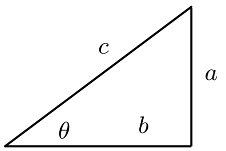
## Mathematics

quadratic equation, $ax^2 + bx + c = 0$ :

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

trigonometry: $a^2 + b^2 = c^2$




$$\begin{aligned} \sin\theta &= a/c \\ \cos\theta &= b/c \\ \tan\theta &= a/b \\ \tan\theta &= \frac{\sin\theta}{\cos\theta} \end{aligned}$$

circular arc ($\theta$ in rad): $s = \theta r$
for a full circle, $\theta = 2\pi$ and $s = 2\pi r$

## Fundamental Constants

$g =$ 9.80 m s$^{-2}$
$c =$ speed of light $= 2.99792458 \times 10^8 \approx 3.00 \times 10^8$ ms$^{-1}$
$m_e =$ mass of electron $= 9.11 \times 10^{-31}$ kg
$m_p =$ mass of proton $= 1.6726 \times 10^{-27}$ kg
$m_n =$ mass of neutron $= 1.6749 \times 10^{-27}$ kg
$G =$ gravit. constant $= 6.674 \times 10^{-11}$ N m$^2$kg$^{-2}$
$R_E =$ radius of Earth $= 6.376 \times 10^6$ m
$M_E =$ mass of Earth $= 5.9736 \times 10^{24}$ kg
$R_L =$ radius of Moon $= 1.74 \times 10^6$ m
$M_L =$ mass of Moon $= 7.35 \times 10^{22}$ kg
$T_L =$ orbital period $= 2.36 \times 10^6$ s
$R_{E-L} =$ Earth-Moon distance $= 3.84 \times 10^8$ m
$R_S =$ radius of Sun $= 6.96 \times 10^8$ m
$M_S =$ mass of Sun $= 1.99 \times 10^{30}$ kg
$R_{E-S} =$ Earth-Sun distance $= 1.496 \times 10^{11}$ m

## Mechanics

uniform linear acceleration $a =$ const:

$$\begin{aligned} v &= v_0 + at \\ \bar{v} &= {}^1\!/_2\,(v_0 + v) \\ \Delta x = d &= \bar{v}t = {}^1\!/_2\,(v_0 + v)t = v_0 t + {}^1\!/_2\,at^2 \\ v^2 &= v_0^2 + 2ad \end{aligned}$$

Newton's Laws:

N2L: $\vec{a} = \frac{1}{m}\vec{F}_{\text{total}}$

N3L: $\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$

Gravity: $F = G\,\frac{m\,m'}{r^2}$

friction:

$$\begin{aligned} f_s &\leq f_s^{(max)} = \mu_s F_N \\ f_k &= \text{const} = \mu_k F_N, \qquad \mu_k < \mu_s \end{aligned}$$

kinematics of uniform angular acceleration:
(angular displacement $\Delta\theta$, velocity $\omega$, acceleration $\alpha$)

$$\begin{aligned} \omega &= \omega_0 + \alpha t \\ \overline{\omega} &= {}^1\!/_2\,(\omega + \omega_0) \\ \Delta\theta &= \overline{\omega}t = {}^1\!/_2\,(\omega + \omega_0)t = \omega_0 t + {}^1\!/_2\,\alpha t^2 \\ \omega^2 &= \omega_0^2 + 2\alpha\,\Delta\theta \end{aligned}$$

rolling without slipping: $d = s = \Delta\theta r$, $v_T = \omega r$, $a_T = \alpha r$
centripetal force and acceleration (for $\alpha = 0$):

$$F_c = ma_c = m\,\frac{v_T^2}{r} = m\omega^2 r$$

torque $\tau =$ force $\times$ lever arm $= Fl$
moment of inertia:

$$I = mr^2 \quad \text{or} \quad I_{\text{body}} = \textstyle\sum_i m_i r_i^2$$

about an axis through the geometrical centre:

$$I_{\text{hoop}} = mr^2 \quad I_{\text{disk}} = \frac{1}{2}\,mr^2 \quad I_{\text{rod}} = \frac{1}{12}\,ml^2$$

$$I_{\text{sphere}} = \frac{2}{5}\,mr^2 \quad I_{\text{sph.shell}} = \frac{2}{3}\,mr^2$$

N2L for rotations

$$\tau = I\alpha \quad \text{or} \quad \textstyle\sum\tau = I_{\text{body}}\alpha$$

work $\mathcal{W}_t = Fd$, or $\mathcal{W}_r = \tau\,\Delta\theta$
work done by a conservative force:

$$\mathcal{W}_{\text{c.f.}}(\vec{r} \to \vec{r}\,') = -\left[U(\vec{r}\,') - U(\vec{r})\right] = -\Delta U(\vec{r})$$

gravitational potential energy

$$F_{\text{gravity}} = -mg \quad \rightsquigarrow \quad U_g = mgy$$

kinetic energy of translation: $K_t = \frac{1}{2}\,mv^2$
kinetic energy of rotation: $K_r = \frac{1}{2}\,I\omega^2$
conservation of total mechanical energy

$$\Delta\,(U + K_t + K_r) = \mathcal{W}_{\text{non-c.f.}}$$

power = rate of doing work $= \mathcal{W}/\Delta t = Fv$ or $= \tau\omega$
1 Watt = 1 J/1 s and 1 h.p. = 746 W

N2L in terms of linear momentum $\vec{p} = m\vec{v}$

$$\text{Impulse} = \overline{\vec{F}}\,\Delta t = \Delta\vec{p} \quad \text{or} \quad \vec{F} = \lim_{\Delta t \to 0}\frac{\Delta\vec{p}}{\Delta t}$$

conservation of momentum for a system as a whole:

$$\sum \vec{F}_{\text{ext}} = \lim_{\Delta t \to 0}\frac{\Delta\vec{p}_{\text{total}}}{\Delta t} = 0 \;\rightsquigarrow\; \vec{p}_{\text{total}} = \text{const}$$

elastic collision ($\Delta E = 0$): C.o.M. **and** C.o.K.E.

$$\begin{aligned} \vec{v}_{f1} &= \vec{v}_{i1}\,\frac{m_1 - m_2}{m_1 + m_2} + \vec{v}_{i2}\,\frac{2m_2}{m_1 + m_2} \\ \vec{v}_{f2} &= \vec{v}_{i1}\,\frac{2m_1}{m_1 + m_2} + \vec{v}_{i2}\,\frac{m_2 - m_1}{m_1 + m_2} \end{aligned}$$

inelastic collision: C.o.M. **only**

$$\vec{v}_f = \vec{v}_{i1}\,\frac{m_1}{m_1 + m_2} + \vec{v}_{i2}\,\frac{m_2}{m_1 + m_2}$$

angular momentum $\vec{L} = I\vec{\omega}$, and $\vec{\tau} = \lim_{\Delta t \to 0}\frac{\Delta\vec{L}}{\Delta t}$
conservation of angular momentum:

$$\vec{\tau}_{\text{net}} = \sum_i \vec{\tau}_i = 0 \;\rightsquigarrow\; \vec{L}_{\text{total}} = \text{const} \;\rightsquigarrow\; I_1\omega_1 = I_2\omega_2$$

## Waves and Sound

simple harmonic oscillator, $F = -kx$, $U = {}^1\!/_2\,kx^2$

$$f = 1/T \quad \omega = 2\pi f = \sqrt{k/m}$$

$$\begin{cases} x = A\cos\theta = A\cos\omega t \\ v = -A\omega\sin\omega t \\ a = -A\omega^2\cos\omega t \end{cases}$$

travelling wave $y = A\cos(\omega t \mp \frac{2\pi}{\lambda}x)$ with $v = \lambda/T = \lambda f$
string of mass $m$, length $l$, volume density $\rho$:

$$v_{\text{string}} = \sqrt{F/(m/l)} = \sqrt{F/(\rho A)}$$

intensity level: $\beta = 10\,\log\frac{I}{I_0}$ dB, $I_0 = 1 \times 10^{-12}$ W/m$^2$
point source intensity: $I = \frac{P}{A} = \frac{P}{4\pi r^2}$
Doppler (upper sign = approach, lower = recede)

$$f = f_s\left(\frac{1 \pm \frac{v_o}{v}}{1 \mp \frac{v_s}{v}}\right)$$

$v$ = speed of sound
$v_s$ = speed of source
$v_o$ = speed of observer