

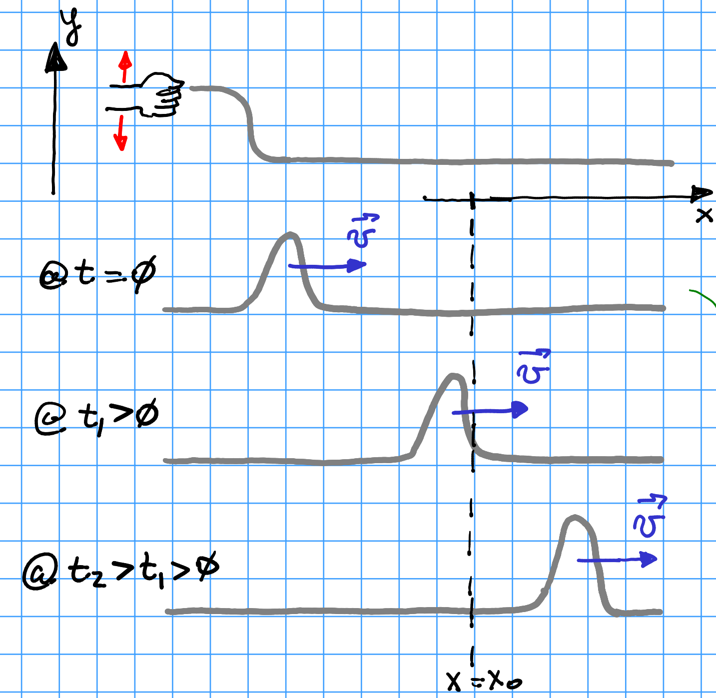
Waves : traveling < periodic or non-periodic > displacements

Ex
- transverse ($\perp$ to travel direction)
- longitudinal ($\parallel$ to travel direction)

} waves on a slinky

Ex longitudinal & transverse waves in the same medium travel at different speeds – earthquakes, scorpions.

Ex a non-periodic transverse wave (a pulse)

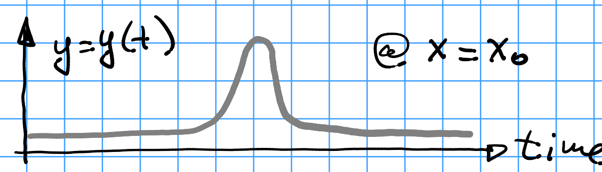


$y(x,t)$ = displacement of the element of the string located at x, at time t

$\vec{v}$ = velocity of propagation of the wave

$y=y(x)$ snapshots for given times

But, also:

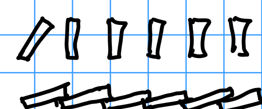


for a given location x_0

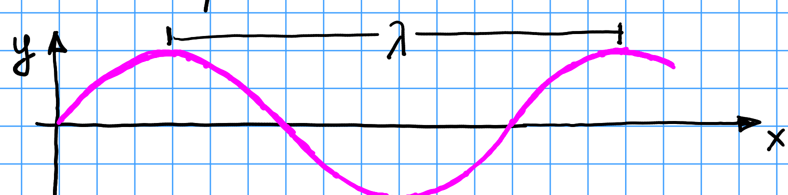
Each element of the string, in turn, experiences an up-and-down motion, but stays at the same x — yet the wave travels!

There is long-range transport of energy and momentum without large-scale displacement of the element of the medium

Ex. the falling dominoes analogy:



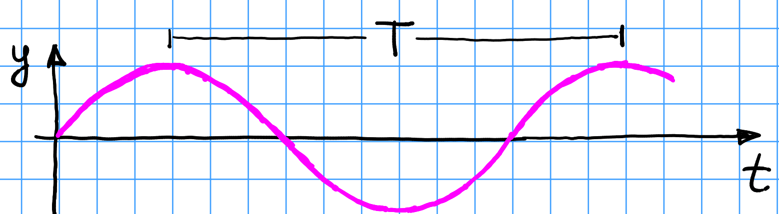
Ex a periodic transverse wave



$y=y(x)$, @ t = fixed

λ = wavelength, a repeat distance in x

$y(x+\lambda)=y(x), \forall x$



$y=y(t)$, @ $x=x_0$

T = period, a repeat distance in t

$y(t+T)=y(t), \forall t$

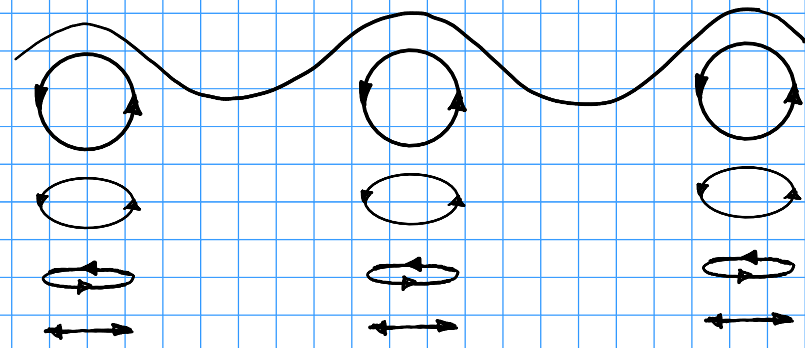
v = velocity of the wave $=\dfrac{\lambda}{T}=\lambda f$ $T=\frac{1}{f}$

Ex "A sinusoidal water wave, with 1.6 m between crests, laps againsth the pier every 4.0 s." $\Rightarrow \lambda = 1.6\,\text{m}, \quad T = 4.0\,\text{s}$

$$\Rightarrow \quad f=\frac{1}{T}=\frac{1}{4.0\,\text{s}}=0.25\,\text{Hz}, \quad v=\frac{\lambda}{T}=\frac{1.6\,\text{m}}{4.0\,\text{s}}=0.40\,\frac{\text{m}}{\text{s}}$$

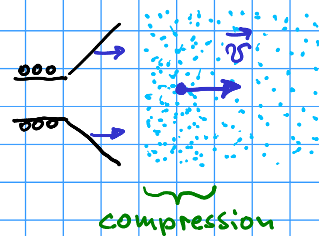
Ex water waves

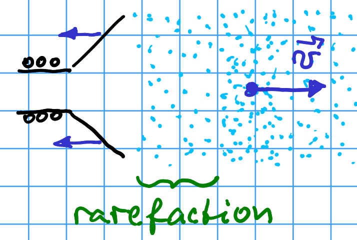
- near surface: a combination of longitudinal and transverse motions
- at depth, essentially longitudinal



Ex sound waves in air

- longitudinal
- a series of peaks and valleys of pressure





Mathematics of waves

$y = A \cos\theta$, θ = phase of the wave

- phase changes in time, by 2π every period, by $\left(2\pi \frac{t}{T}\right)$ at time t

  Note: $2\pi \frac{t}{T} = 2\pi f t = \omega t$

- phase changes in space, by 2π every wavelength λ

  $\Rightarrow$ by $\left(2\pi \frac{x}{\lambda}\right)$ at location x

Thus: $\boxed{y = A \cos\left(\frac{2\pi}{T} t - \frac{2\pi}{\lambda} x\right)}$ is the most general wave equation

- why the minus sign?

  as t increases, x must increase for the phase to remain the same.

  $\Rightarrow$ the point of same phase (wavefront) propagates in the +ve direction (−ve direction for a plus sign, $\ldots \underline{+} \frac{2\pi}{\lambda} x$)

- Note: $\theta = \frac{2\pi}{T} t - \frac{2\pi}{\lambda} x = \frac{2\pi}{\lambda}\left(\frac{\lambda}{T} t - x\right) = \frac{2\pi}{\lambda}(v t - x)$

  where $v = \lambda / T$

- the principle of superposition

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

Demo using the applet

Ex same amplitude and wavelength

$$\left.\begin{aligned} y_1 &= A\cos\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right) \\ y_2 &= A\cos\underbrace{\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right)}_{\Delta\theta = \emptyset} \end{aligned}\right\} \quad y = y_1 + y_2 = 2A\cos\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right)$$

same phase $\Rightarrow$ constructive interference

Ex when two waves do not start at the same place, or at the same time, there is a fixed phase difference between them

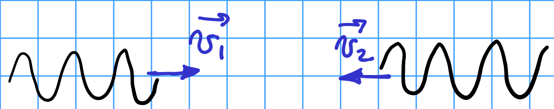
$$\Rightarrow y = A\cos\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right) + A\cos\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x + \Delta\theta\right)$$

Recall : $\cos(\pi + \alpha) = -\cos\alpha$

$\Rightarrow$ for $\Delta\theta = \pi$, $y = \emptyset$ everywhere! $\rightarrow$ destructive interference

e.g. noise canceling earphones

Ex



$$y = A\cos\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right) + A\cos\left(\frac{2\pi}{T}t + \frac{2\pi}{\lambda}x\right)$$

$$\Rightarrow y = 2A\cos\left(\frac{2\pi}{T}t\right)\cos\left(\frac{2\pi}{\lambda}x\right)$$

recall:

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\begin{aligned}&\cos(\alpha+\beta)\\ &+\cos(\alpha-\beta)\end{aligned} = 2\cos\alpha\cos\beta$

This is a standing wave of the same T and λ.

For example, when $\cos\left(\frac{2\pi}{\lambda}x\right) = \emptyset$, $y = \emptyset$ $\forall t$ $\rightarrow$ a node, a point that is stationary at all times.

Note:

$$y = 2A\cos\left(\frac{2\pi}{T}t\right)\cos\left(\frac{2\pi}{\lambda}x\right)$$

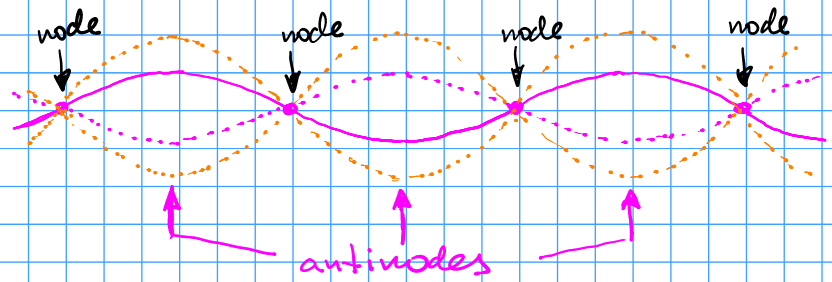
time- and space-dependence decoupled

e.g. @ $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4} \ldots$ $\quad \cos\left(\frac{2\pi}{\lambda}x\right) = \emptyset$, $\forall t$ $\rightarrow$ nodes

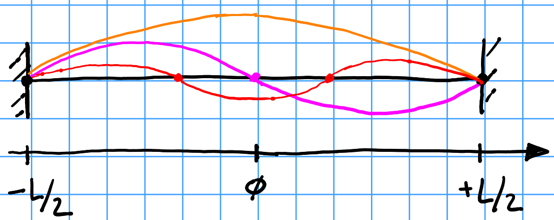
@ $x = \emptyset, \frac{\lambda}{2}, \lambda, \ldots$ $\quad \cos\left(\frac{2\pi}{\lambda}x\right) = \pm 1$ $\rightarrow$ antinodes (max of intensity)

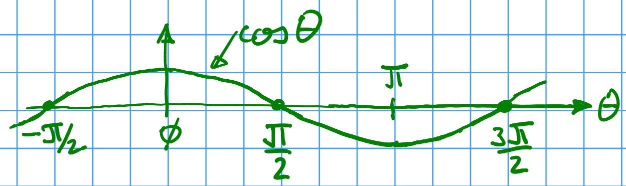
@ antinodes, y oscillates (with time) between $-2A$ and $+2A$

On the other hand, $\forall t$ $y = (\dots)\cos\left(\frac{2\pi}{T}t\right)$, i.e. the entire string oscillates together, though the amplitude $(\dots)$ depends on x.



Ex string instruments, part I





Length of the string, L, selects the wavelength of the standing wave.

To match the boundary conditions, need

$$y = 2A\cos\omega t \cdot \cos\frac{2\pi}{\lambda}x = \phi$$

at $x = \pm\frac{L}{2}$, $\forall t$

$$\Rightarrow \cos\left(\frac{2\pi}{\lambda}\frac{L}{2}\right) = \cos\left(-\frac{2\pi}{\lambda}\frac{L}{2}\right) = \phi$$

$$\Rightarrow \pm\frac{2\pi}{\lambda}\frac{L}{2} = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \dots$$

$$\Rightarrow \frac{2L}{\lambda} = 1, 3, 5, \dots \quad \Rightarrow \quad L = \underset{\substack{\uparrow \\ \text{fundamental} \\ \text{nodes only at the ends}}}{\frac{\lambda}{2}}, \underbrace{\frac{3\lambda}{2}, \frac{5\lambda}{2}, \dots}_{\substack{\text{overtones} \\ \text{nodes in the interior}}}$$

Ex. string instruments, part II

To create a standing wave, need to match the speed of the wave, so that the phase difference $\Delta\theta = \text{const}$ for the forward and reflected wave. This happens when:

$$vt\Big|_{t=\frac{T}{2}, T, \frac{3T}{2}, 2T, \frac{5T}{2}, \dots} = L$$

$$\Rightarrow v n\frac{T}{2} = L \quad \text{or} \quad v n\frac{1}{2f} = L$$

$$\Rightarrow f = n\frac{v}{2L}, \quad n = 1, 2, 3, \dots$$

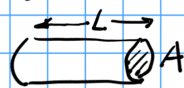
$n = 1$ is the fundamental

Also: https://radiopaedia.org/cases/resonance-imparted-periostitis-rip

Speed of wave on a string $v = \sqrt{\frac{F}{m/L}}$ where F = tension in the string, m/L = linear mass density, so can adjust v by changing the tension.

<u>Summary</u> 1) tension (and, hence, v), and
2) length

matter for creating a standing wave of just the right frequency. Pitch is changed by adjusting L (e.g. frets on the guitar) and F (e.g. tuning pegs on guitar/piano)

- In general, properties of the medium determine v, e.g. for transverse waves on a string, $v = \sqrt{\frac{F}{m/L}}$. Now, since mass $m = \underset{\text{density}}{\rho}\,\underset{\text{volume}}{V} = \rho L A$  $\Rightarrow$ linear density $\frac{m}{L} = \rho A$

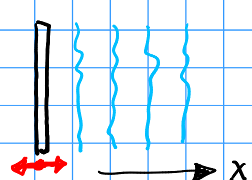
$$\Rightarrow \quad v = \sqrt{\frac{F}{\rho A}},$$ waves propagate faster in a thinner string (and $f \propto v$, so pitch is higher)

- waves transport energy

Ex power of a sound wave

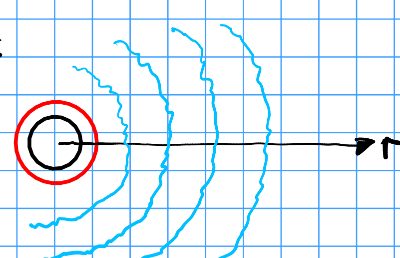
$\left.\begin{array}{l} P = \text{power} \\ A = \text{area} \end{array}\right\}$ $I \equiv \frac{P}{A}$ = intensity of the sound, $[I] = \frac{W}{m^2}$

- planar wave



$A = \text{const}$, $I = \text{same}\ \forall x$

- spherical wave



$$A = 4\pi r^2$$

$$I = \frac{P_{source}}{4\pi r^2} = I(r)$$

a function of distance from the source.

$$\Rightarrow \quad \frac{I_1}{I_2} = \frac{P/4\pi r_1^2}{P/4\pi r_2^2} = \left(\frac{r_2}{r_1}\right)^2$$

e.g. $r_2 = 2r_1 \Leftrightarrow I_2 = \frac{1}{4} I_1$

Ex. humans hear a very wide range of sound intensities

$I_{min} = 10^{-12}\ \frac{W}{m^2}$ ← threshhold of hearing

$I_{max} = 10^{0}\ \frac{W}{m^2}$ ← threshhold of pain

$\frac{I_{max}}{I_{min}} = 10^{12}$ (!!)

For such a wide range, it is convenient to use a log scale

1) reference to threshhold of hearing:

$I_0 = I_{min} = 1 \times 10^{-12}\ \frac{W}{m^2}$ ⟷ "zero" Bels, ∅B (after A.G.Bell)

2) define "intensity level" as

$\log\left(\frac{I}{I_0}\right) = \log I - \log I_0$ ⟷ e.g. $I_{max} = 1\ \frac{W}{m^2}$

$\log\left\{\frac{1\ \frac{W}{m^2}}{10^{-12}\frac{W}{m^2}}\right\} = 12\,B$

A more convenient unit is decibel, dB:

$$\boxed{\beta = 10 \log \frac{I}{I_0}}\ ,\ \text{in dB}$$

recall: $10^{\log x} = x$

Note 1: $[\beta] = 1$ (dimensionless, a ratio)

∅dB	20dB	50dB	90dB	110dB	180dB	210dB
hearing threshhold	whisper	average home	subway train	rock concert	shuttle launch	loudest sound ever made

→ pain

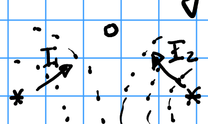
Note 2: dB scale approximately reflects how human ears perceive loudness: it takes at least $I_2 = 10 I_1$ (i.e. +10dB) for us to notice a significant difference in loudness.

E.g. doubling the intensity:

$$\beta_2 - \beta_1 = 10 \log \frac{I_2}{I_0} - 10 \log \frac{I_1}{I_0} = 10 \log \frac{I_2/I_0}{I_1/I_0} = 10 \log \frac{I_2}{I_1} \quad (I_2 = 2I_1)$$

$$= 10 \log 2 \simeq 3\,dB$$ is not much louder.

Note 3:



$I_{total} = I_1 + I_2$ but $\beta_{total} \neq \beta_1 + \beta_2$

$$\beta_{total} = 10 \log \frac{I_1 + I_2}{I_0} \neq 10 \log \frac{I_1}{I_0} + 10 \log \frac{I_2}{I_0} = 10 \log \frac{I_1 \cdot I_2}{I_0^2}$$

Ex An amplified guitar has a sound intensity level that is 14 dB greater than the same unamplified sound. What is the ratio of the amplified intensity to the unamplifeid intensity?

$$\beta_2 - \beta_1 = 10 \log \frac{I_2}{I_0} - 10 \log \frac{I_1}{I_0} = 10 \log \frac{I_2/I_0}{I_1/I_0} = 10 \log \frac{I_2}{I_1} = 14\,dB$$

$$\Rightarrow \frac{I_2}{I_1} = 10^{\frac{\beta_2 - \beta_1}{10}} = 10^{1.4} \simeq 25$$

$$I_2 \simeq 25\, I_1$$

recall: $\log(AB) = \log A + \log B$

$\log \frac{A}{B} = \log A - \log B$

Ex a log-scale comparison

flute: $\beta_1 = 67\,dB$

rock concert: $\beta_2 = 115\,dB$

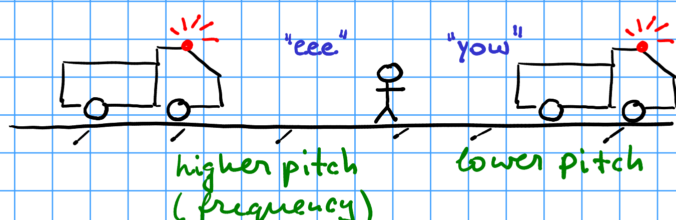
$\beta_2 - \beta_1 = 48\,dB \Rightarrow \frac{I_2}{I_1} = 10^{4.8} = 63 \times 10^3$

i.e. need >63,000 flutes to drown out a rock band

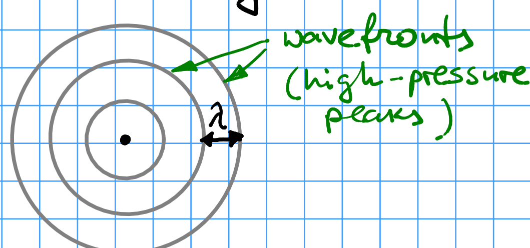
- since β is a ratio, dB is often used to describe a change, or a relationship between two intensities, not necessarily of sound. E.g. "gain of 20 dB", or "noise floor of −90 dB", etc.

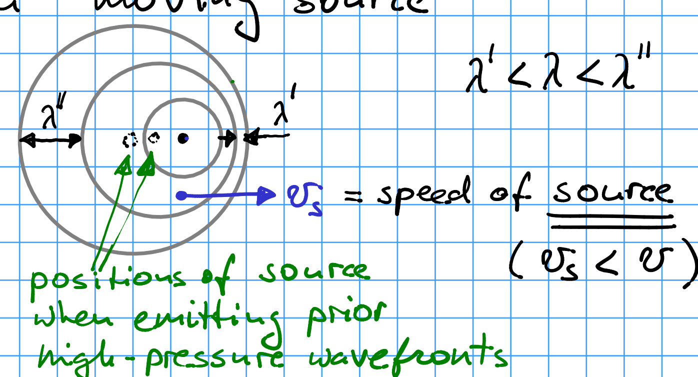
Doppler effect

- a moving source



– a stationary source vs. a moving source





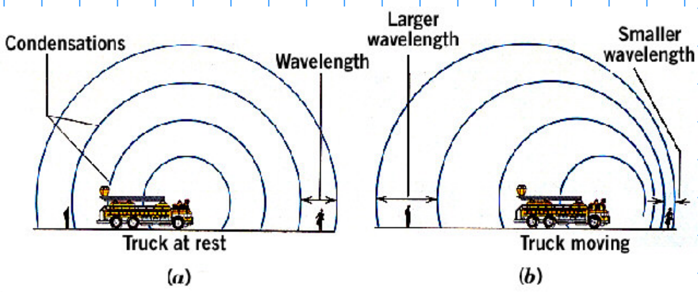
wavelength $\lambda = vT = \frac{v}{f} \Leftrightarrow f = \frac{v}{\lambda}$

where v = speed of sound

By the time the source emits the next wavefront, it has moved the distance $v_s T$

$\Rightarrow$ apparent wavelength $<$ $\lambda'' = \lambda + v_s T$, source receding

$\lambda' = \lambda - v_s T$, source approaching



Combining both cases:

$$\lambda' = \lambda \pm v_s T = vT \pm v_s T$$
$$= (v \pm v_s) T = \frac{v \pm v_s}{f}$$

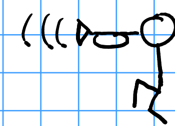
Thus the observer hears $\boxed{f = \frac{v}{\lambda'} = \frac{v}{v \pm v_s} f_s}$

$\oplus$ receding source, $f < f_s$ "yaw"

$\ominus$ approaching source, $f > f_s$ "eee"

or $\boxed{f = \frac{f_s}{1 \pm v_s/v}}$

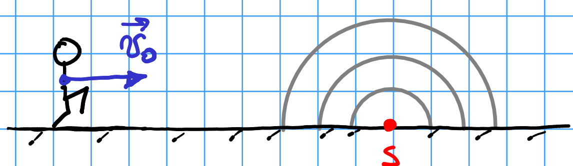
Ex. At a football game, a stationary spectator is watching the halftime show. A trumpet player in the band is playing a 784-Hz tone while marching directly toward the spectator at a speed of 0.830 m/s. On a day when the speed of sound is 343 m/s, what frequency does the spectator hear?



$v_s = 0.830 \, \frac{m}{s}$ $\quad f_s = 784 \, Hz$ $\quad v = 343 \, \frac{m}{s}$

$$\Rightarrow f = \frac{784 \, Hz}{1 - \frac{0.831 \, \cancel{m/s}}{343 \, \cancel{m/s}}} \simeq 786 \, Hz$$

- a moving observer



v_o = speed of the observer

Observer encounters extra waveforms, $\frac{v_o}{\lambda}$ of them per second

$$\Rightarrow f = f_s + \frac{v_o}{\lambda} = f_s + f_s \frac{v_o}{v} = f_s \left(1 + \frac{v_o}{v}\right)$$

$\lambda = v T_s = \frac{v}{f_s}$

If the observer is moving away from the source: $\frac{v_o}{\lambda}$ fewer wavefronts per second, i.e. $f = f_s \left(1 - \frac{v_o}{v}\right)$

$$\Rightarrow \boxed{f = f_s \left(1 \pm \frac{v_o}{v}\right)}$$

Note: – source moving changes the wavelength of the wave
– observer moving does not! – relative to ground, $\lambda = \frac{v}{f_s}$

Ex. riding away from the siren

A motorcycle starts from rest and accelerates along a straight line at 2.81 m/s^2, on a day when the speed of sound is 343 m/s. A siren is located at the starting point and remains stationary. How far has the motorcycle gone when the driver hears the frequency of the siren at 90% of the value it has when the motorcycle is stationary.

$$f = f_s\left(1-\frac{v_o}{v}\right) = \frac{9}{10} f_s \Rightarrow 1-\frac{v_o}{v} = \frac{9}{10} \Rightarrow \frac{v_o}{v} = \frac{1}{10} \Rightarrow v_o = \frac{1}{10} v$$

The rider moves with constant acceleration, so

$$v^2 = v_{initial}^2 + 2ad \quad , \text{ where } v_{initial} = \emptyset, \; v = v_o \; (o = \text{observer})$$

$$\Rightarrow d = \frac{v_o^2}{2a} = \frac{\left(\frac{1}{10} v\right)^2}{2a} = \frac{343^2 \, \frac{\cancel{m^2}}{\cancel{s^2}}}{100 \cdot 2 \cdot 2.81 \, \frac{\cancel{m}}{\cancel{s^2}}} \simeq 209\,m$$

- both source and observer moving

$$\boxed{f = f_s \frac{1 \pm \frac{v_o}{v}}{1 \mp \frac{v_s}{v}}}$$

upper sign: toward (s, o) "eee"

lower sign: away from (s, o) "yow"

where v = speed of sound in air
v_o = speed of observer
v_s = speed of source
f_s = frequency of source

Ex red-shifted starlight (i.e. not just sound waves)