Brock University Physics Department

St. Catharines, Ontario, Canada L2S 3A1

PHYS 1P91 Laboratory Manual

Physics Department

Contents

Data fitting and error analysis

During this first lab session you will meet your lab-mates and lab demonstrators, and to learn about the `physicalab` software that you will use to collect, analyze, and plot data in all subsequent labs. We will also review essential data analysis concepts, and introduce you to error-analysis calculations.

The Cartesian coordinate system for plotting data

The Cartesian coordinate system specifies each point in a two-dimensional grid by a unique pair of coordinates, x and y. The horizontal x-axis and the vertical y-axis cross at the point $(0, 0)$, known as the origin of the coordinate system.

By marking the x and y axes at regular intervals determined by the data to be plotted, the plane is divided into a scaled array of intersecting lines, as shown in Figure 1. To plot a point at coordinates (x, y),

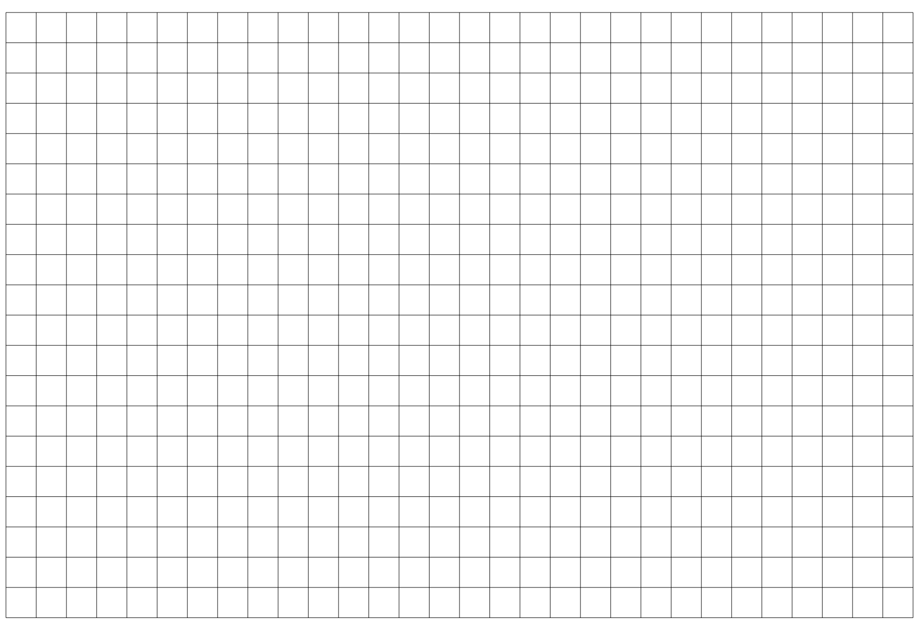


Figure 1: Cartesian coordinate grid

move horizontally along the x-axis to position x and then vertically parallel to the y-axis to position y.

What is curve fitting?

Suppose that you perform an experiment and collect some data. How would you analyse this set of measurements, or extract useful conclusions from t? For example, you might expect that the measured variables are related in some specific way; how would you decide whether the data supports the hypothesized relationship?

One can review the data and make a qualitative judgement regarding the trend that the data follows. Curve fitting attempts to quantify the behaviour of a set of data points by determining a function that represents this behaviour.

The fitting process uses a trial function that includes one or more variable parameters, called fitting parameters. The fitting parameters are adjusted by the fitting algorithm to yield a curve of best fit. The curve of best fit does not necessarily pass through any of the data points, although it might do so. Instead, the curve of best fit is placed so that a certain kind of average distance from the curve to the points is minimized. (You might like to think about what kind of average might be helpful.)

The failure of a fitted curve to pass through the data points is typically due to uncertainties associated with the values of these data points; e.g., due to the limited precision of a measuring instrument, or the presence of noise.

A mis-fit may also occur if the fit function is inappropriate for this particular data. For example, trying to fit the equation of a straight line to a periodic data set of pendulum oscillations would yield meaningless results.

Once the data is properly fitted, the fitting formula can be used to evaluate various properties inherent in the data. For example, fitting a sine function to a series of points that describe the periodic change in distance with time of a pendulum bob allows the fitting algorithm to precisely determine the period and amplitude of the pendulum oscillation.

Graphing and fitting to a linear data set

A linear data set consists of a series of coordinate points (x, y) where the relationship between the x and y cordinates is always such that a change Δy in the variable y is directly proportional to a change Δx in the variable x.

This relationship can be represented by a function $y = m * x + b$. This is the equation of a straight line with constant slope $m = \Delta y/\Delta x$. The y-intercept b is the value of y when $x = 0$.

Plotting and fitting a linear data set by hand

Here are some (x, y) coordinate points that follow a linear trend:

$$(0.2, 2.4)\ ,\ (1.1, 3.6)\ ,\ (2.2, 4.4)\ ,\ (3.1, 5.7)\ ,\ (4.1, 6.6)\ ,\ (4.9, 7.3)\ ,\ (6.0, 8.4)$$

Proceed as follows to plot these points on a Cartesian grid, then sketch an estimated line of best fit by hand, and finally determine the slope and y-intercept of your estimated line of best fit.

1. Review the data set and determine the x-coordinate minimum and maximum values. Subtract the minimum value from the maximum value, then divide this difference by the number of horizontal grid divisions. Round up the result, if necessary, to get a nice step increment that you can apply to the horizontal axis. (i.e., $x_{max} - x_{min} = 6.0 - 0.2 = 5.8$, $5.8/30 = 0.193$, so use 0.2 for the step size)

   Label the bottom of the grid with these incremental step values, starting from the left side, so that all the x values fit on the grid. (i.e., $x_{min} = 0.2$ so start with 0.2, then 0.4, 0.6, etc.)

2. Repeat the above procedure using the y-coordinates, dividing by the number of vertical grid divisions and labelling the left edge of the grid. Data graphs need not include the origin $(0, 0)$.

3. With the grid now scaled, plot each of the data points onto the grid. Carefully estimate the position of points that do not lie exactly on the intersection of two grid lines.

   You will note that the points appear to follow a linear behaviour; they appear to lie on a straight line. This observation gives you the hint that if you were going to try and fit some function to this data set, then the a straight line might be a good choice.

4. Now use a straight edge to draw a straight line that you think best estimates your data points, extending it past the left and right edges of the grid.

? How did you choose where to place your *line of best fit*?

5. Estimate the coordinates of these two edge points (x_1, y_1) on the left edge and (x_2, y_2) on the right edge, then determine the slope and y-intercept of your line of best fit, as described above. This choice of points will typically yield the least relative error.

$$\begin{aligned} \textit{slope}:\ m &= \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} &&= \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots \\ y - \textit{intercept}:\ b &= y_1 - m x_1 \equiv y_2 - m x_2 &&= \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}$$

Plotting and fitting a linear data set using `Physicalab`

Now that you have estimated a line of best fit by hand, use the `Physicalab` software to determine a more precise line of best fit for the same set of data.

- In `Physicalab`, click **File**, then **Load linear data** to place some points in the data window.

- Select **fit to: y=** and enter **A*x+B** in the fitting equation box then click **Draw** to generate a graph of the data with a straight line fitted to it.

  The slope **A** and y-intercept **B** are the *fitting parameters* adjusted by the fitting algorithm to yield the equation of the line of best fit that is drawn over your data points. The resulting values of the fitting parameters appear below the graph.

  Note that the points do not lie exactly on the line. This total difference between the y-values of the points and the y-values predicted by the fitted line is used by the fitting program to determine the uncertainty in the values of the fit parameters.

  The uncertainty, or error, in **A** is denoted by δ**A** and the error in **B** is denoted by δ**B**. These results are typically displayed in the form **A**$\pm\delta$**A**, **B**$\pm\delta$**B**. The smaller the error values, the better the fit of the equation to the data set; if all the points were to lie exactly on the line, then these uncertainties would be zero.

? Record below the resulting values from the `Physicalab` fit of the linear data set. How do these results compare with your fit estimates for the slope and the y-intercept?

$$\begin{aligned} \textit{slope}:\ \mathbf{A} &= \ldots\ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots\ldots \\ y - \textit{intercept}:\ \mathbf{B} &= \ldots\ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}$$

Graphing and fitting to a periodic data set

A data set that is periodic has y values that repeat regularly after an interval in x. Many such periodic data sets can be fitted to a sine function, or sine wave. The basic sine function has formula $y = \sin(x)$, and a general sine function has formula $y = A\sin(Bx + C) + D$. Do you recall from high school how changing the values of A, B, C, and D influence the graph?

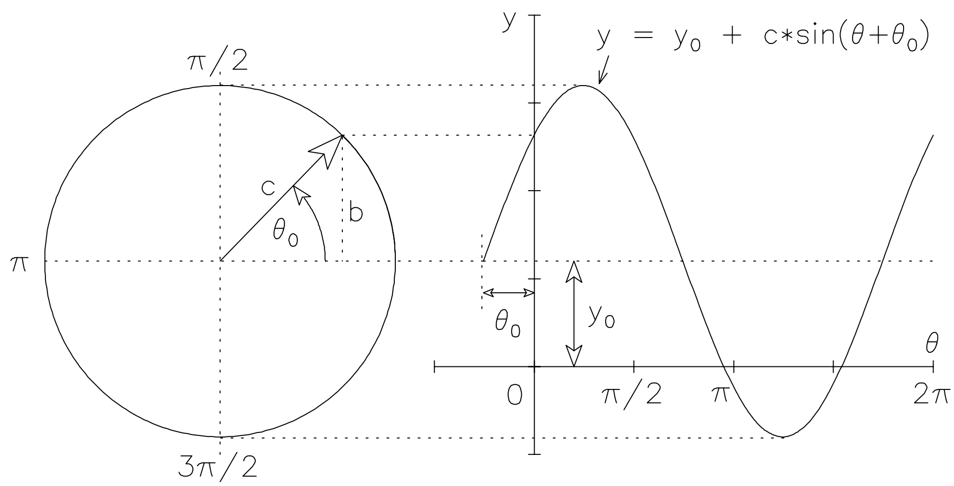


Figure 2: Projection of a circular motion to an xy-plane to generate a sine wave

Consider the radius vector of magnitude c that describes the circumference of a circle, as shown in Figure 2. Recall that a circle of radius r has a circumference of $2\pi r$. If we sweep the radius vector along this circumference by an arc distance equal to the circle radius, then the angle θ is increased by one radian. If we increase θ at a constant rate ω from 0 to 2π radians and plot the magnitude of the vertical line segment $b = c\ \sin\theta$ as a function of θ, a sine wave of amplitude c and period of 2π radians is generated.

You can see this in Figure 2 if you imagine a spotlight shining on the radius vector from the left so that the radius vector casts a shadow on a screen located towards the right (say, on the y-axis). As the radius vector rotates in a circle, the shadow of its tip just oscillates up and down on the screen. The location of the shadow of the tip of the radius vector is plotted on the sine curve at the right of the figure as time passes.

The variable $\omega = \Delta\theta/\Delta t$, represents the angular velocity of the radius vector as it rotates in a circle on the left diagram of Figure 2. This same quantity determines how quickly the sine wave evolves in time. For motions that are periodic, but are not circular, the quantity ω is called the angular frequency. An example of such a motion is the back-and-forth motion of the shadow of the tip of the radius vector.

If the sine wave advances by one cycle then the angle θ changes by $\Delta\theta = 2\pi$ radians. The time needed for the sine wave to advance by one cycle is called the period of the motion. Thus, the period T is related to the angular frequency ω by

$$\omega = \frac{2\pi}{T} \qquad \text{which is equivalent to} \qquad T = \frac{2\pi}{\omega}$$

Relative to some arbitrary coordinate system, in this case the xy-axes shown, the sine wave is shifted by an *offset distance* of y_0 from the horizontal axis and by a *phase angle* of θ_0 from the vertical axis. In this (θ, y) coordinate system, the sine wave is represented by the formula $y = c\sin(\theta + \theta_0) + y_0$. Expressing the formula using ω we get

$$y = c\sin(\omega t + \theta_0) + y_0 \tag{1}$$

`Physicalab` uses the same format, but different symbols, as follows:

$$y = A\sin(Bx + C) + D$$

By referencing Figure 2 you will be able to provide graphical interpretations for the `Physicalab` fitting parameters A, B, C, and D.

Fitting to a sine wave using `Physicalab`

- Click **File**, then **Load sinusoidal data**. This is some sample data obtained from the pendulum experiment (Experiment 1) that you will perform later in the course.

- Click **Draw** to generate a graph of the data. Always check that the data points follow the general shape of the function that you are going to fit to the data. In this case, the set of points should resemble a nice smooth sine wave, without spikes, stray points or flat spots.

- Select **fit to: y=** and enter **A*sin(B*x+C)+D** in the fitting equation box. Comparing this equation with Equation 1, we see that **x** represents the independent variable (typically in units of time), **A** is the amplitude of the sine wave, **C** is the initial phase angle (in radians) of the wave when $\mathbf{x} = 0$, and **D** is the offset distance of the wave measured from the x-axis, so that when $\mathbf{A} = 0$, $y = \mathbf{D}$.

  The fit parameter **B** (in radians/s) is the rate of change in angle with time, so that **B*x** is an angle in radians. If this angle is advanced by 2π radians, so that $\mathbf{B}{*}\mathbf{x} = 2\pi$, then the time changes by an amount $\mathbf{x} = T$, where T represents the period of the sine wave in seconds. Then $\mathbf{B} = 2\pi/T$.

- Look at your graph and enter some reasonable *approximate* values for the fitting parameters. You can get an initial guess for **B** by estimating the time **x** between two adjacent minima, or one period, of the sine wave and approximate the value of $2\pi = 6.2831\ldots$ with the number 6.

$$\begin{aligned} amplitude: &\quad \mathbf{A} \approx \ldots\ldots\ldots\ldots\ldots\ldots \\ angular\ velocity: &\quad \mathbf{B} \approx \ldots\ldots\ldots\ldots\ldots\ldots \\ initial\ phase\ angle: &\quad \mathbf{C} \approx \ldots\ldots\ldots\ldots\ldots\ldots \\ offset: &\quad \mathbf{D} \approx \ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}$$

- Click **Draw**. If you get an error message the initial guesses for the fitting parameters may be too distant from the required values for the fitting program to properly converge. Try again.

Introduction to the analysis of uncertainty (error analysis)

Beware! In general conversation, the term **error** usually refers to some sort of **mistake**. In sciences the term **error** refers specifically to the **uncertainty** δX in the **magnitude** of a measured quantity X. A measured quantity X without an associated error δX is meaningless, as there is no way to establish how reliable the value of X is. A proper result is expressed as a pair of numbers $X \pm \delta X$.

A sample data set

Table 1 contains a sample data set obtained from the Pendulum experiment. In the experiment, the length s of a string supporting a ball of diameter d that acts as the pendulum bob is adjusted and the length s is

Run, i	mass	m (kg)	d (m)	s (m)	L (m)	B (rad/s)	g_i (m/s^2)
1	m_1	0.0225	0.02540	0.300	0.3127	5.59641±0.00267	9.79370
2	m_1	0.0225	0.02540	0.450	0.4627	4.60396±0.00224	9.80760
3	m_1	0.0225	0.02540	0.600	0.6127	4.00688±0.00163	9.83695
4	m_1	0.0225	0.02540	0.750	0.7627	3.58703±0.00182	9.81350
5	m_1	0.0225	0.02540	0.900	0.9127	3.27880±0.00229	9.81201
1	m_2	0.0095	0.01904	0.500	0.5095	4.38192±0.00240	9.78344

Table 1: Table of experimental results

measured and recorded. The pendulum length L is the string length plus the radius of the ball: $L = s + \frac{1}{2}d$. (Note that the radius of the ball is half of its diameter.)

The pendulum ball is set swinging and the motion is recorded and fitted to a sine function. The fit parameter $\mathbf{B}$ is obtained from the fit and the acceleration due to gravity g is obtained from $g = \mathbf{B}^2 L$.

There were five trials done using a ball of mass m_1 and a single trial using a ball of mass m_2. The balls have a different diameter d. We will use this data to determine by different methods values for g and their associated errors δg.

Determining the uncertainty in a single measurement

Because there was only one trial using the ball of mass m_2, the only option is to use error propagation rules to determine error estimates for L and g.

First, we need to determine the magnitude of the uncertainties in the measured values of the string length s and the diameter of the ball d. The **measurement errors** in s and d, represented by δs and δd respectively, are determined from the **precision** of the scales of the measuring instruments. This error is expressed as ± one-half of the smallest increment, or **resolution**, of the scale used to make the measurement. The scale used to set the string length s had a resolution of 0.001 m, while the micrometer used to measure the ball diameter d had a scale increment, of 0.00001 m. The errors are:

$$\delta s = \pm \ldots\ldots\ldots\ldots\ldots \qquad\qquad \delta d = \pm \ldots\ldots\ldots\ldots\ldots$$

The proper way to show a calculation is in three steps: first display the relevant equation, then replace the variables by their unrounded values and finally show the numerical result. Do not include units. The error equation for L is obtained from the error rules in the Appendix. Then for m_2,

$$L = s + \tfrac{1}{2}d = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\delta L = \sqrt{(\delta s)^2 + \left(\tfrac{1}{2}\delta d\right)^2} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Once these two values are obtained, the final result for $L \pm \delta L$, is obtained as follows:

1. round the error term, in this case δL, to one significant digit. Round up this digit by adding one to it if the digit that followed it was greater than four, i.e. 0.0149 → 0.01 and 0.0150 → 0.02;

2. round the result, in this case L, to the same precision (or decimal place) as the error term, rounding up this digit if the digit that followed it was greater than four.

   For example, if $\delta X = 0.014$ and $X = 4.354$ then $X \pm \delta X = 4.35 \pm 0.01$. If $\delta X = 0.015$ and $X = 4.355$ then $X \pm \delta X = 4.36 \pm 0.02$.

Finally, use the format below to show the properly rounded pair of values, with the appropriate units:

$$L = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$$

To get a final result for g using the ball of mass m_2, the error equation for $g = \mathbf{B}^2 L$ is needed. This time it is a good idea to perform a change of variables to determine δg.

Note that the error equation is a product of two terms, $\mathbf{B}^2$ and L. Let $A = \mathbf{B}^2$ then $g = AL$. Using the power rule and the product rule respectively:

$$F = X^2 \longrightarrow \frac{\delta F}{F} = 2\frac{\delta X}{X}, \qquad F = XY \longrightarrow \frac{\delta F}{F} = \sqrt{\left(\frac{\delta X}{X}\right)^2 + \left(\frac{\delta Y}{Y}\right)^2} \tag{2}$$

we replace the variables and substite the $\delta A/A$ term in the g error equation:

$$A = \mathbf{B}^2 \longrightarrow \frac{\delta A}{A} = 2\frac{\delta \mathbf{B}}{\mathbf{B}}, \qquad g = AL \longrightarrow \frac{\delta g}{g} = \sqrt{\left(\frac{\delta A}{A}\right)^2 + \left(\frac{\delta L}{L}\right)^2} = \sqrt{\left(2\frac{\delta \mathbf{B}}{\mathbf{B}}\right)^2 + \left(\frac{\delta L}{L}\right)^2} \tag{3}$$

$$g \quad = \quad \mathbf{B}^2 L = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\delta g \quad = \quad g\sqrt{\left(\tfrac{2\delta \mathbf{B}}{\mathbf{B}}\right)^2 + \left(\tfrac{\delta L}{L}\right)^2} = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

Finally,

$$g = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$$

Determining the uncertainty from a series of measurements

Note: the symbol Δ is traditionally used to represent a difference of two values, as with $\Delta x = x_2 - x_1$, whereas the symbols δ and σ represent the uncretainty associated with a result.

In Table 1, there are five trials ($i = 1, \ldots, N = 5$) using the large ball of mass m_1. These five results for g are *expected to have the same value.* In this case, you can invoke the theory of statistics to evaluate a sample average $\langle g \rangle$, or mean value, of the five trials as well as the standard deviation of the sample $\sigma(g)$ that gives a measure of the scattering of the trials around $\langle g \rangle$.

The sample average of N values g_i is given by the sum of the samples divided by the number of samples:

$$\langle g \rangle = \frac{1}{N}\sum_{1}^{N} g_i = \frac{g_1 + g_2 + \ldots + g_N}{N} \tag{4}$$

To get a feel for what is involved, let's perform a manual standard deviation calculation.

- To begin, we use Equation 4 to calculate $\langle g \rangle$.
- Then for each g_i we calculate the difference Δg_i and $(\Delta g_i)^2$.

i	g_i	$\Delta g_i = g_i - \langle g \rangle$	$(\Delta g_i)^2$
1			
2			
3			
4			
5			
$\langle g \rangle =$		$variance =$ $\sigma(g) = \sqrt{variance} =$	

Table 2: Calculation template for g and $\sigma(g)$.

- The variance of the sample is a sum of the $(\Delta g_i)^2$ terms, this time divided by $N - 1$:

$$\text{variance} = \sigma(g)^2 = \frac{1}{N-1} \sum_{1}^{N} (\Delta g_i)^2 = \frac{(\Delta g_1)^2 + (\Delta g_2)^2 + \ldots + (\Delta g_N)^2}{N-1}.$$

- Finally, we determine the standard deviation $\sigma(g)$ from the variance in the sample of g values:

$$g = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$$

- You can also enter the five g values as a column in `Physicalab`, then from the **Edit** menu select **Insert X Index** to add a column of index values. Check **bellcurve** to view your data as a distribution.

  Compare the mean and standard deviation values from the graph with your results from Table 2. They should be the same.

  ....

- To view a distribution of g values with a larger number of samples, click **File** and **Load distribution data**. Check the **bargraph** box to display the number of samples in each bin as a series of vertical bars. See how the graph changes as you partition the data into a different number of bins.

$$g = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$$

  How does this result for a larger data sample compare with the result for a sample of five g values?

  ....

(TA INITIALS)

FIRST NAME (PRINT) LAST NAME (PRINT) BROCK ID (AB17CD) (LAB DATE)

Experiment 1

The pendulum

A simple pendulum consists of a compact object of mass m suspended from a fixed point by a string of length L, as shown in Fig. 1.1. The gravitational force exerted on the object of mass m is $F = mg$, where g is the acceleration due to gravity.

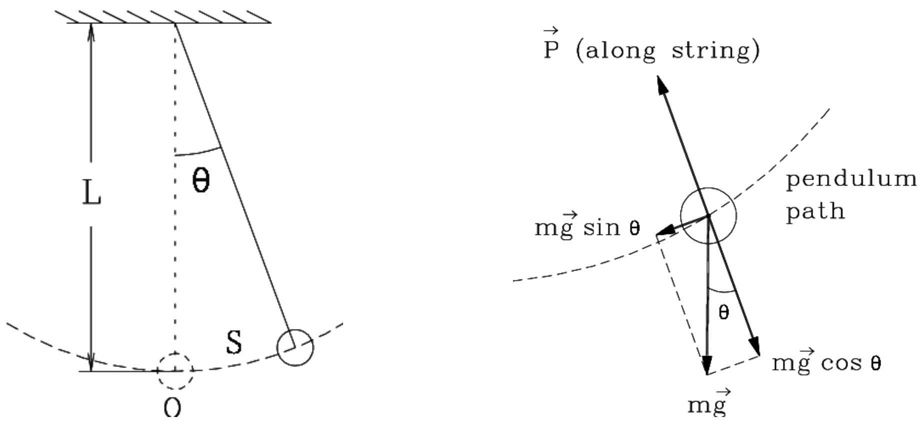


Figure 1.1: Two-dimensional (plane) trajectory of the simple pendulum

If the suspended object (often called a pendulum bob) is displaced slightly from its equilibrium vertical position by an angle θ, it will swing back and forth. The motion of the pendulum can be predicted using Newton's laws of motion and Newton's law of gravity. Assuming that there is no air resistance or other kinds of resistance, that the string has zero mass, and θ is a limited to few degrees, (the smaller the better) this analysis yields that the period T of the motion of this ideal pendulum satisfies to a good approximation

$$T = 2\pi\sqrt{\frac{L}{g}}. \tag{1.1}$$

$\boxed{?}$ How would the period T of the pendulum change if the length L were doubled and everything else remained the same?

. . . .

$\boxed{?}$ How would T change if the mass m were doubled and everything else remained the same?

. . . .

$\boxed{?}$ How would T change if the gravitational acceleration g were less; for example, at the surface of the Moon instead of the Earth?

. . . .

Procedure

In this experiment, you will explore the relationship between the period T and length L of a swinging pendulum. The pendulum apparatus consists of:

- a vertical post and fixed arm from which the pendulum bob, an aluminum ball of diameter d, is suspended by a light nylon string of negligible mass ($m \approx 0$);
- a sliding arm used to adjust the pendulum swing length. If the arm is properly calibrated to the scale on the pendulum post, so that the scale reads '0' when the arm touches the top of the ball, then the scale can be used to directly measure, or set, this length;
- a scale that, when calibrated to the sliding arm, displays the length of the string s from **the bottom of the sliding arm**, the pivot point of the pendulum, to the **top of the ball**, to a precision of one millimetre (mm).

This length s is not exactly equal to the pendulum length L; the pendulum length is measured from the top of the string (at the fixed point) to the centre of the ball. Therefore, L is the sum of the length of the string s and half the ball's diameter d given in Table 1.1:

$$L = s + \frac{1}{2}d. \tag{1.2}$$

To calibrate the pendulum:

Click the "pendulum calibration" links (Parts 1 and 2) in "Lab Documents" to view a graphical description of the following steps:

1. Loosen the clamping nut to release the string and lower the ball to the table.
2. Align the bottom of the sliding arm, labelled **Index**, with the zero mark on the scale.
3. Adjust the string length so that the top of the ball just contacts the bottom of the arm, ensuring that the string is not stretched.
4. Gently tighten the string under the clamping nut. Do not just wrap the string around the nut; it will slip and result in length measurements that are incorrect.

To check the calibration:

1. Raise the arm away from the ball and carefully reposition the arm until it once again just contacts the top of the ball.
2. The index at the bottom of the sliding arm should be at the zero mark on the scale. If it is not, repeat the calibration adjustments until the bottom of the arm is in line with the zero mark of the scale *and* the arm lightly contacts the top of the pendulum ball.

Data gathering and analysis using Physicalab

(!) At the start of every lab session, click on the desktop icon to open a new Physicalab application, then enter your *Brock* email address. Without a valid email address, you will not be able to send yourself the graphs made during the experiment for inclusion in your lab report.

(!) At the end of the lab session, be sure to close the Physicalab application, otherwise your email address will be accessible to the next person using the work station.

As shown in the short video "Introduction to using rangefinder and Physicalab," you are going to use a computer-controlled range-finder and the Physicalab software to monitor the change in distance over time of the pendulum bob from the device.

- Mount the **larger ball** m_1 and **calibrate the pendulum**.
- Adjust the sliding arm so that the string length s is approximately 0.3 m.

  Record the actual length to a precision of 1 mm (0.001 m).

Acquire distance/time data of the pendulum motion

1. Set the pendulum swinging in a straight line, keeping θ small (less than approximately 15°). Wait several seconds to allow for any stray oscillations present in the bob to dissipate before beginning to collect data.

[?] Does the angle of swing need to be precisely 15°? How might other choices of angle affect the results?

. . . .

2. Shift focus to the Physicalab software. Check the **Dig1** box and choose to collect **50** points at **0.1** s/point. Click **Get data** to acquire a set of data points $(\mathbf{x}, \mathbf{y})$ of distance $\mathbf{y}$ as a function of time $\mathbf{x}$.

3. Click **Draw** to graph your data. Your points should look like a smooth sine wave, without spikes, stray points or flat spots. If any of these are noted, adjust the position of the range-finder and acquire a new data set. Flat spots at the bottom of the graph occur when the ball is too close to the range-finder.

[?] If the pendulum was not swinging in-line with but at a significant angle β to the range-finder, how would the appearance of your graph change? Try to explain what is going on mathematically. Would this likely affect your results?

. . . .

Fit the pendulum distance/time data to a sine wave

1. Select **fit to: y=** and enter **A*sin(B*x+C)+D** in the fitting equation box.

   As reviewed in the Appendix, **x** represents the independent variable (here in units of time), **A** is the amplitude of the sine wave, **C** is the initial phase angle (in radians) of the wave when $\mathbf{x} = 0$, and **D** is the average distance of the pendulum from the detector; i.e. **D** is the distance from the pendulum to the detector when the pendulum is vertical or motionless.

   The fit parameter **B** (in radians/s) is the rate of change in angle with time, which is also called the angular frequency, so that **B*x** is an angle in radians. After one period of oscillation, where

T represents the period in seconds, **x** increases by T and the angle **B***x** increases by 2π radians. Therefore $\mathbf{B}T = 2\pi$, and this allows us to relate the fitting parameter **B** to the period T of the pendulum's oscillation:

$$\mathbf{B} = \frac{2\pi}{T} \qquad \text{which is equivalent to} \qquad T = \frac{2\pi}{\mathbf{B}}$$

2. Click **Draw**. If you get a **Fit timed out** message on the bottom left of the screen, the initial guesses for the fitting parameters may be too distant from the required values for the fitting program to properly converge.

   Look at your graph and enter some reasonable *approximate* values for the fitting parameters. You can get an initial guess for **B** by estimating the time **x** between two adjacent minima, or one period, of the sine wave.

? Why are you fitting an equation to your data when you can estimate the period T of the pendulum directly from the graph?

. . . .

3. Label the axes and give your graph a descriptive title that includes the length s of the string. Click **Send to:** to email yourself a copy of the graph for later inclusion in your lab report.

4. Record in Table 1.1 the trial length s, the fitting parameter B, the values of T and T^2, and a value for g using Equation 1.1. Do not round values at this time.

? Is your value for g reasonable? Explain.

. . . .

Run, i	mass	m (kg)	d (m)	s (m)	L (m)	B (rad/s)	T (s)	T^2 (s^2)	g_i (m/s^2)
1	m_1	0.0225	0.02540						
2	m_1	0.0225	0.02540						
3	m_1	0.0225	0.02540						
4	m_1	0.0225	0.02540						
5	m_1	0.0225	0.02540						
1	m_2	0.0095	0.01904						

Table 1.1: Table of experimental results

- Repeat the above steps for m_1 with $s = 0.45$ m, 0.60 m, 0.75 m, and 0.90 m.

? Do you have to use these specific lengths? Could another set of values be used just as well? In terms of measurement errors, how might your choice of length affect the results?

. . . .

- Mount the second ball m_2, **recalibrate the pendulum** and verify the calibration. Set the string length to approximately $s = 0.5$ m, then repeat steps 3–9 for m_2 to complete Table 1.1.

Determining δg from a single value of g

Because you made only one trial using the small ball of mass m_2, error propagation rules need to be used to determine error estimates for L and g. The *measurement errors* in s and d, represented by δs and δd, are determined from the scales of the measuring instruments. The micrometer used to measure the ball diameter d has a resolution, or scale increment, of 0.00001 m.

$$\delta s = \pm \ldots\ldots\ldots\ldots\ldots \qquad\qquad \delta d = \pm \ldots\ldots\ldots\ldots\ldots$$

- Equation 1.2 is used to calculate L and derive δL. Show in three steps below the relevant equation, then the variables replaced by the appropriate unrounded values, and finally show the numerical result. Do not include units at this step.

$$L = s + \tfrac{1}{2}d = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\delta L = \sqrt{(\delta s)^2 + \left(\tfrac{1}{2}\delta d\right)^2} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Present the final result for $L \pm \delta L$, properly rounded and with the correct units

$$L = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$$

- Equation 1.1 expresses the relationship between g, L, and T. You now have δL but not δT, the error in T. Because the value of T is derived from the fit parameter **B** and $\delta\mathbf{B}$ is given by the fit, you could derive δT from $\delta\mathbf{B}$. However, a more direct approach is to use the relationship $\mathbf{B} = 2\pi/T$ to rewrite Equation 1.1 in terms of **B** instead of T and solve for g to get:

$$g = \mathbf{B}^2 L = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\delta g = g\sqrt{\left(\tfrac{2\delta\mathbf{B}}{\mathbf{B}}\right)^2 + \left(\tfrac{\delta L}{L}\right)^2} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$g = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$$

Determining δg from a set of g values

You performed five trials ($i = 1, \ldots, N = 5$) using the large ball of mass m_1 to obtain five results for g that are *expected to have the same value* if Equation 1.1 is valid. In this case, you can invoke the theory of statistics to evaluate a sample average $\langle g \rangle$ of the five trials as well as the standard deviation of the sample $\sigma(g)$.

- In Physicalab, enter in a column your five g values, then from the **Edit** menu select **Insert X Index column 1** to add a column of index values. Check **bellcurve** to view your data as a distribution. Also check **Bargraph** to display the data in bins.
- Copy $\langle g \rangle$ and $\sigma(g)$ below and remember to email yourself a copy of the graph.

$$g = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots, \qquad N = \texttt{5 samples}$$

- Now, click **File, Upload your g data** to add your five g values to the pendulum database. As the data from all the different groups of students doing the experiment is accumulated, a nice statistical distribution of the value of g should evolve and the standard deviation $\sigma(g)$ should systematically decrease.

- Click **File, Get class g data** to download the list of N values of g so far collected, then click **Draw** to display a distribution of the data. Record below the values for the group mean $\langle g \rangle$ and the group standard deviation $\sigma(g)$.

$$g = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots, \qquad N = \ldots\texttt{samples}$$

? How do $\langle g \rangle$ and $\sigma(g)$ for the N currently accumulated group values compare with your result? Is this what you expect?

. . . .

Determining g and δg from the slope of a graph

A graphical method can also be used to determine g and δg. Here, a line of best fit that represents the relationship between the x and y coordinates is drawn through your data. The software chooses the line of best fit so that it minimizes some quantity related to the distances of the data points from the line of best fit. (As a challenge, you might like to think about the details; specifically which quantity is minimized?) Rewriting Equation 1.1 in terms of L as a function of T^2 as follows

$$L = \left(\frac{g}{4\pi^2}\right) T^2 \tag{1.3}$$

yields a linear relationship $y = mx + b$ between $y = L$ and $x = T^2$, with slope $m = g/(4\pi^2)$ and y-intercept $b = 0$.

- Enter the five coordinates (T^2, L) in the Physicalab data window. Select **scatter plot**. Click **Draw** to generate a graph of your data. Select **fit to: y=** and enter **A*x+B** in the fitting equation box. Click **Draw**. The computer will evaluate a line of best fit through the data points and output the fit results. The χ^2 (chi square) value is a measure of the goodness of the fit; the value of χ^2 is smaller for data that more closely lies along a straight line.

- Send yourself the graph, then record the slope A and error δA below:

$$A = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$$

- Calculate values for g and δg. Refer to the Appendix to determine the error equation for δg.

$$g = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\delta g = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$g = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$$

- Redraw the preceding graph, this time including the data point from the single trial using mass m_2 and repeat the linear fit. This graph can provide you with insight on the dependence of g on the mass of the pendulum bob. Record the slope for comparison with the previous result.

$$A = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$$

(!) Important! Be sure to have this printout signed and dated by a TA before you leave at the end of the lab session. All your work needs to be kept for review by the instructor, if so requested.

Lab report

Go to the "Lab Documents" web page to access the online lab report template for this experiment. Complete the template as instructed and submit it to Turnitin before the lab report submission deadline, late in the evening six days following your scheduled lab session. Do not wait until the last minute. Turnitin will not accept overdue submissions. Unsubmitted lab reports are assigned a grade of zero.

Pendulum prelab preparation

The Pendulum worksheets, videos and all other lab-related content is found at:

`http://www.physics.brocku.ca/Courses/1P91/lab-manual`

Your quest is to apply the scientific method to explore the relationship between a pendulum's period of oscillation and other experimental parameters (variable quantities such as pendulum colour, temperature, mass, ambient temperature, density etc.)

1. **Observation:** watch the short video introduction to the Pendulum and play with the PhET pendulum simulation to get a feel for the factors that might affect the period of a pendulum.

   `https://phet.colorado.edu/en/simulation/pendulum-lab`

2. **Hypothesis:** list some parameters that you think might influence the pendulum's period of oscillation. How do you think that changing each of these parameters might change the period?

   Formulate hypotheses as specifically as you can. For example, you might hypothesize that the colour of the pendulum influences the period; specifically, the darker the colour, the greater the period.

3. **Experiment:** now make your own pendulum and play with it, with your hypothesis in mind. For example,take your shoelace and tie one end to your water bottle and the other end to a tree branch and you have a pendulum! Vary each of the experimental parameters, one at a time, and note how the pendulum period changes. How would you determine if the period changes?

4. **Conclusion:** did your observations confirm or contradict your hypotheses? Write summary statements about your thinking about pendulums and how it has changed based on your prelab activities.

5. **Publish your results:** Login to Turnitin and submit your insights and observations in your Pendulum Prelab assignment before the "Due" time and date shown. Do not wait until the last minute to submit your report. Turnitin will not accept submissions after the set due date/time. Note that overdue prelab reports are assigned a grade of zero.

☐ Print a copy of the Pendulum experiment to bring to your scheduled lab session. The data, observations and notes entered on these pages will be needed when you write your lab report. Compile these printouts to create a lab book for the course.

CONGRATULATIONS! YOU ARE NOW READY TO PROCEED WITH THE EXPERIMENT!

(TA INITIALS)

FIRST NAME (PRINT) | LAST NAME (PRINT) | BROCK ID (AB17CD) | (LAB DATE)

Experiment 2

Harmonic motion

When an object of mass m is attached to a hanging spring, the object experiences a gravitational force $F_g = mg$ and a force due to the additional stretch of the spring. The latter force is described by Hooke's law,

$$F_s = -ky \; , \tag{2.1}$$

where y is the magnitude of the additional stretch of the spring. nd k is called the stiffness constant of the spring (also known as the spring constant, or the spring's force constant). By drawing a free-body diagram for the hanging object, and doing a bit of algebraic manipulation, you can arrive at the following conclusions:

$$F_g = -F_s \quad \Rightarrow \quad mg = ky \quad \Rightarrow \quad m = \frac{k}{g}\, y. \tag{2.2}$$

If the mass is displaced from its new equilibrium position and released, it will begin to oscillate according to

$$y = A_0 \cos(\omega_0 t + \phi), \qquad \omega_0 = \sqrt{\frac{k}{m}} = 2\pi f_0 = \frac{2\pi}{T_0} \tag{2.3}$$

where A_0 and ϕ are the initial amplitude and phase angle of the oscillation, T_0 is the period in seconds, and ω_0 is the angular speed in radians/second. If a damping force F_d is present, the oscillation decays exponentially at a rate determined by the damping coefficient γ:

$$y = A_0 e^{(-\gamma t)} \cos(\omega_d t + \phi), \qquad \omega_d = \sqrt{\omega_0^2 - \gamma^2} \tag{2.4}$$

Procedure and analysis

The experimental setup consists of a vertical stand from which hangs a spring. A platform attached to the bottom of the spring accepts various mass loads. The spring-mass system is free to move in the vertical direction.

When the platform is static, its height above the table can be measured with a ruler.

When the system is in motion, Physicalab is used to record the time-dependent change in distance from a rangefinder to the bottom of the platform. The rangefinder measures distance by emitting an ultrasonic pulse and timing the delay for the echo to return.

Static determination of the stiffness constant k

In the following exercise you will determine the stiffness constant of a spring. **Note:** Use *positive k values* in all the following steps.

Equation 2.2 gives the relationship between the mass m and displacement y for a series of points (y, m). This is the equation of a straight line with slope $k/g = \Delta m/\Delta y$, so that $k = |\Delta m/\Delta y| * g$.

By varying m, measuring y_0, and fitting the resulting data to the equation of a straight line, the stiffness constant k for the spring can be determined. Note that the absolute distance y_0 does not matter, but the change in distance Δy with the change in mass Δm is critical.

- Suspend the spring from the holder and load it with the 50 g platform. Use a ruler to carefully measure the distance y from the top of the table to the bottom of the platform to a precision of 1 mm. Record your result in Table 2.1.

- Add masses to lower the platform until it is close to but is not touching the top of the table. Record this distance, then select several intermediate masses and fill in Table 2.1.

$\boxed{?}$ Why do you want to determine a mass that brings the platform near to the table?

. . . .

- Enter the data pairs (y, m) in the Physicalab data window. Select **scatter plot** and click **Draw** to generate a graph of your data.

m (kg)							
y (m)							

Table 2.1: Data for determining the spring force constant k

$\boxed{?}$ Did you choose to include the mass of the platform (50 g) as part of the stretching mass m in Table 2.1? How would the graph and the resulting value for the stiffness constant k be affected by your choice?

. . . .

- Select **fit to: y=** and enter **A+B*x** in the fitting equation box. Click **Draw** to perform a linear fit on your data, then evaluate k and the associated error $\delta(k)$. Click **Send to:** to email yourself a copy of the graph for later inclusion in your lab report.

B = ... ± . . . . A = ... ± . . . .

k = ... = ... = ...

δk = ... = ... = ...

k = ... ± . . . .

Damped harmonic oscillator

You will now explore the behaviour in time of an oscillating mass with the aid of a computer-controlled rangefinder. This device sends out an ultrasonic pulse that reflects from an object in the path of the conical beam and returns to the rangefinder as an echo. The rangefinder measures the elapsed time to determine the distance to the object, using the speed of sound at sea level as reference.

By accumulating a series of coordinate points (ω, m) and fitting your data to Equation 2.4, you can use several methods to determine the spring constant for the oscillating mass system.

- With each of the masses used previously with this spring, raise the platform a small distance, then release it to start the mass oscillating vertically. Wait until the spring/mass system no longer exhibits any erratic oscillations.
- Login to the Physicalab software. Check the **Dig1** box and choose to collect **200** points at **0.05** s/point. Click **Get data** to acquire a data set.
- Select **scatter plot**, then Click **Draw**. Your points should display a smooth slowly decaying sine wave, without sharp peaks, stray points, or flat spots. If any of these are noted, adjust the position of the rangefinder and acquire a new data set.
- Select **fit to: y=** and enter **A*cos(B*x+C)*exp(-D*x)+E** in the fitting equation box. Click **Draw**. If you get an error message the initial guesses for the fitting parameters may be too distant from the required values for the fitting program to properly converge.

  The fitting equation is equivalent to Equation 2.4. Look at your graph and enter some reasonable initial values for the amplitude **A** of the wave and the average (equilibrium) distance **E** of the wave from the detector. **C** corresponds to the initial phase angle of the sine wave when $\mathbf{x} = 0$.

  The angular speed **B** (in radians/s) is given by $\mathbf{B} = 2\pi/T$. Estimate the time x $= T$ between two adjacent minima of the sine wave then estimate and enter an initial guess for **B**.

  The damping coefficient **D** determines the exponential decay rate of the wave amplitude to the equilibrium distance **E**. When $\mathbf{D} = 1/\mathbf{x}$, the envelope will have decreased from $\mathbf{A}_0$ to $\mathbf{A}_0 e^{-1} = \mathbf{A}_0/e = \mathbf{A}_0/2.718 \approx \mathbf{A}_0/3$.

  Make an initial guess for **D** by estimating the time $t = \mathbf{x}$ required for the envelope to decrease by a factor of 2/3.
- Check that the fitted waveform overlaps your data points well, then label the axes and include as part of the title the value of mass m used. Email yourself a copy of all graphs as part of your lab data set.
- Record the results of the fit in Table 2.2, then complete the table as before.

m (kg)							
y_0 (m)							
ω_d (rad/s)							
γ (s^{-1})							

Table 2.2: Experimental results for damped harmonic oscillator

- The stiffness constant k can be determined as before from the slope of a line fitted through the (y_0, m) data in Table 2.2. Generate and save the graph, then record the results below:

B = ... ± .... A = ... ± ....

k = ... = ... = ...

δk = ... = ... = ...

$k(y_0)$ = ... ± ....

The stiffness constant k can also be determined from the period of oscillation of the mass. Rearranging the terms in Equation 2.3 yields $m = k/\omega_d^2$.

- Enter the coordinate pairs from Table 2.2 in the form (ω_d, m) and view the scatter plot of your data.

- Select **fit to: y=** and enter **A+B/x**2** in the fitting equation box. This is more convenient than squaring all the ω_0 values and fitting to **A+B/x**.

- Click **Draw** to perform a quadratic fit on your data. Print the graph, then enter the value for the stiffness constant below:

$k(\omega_d)$ = ... ± ....

The damped harmonic oscillator equation (Equation 2.4) predicts that the frequency of oscillation depends on the damping coefficient γ so that your experiment actually yields values for ω_d rather than ω_0. Calculate ω_0 from ω_d using an average value for γ.

? Which ω_d value should be used? Why?

....

ω_0 = ... = ... = ...

? Is the difference between ω_d and ω_0 significant? Explain.

....

Calibration of rangefinder

It was assumed that the rangefinder is properly calibrated and measuring the correct distance. It is always a good practice to check the calibration of instrument scales against a known good reference, in this case the ruler scale. Calibration errors are systematic errors and can be corrected. Once the amount of mis-calibration is determined, the mis-calibrated data can be adjusted to represent correct values.

- Check the calibration of your rangefinder against the ruler. Compare the slope values from the (y_0, m) data. If the rangefinder is calibrated, the slope results should agree within experimental error.

$$\texttt{ruler :} \quad \texttt{slope} \; = \; \ldots\ldots\ldots \; \pm \; \ldots\ldots\ldots \quad y\texttt{-int =} \; \ldots\ldots\ldots \; \pm \; \ldots\ldots\ldots$$

$$\texttt{rangefinder :} \quad \texttt{slope} \; = \; \ldots\ldots\ldots \; \pm \; \ldots\ldots\ldots \quad y\texttt{-int =} \; \ldots\ldots\ldots \; \pm \; \ldots\ldots\ldots$$

$\boxed{?}$ Do your two results for the slopes **B** agree? Explain.

....

$\boxed{?}$ Should the values of the y-intercepts **A** be the same? Explain.

....

(!) Important! Be sure to have this printout signed and dated by a TA before you leave at the end of the lab session. All your work needs to be kept for review by the instructor, if so requested.

Lab report

Go to the "Lab Documents" web page to access the online lab report template for this experiment. Complete the template as instructed and submit it to Turnitin before the lab report submission deadline, late in the evening six days following your scheduled lab session. Do not wait until the last minute. Turnitin will not accept overdue submissions. Unsubmitted lab reports are assigned a grade of zero.

Harmonic motion prelab preparation

The worksheets, videos and all other lab-related content is found at:

`http://www.physics.brocku.ca/Courses/1P91/lab-manual`

1. Watch the "video introduction" and "tips on fitting using Physicalab".
2. Think about a back-and-forth motion that you remember seeing or experiencing. Briefly describe the motion. In particular, what happened to the amplitude of the motion as time passed?
3. Sketch a rough position-time graph of the motion that you thought about in the previous question. It's not necessary to label the axes with numbers; what we are most interested in is that you can approximately represent the long-term behaviour of a realistic back-and-forth motion.
4. Guess what kind of function might accurately model the graph that you roughly sketched in the previous question. For instance, is it a linear function? Is it a quadratic function? Is it some combination of functions?
5. Play with the simulation found here:

   `https://phet.colorado.edu/sims/html/masses-and-springs/latest/masses-and-springs_en.html`

   Experiment by varying the amount of damping in each run. After you have explored a few runs of the simulation, briefly describe any additional understanding that you have gained beyond what you wrote in Question 2.
6. Also briefly describe how increasing the friction changes the motion. If any new understanding that you have gained from the simulation causes you to revise your approximate graph of such a motion, sketch the new graph. If any new understanding that you have gained from the simulation causes you to revise your guess about what kind of function models this kind of motion, revise your guess here.
7. Briefly summarize what you have learned about this kind of motion from doing the prelab exercises.

☐ Print a copy of the experiment to bring to your scheduled lab session. The data, observations and notes entered on these pages will be needed when you write your lab report. Compile these printouts to create a lab book for the course.

CONGRATULATIONS! YOU ARE NOW READY TO PROCEED WITH THE EXPERIMENT!

The simple harmonic oscillator

A simple harmonic oscillator (SHO) is a model system that is used to describe numerous real physical systems. The reason for this is profound: the same fundamental equations that describe the motion of a mass-on-a-spring also describe, to a very good approximation, the inter-atomic forces that hold all matter together. At least in a first approximation, we can fairly accurately pretend that *every* solid object we touch is held together by springs connecting pairs of atoms.

With no external forces applied to a solid material, the "inter-atomic springs" are at their equilibrium lengths, neither stretched nor compressed. The application of an external stretching force to the material will cause these springs to extend, thereby increasing the bulk length of the material. When the applied external force is removed, the springs return to their equilibrium lengths, restoring the material to its original dimensions. Such restoring forces may be overcome by a large enough applied external force that will cause the object to deform permanently or to break. The maximum force applicable without permanent distortion is called the *elastic limit* of the material.

Hooke's law states that the stretch or compression x of a material is directly proportional to the applied force F. The proportionality constant, called the *stiffness constant* k, has units of newtons per metre (N/m), and is also frequently called the spring constant or the spring's force constant. This proportionality between force and deformation has been found to be an excellent approximation for any solid object, as long as the elastic limit of the material is not exceeded.

For an object attached to a spring, Hooke's law is:

$$F_s = -kx \ ,$$

where F_s is the force exerted by the spring on the object and x is the displacement of the object from its equilibrium position. The negative sign expresses the fact that the force exerted *by the spring on the object* is in the direction *opposite* to the object's displacement.

For this reason, the force exerted by a spring on an attached object is often described as a *restoring force*, because it tends to restore equilibrium. That is, when the object is not at its equilibrium position, the force that the spring exerts on the object is directed towards the equilibrium position.

If the spring force and gravity are the only forces acting on the object, the system is called a simple harmonic oscillator, and the object undergoes simple harmonic motion — sinusoidal oscillations about the equilibrium point, with a constant amplitude, and a constant frequency that does not depend on the amplitude.

Simple harmonic motion is equivalent to an object moving around the circumference of a circle at constant speed, in the sense that the same formulas describe each kind of motion.

In the absence of friction, the oscillations will continue forever. Friction robs the oscillator of its mechanical energy, transferring it to thermal energy, and so the oscillations decay, and eventually stop altogether. This process is called damping, and so in the presence of friction, this kind of motion is called damped harmonic oscillation.

If the mass is displaced from its new equilibrium position and released, it will begin to oscillate according to

$$y = A_0 \cos(\omega_0 t + \phi), \qquad \omega_0 = \sqrt{\frac{k}{m}} = 2\pi f_0 = \frac{2\pi}{T_0}$$

where A_0 and ϕ are the initial amplitude and phase angle of the oscillation, T_0 is the period in seconds, and ω_0 is the angular speed in radians/second.

Recall that pendulum motion was also analyzed using this same equation. In essence, the pendulum was assumed to be undergoing simple harmonic motion during the short time interval that was sampled.

If a damping force F_d is present, the oscillation decays exponentially at a rate determined by the damping coefficient γ:

$$y = A_0 e^{(-\gamma t)} \cos(\omega_d t + \phi), \qquad \omega_d = \sqrt{\omega_0^2 - \gamma^2}, \qquad \gamma = \frac{R}{2m}$$

Note an interesting detail; the damped frequency of oscillations, ω_d, is smaller than ω_0 because of the subtraction of γ^2 under the square root. This reduction is not all that noticeable, even though the decrease in the amplitude due to the $e^{-\gamma t}$ term may be readily observed.

Note also that the hanging spring is stretched by its own weight and may exhibit twisting as well as lateral oscillations when stretching, factors that are neglected in our analysis.

One other simplifying assumption: this experiment assumes an ideal massless spring connected to a point mass m. Even with all these approximations, the damped harmonic oscillator lends itself very nicely to an experimental investigation.

Note further that the pendulum motion would, if given enough time, have decreased in amplitude so that for a long data sample of the pendulum motion over time, the damped harmonic oscillator equation would have to be used to make a proper fit of the data set.

(TA INITIALS)

FIRST NAME (PRINT) LAST NAME (PRINT) BROCK ID (AB17CD) (LAB DATE)

Experiment 3

The ballistic pendulum

Procedure

The launcher component of the ballistic pendulum consists of a sliding metal rod surrounded by a precision spring. When the lever that is connected to the sliding rod is pulled to the left, the spring is compressed and the rod extends out of the left side of the launcher. This extension can be measured to determine the distance that the spring was compressed.

There are four slots on the side of the launcher. The lever is moved from its relaxed position at the rightmost slot of the launcher and is then lowered into one of the other three slots to lock the spring at one of three compression settings that we will name, from right to left: *short*, *medium*, or *long* range.

A cylindrical barrel on the right side of the launcher holds the projectile, a steel ball. As the lever is *slowly* raised in the slot, the launcher suddenly discharges with the spring extending and pushing the rod to strike the ball. The ball exits the launcher and impacts the pendulum.

The pendulum consists of a rod and bob of combined mass M attached to a pivot point. When an impact takes place, the pendulum catches the impacting mass m, changing the total mass of the pendulum to $M_T = M + m$, and swings about the pivot point to a maximum angle of deviation θ, relative to the initial vertical position of $\theta = 0°$.

The pendulum drags with it a pointer that stops at the limit of the swing and identifies the value of θ on a degree scale concentric with the pivot. The pendulum then free falls back to the vertical position to stop against the barrel. A small amount of friction between the pointer and the scale prevents the pointer from falling back along with the pendulum. The effect of this friction and the mass of the pointer on the system is negligible.

$\boxed{?}$ The pointer, initially at rest, is accelerated along with the the pendulum arm on impact. Could it keep moving past the limit of the pendulum arm, after the arm has stopped, and thus give inaccurate angle readings?

. . . .

Note: When the launcher is in the discharged position, the spring is not fully extended but is subjected to a compression preload x_0 and is thus storing some potential energy. You will determine this preload at the end of the experiment.

Caution: Do not place ball in the launcher until the lever is properly lowered and secure in a slot, otherwise an unintended spring discharge could occur, ejecting the ball and possibly causing an injury.

Data gathering and analysis

To determine the physical characteristics of the ballistic pendulum apparatus:

- remove the pendulum arm from the ballistic pendulum assembly by unscrewing the pivot screw. **Replace the screw for safekeeping**;
- measure with a digital scale ($\sigma = \pm 0.01$ g) the mass of the ball m and ball/pendulum assembly M_T;

$$m = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots \text{ kg} \qquad M_T = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots \text{ kg}$$

- determine the centre of mass point of the pendulum/ball combination by balancing it on the edge of the steel ruler and noting the position of the balance point on the scale located on the pendulum arm, then measure with the ruler the distance from this point to the centre of the pendulum pivot (see schematic Figure 4.2).
- The centre-of-mass distance R_{cm}, from the balance point to the centre of the pivot hole on the arm of the pendulum of mass M_T, is

$$R_{cm} = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots \text{ m.}$$

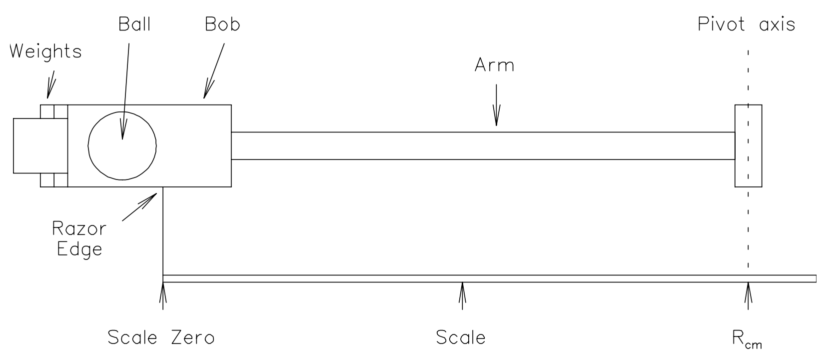


Figure 3.1: Generic diagram for determining a pendulum centre-of-mass

- With the launcher discharged, measure the distance in mm that the rod extends from the left end of the launcher. You will subtract this rod offset from all of the following measurements. The offset is

$$\text{offset} = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots \text{ m.}$$

- Compress and lock the spring at the *short* range setting. Measure the distance that the rod now extends from the end of the launcher. Subtract from this length the rod offset and record the result below and as x in Table 3.2:

$$x_{short} = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots \text{ m.}$$

- Repeat the above step for the *medium* and *long* range settings:

$x_{medium} = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$m $\qquad x_{long} = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$m.

- Finally, replace the pendulum arm, making sure that the pivot screw is tight.

Before you begin to gather ballistic data remember to avoid sitting directly in front of the discharge path of the launcher barrel and

CAUTION: Always wear safety glasses while using the launcher.

- Move the lever from the discharged position to the first slot and push it down to lock the spring at the *short* range setting. Further compression selects the *medium* range and finally, the *long* range setting.
- Load the launcher by raising the pendulum and placing the ball at the end of the barrel, then lower the pendulum to freely hang in the vertical position.
- Hold the pendulum in the vertical position and move the angle indicator to the 0° mark. If the indicator does not reach zero, you will need to subtract this difference from all your angle readings.
- To fire the launcher, gently raise the lever from the retaining slot to release the spring.

(!) **Note:** if the ball falls out of the pendulum when launched, the assembly is not level or the pendulum bob position needs adjustment. Make small adjustments to the base leveling screws to centre the pendulum with the launcher barrel. If the problem persists, see a TA for assistance.

(!) **Note:** be sure that the angle indicator does not move when the pendulum falls back and strikes the launcher barrel, otherwise you will get an incorrect angle reading. If this happens, gently stop the falling pendulum with your finger before it strikes the launcher.

- Perform five *short* range launches, recording the angle θ_i reached in trial $i = 1 \ldots 5$ in the appropriate spaces of Table 3.1.
- Perform five launches using the *medium* and then the *long* range settings.

Because the five θ_i values at a given range setting *are expected to be the same*, you can perform a statistical analysis to get an average value $\langle\theta\rangle$ and the standard deviation $\sigma\theta$ for each of the three settings.

- Enter the five θ values as a column in Physicalab, then from the **Edit** menu select **Insert X Index** to add a column of index values. Check **bellcurve** and click **Draw**. The results for $\langle\theta\rangle$ and $\sigma\theta$ appear at the bottom of the graph as $\langle\theta\rangle \pm \sigma\theta$.
- Convert the degree values $\langle\theta\rangle$ and $\sigma\theta$ to radians values $\theta(rads)$ and $\sigma\theta(rads)$. Recall that $360° = 2\pi$ raduans.

You now need to calculate v and δv for the three range settings. The equation that relates the projectile velocity to the pendulum angle is derived in the Theory lab document and reproduced here:

$$v = \frac{M_T}{m}\sqrt{2gR_{cm}\left(1 - \cos\theta\right)} \tag{3.1}$$

range	θ_1°	θ_2°	θ_3°	θ_4°	θ_5°	$\langle\theta\rangle$°	$\sigma\theta$°	$\theta(rads)$	$\sigma\theta(rads)$
short									
medium									
long									

Table 3.1: Experimental angle values at three force settings

Note that in Equation 3.1 only the $\sqrt{(1-\cos\theta)}$ factor changes with θ. Chances of mistakes in the error propagation process will be minimized if the constant quantities are represented by C and δC and are evaluated only once. Use the radian values of θ and $\sigma\theta$ from Table 3.1 in the following calculations.

$$C = \frac{M_T}{m}\sqrt{2gR_{cm}} \qquad \frac{\delta C}{C} = \sqrt{\left(\frac{\delta M_T}{M_T}\right)^2 + \left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta R_{cm}}{2R_{cm}}\right)^2}$$

....

....

$$v_s = C\sqrt{(1-\cos\theta)} \qquad \delta v_s = |v_s|\sqrt{\left(\frac{\delta C}{C}\right)^2 + \left(\frac{\sin\theta\ \delta\theta}{2\cos\theta}\right)^2}$$

....

....

$v_s =$ .... $\pm$ .... m/s

$\boxed{?}$ What should be the dimensions of C? And those of $\delta C/C$?

....

- Calculate the maximum kinetic energy of the steel ball at the moment that it lost contact with the launcher rod and enter the value in Table 3.2. Show a complete calculation for the *short* range setting:

$$K_s = \frac{1}{2}mv_s^2 \qquad \delta K_s = |K_s|\sqrt{\left(\frac{\delta m}{m}\right)^2 + \left(2\frac{\delta v_s}{v_s}\right)^2}$$

....

. . . .

$$K_s = \ldots\ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots\ldots \text{ J}$$

If no energy is lost (or gained) during the interaction, the kinetic energy K is equal to the potential energy $V = kx^2/2$ stored by the spring before the ball was discharged, and so

$$K = \left(\frac{k}{2}\right) x^2 \tag{3.2}$$

Comparing Equation 3.2 with the equation of a straight line $Y = AX + B$ and matching terms we see that $Y = K$, $X = x^2$ and the slope is $A = k/2$. The Y-intercept B (which is the value of Y when $X = 0$) is $B = 0$ since there is no corresponding term in Equation 3.2.

Plotting K as a function of x^2 and fitting these points to a straight line yields from the slope of the fitted line a value for the stiffness constant k of the launcher spring.

range	v (m/s)	x (m)	x^2 (m^2)	K (J)
short	$\pm$	$\pm$	$\pm$	$\pm$
medium	$\pm$	$\pm$	$\pm$	$\pm$
long	$\pm$	$\pm$	$\pm$	$\pm$

Table 3.2: Parameters for the calculation of the kinetic energy K and the stiffness constant k

- Shift focus to the Physicalab software and enter in the data window the three data pairs and corresponding errors as four space-delimited numbers: x^2 K δK δx^2.
- Select **scatter plot**. Click **Draw** to generate a graph of your data. Your graphed points should well approximate a straight line.

  Draw an imaginary line through the three points; the points should lie close to the line. If they are not, then the computer fit of a straight line through these points will not yield a good result. Look for a mistake in your calculations before proceeding. If the problem persists, consult the TA.
- Select **fit to: y=** and enter **A*x+B** in the fitting equation box. Click **Draw** to perform a linear fit of the data. Label the axes and include a descriptive title. Click **Send to:** to email yourself a copy of the graph for later inclusion in your lab report.
- Record the values of the fit parameters **A**, **B** and their associated errors, then calculate k and δk:

A = ... $\pm$ . . . .   **B** = ... $\pm$ . . . .

k = ... = ... = ...

δk = ... = ... = ...

$$k = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$$

? The stiffness constant k is typically expressed in units of N/m. Using dimensional analysis, verify that your dimensions for k obtained from the graph agree with those of N/m.

. . . .

- From the slope and Y-intercept, calculate the X-intercept (i.e. the value of X when $Y = 0$) by setting Y to zero and solving for X:

$$X = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\delta X = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$X = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots m^2.$$

- From this result estimate the spring preload distance x_0:

$$x_0 = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\delta x_0 = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$x_0 = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots m.$$

! Important! Be sure to have this printout signed and dated by a TA before you leave at the end of the lab session. All your work needs to be kept for review by the instructor, if so requested.

Lab report

Go to the "Lab Documents" web page to access the online lab report template for this experiment. Complete the template as instructed and submit it to Turnitin before the lab report submission deadline, late in the evening six days following your scheduled lab session. Do not wait until the last minute. Turnitin will not accept overdue submissions. Unsubmitted lab reports are assigned a grade of zero.

Ballistic pendulum prelab preparation

The worksheets, videos and all other lab-related content is found at:

`http://www.physics.brocku.ca/Courses/1P91/lab-manual`

1. Click the **Video Introduction** link to get acquainted with Hooke's Law and harmonic motion.
2. Visit the following site to get a feel for the concepts behind the calculations you will have to do in analyzing this experiment and try a few sample calculations for yourself.

   `https://www.thephysicsaviary.com/Physics/APPrograms/EnergyBallisticPendulum`

   The formula that relates the ball speed to the maximum angle that the box reaches can be found in the Theory link for the Ballistic Pendulum experiment. Enter it here:

   ....

   Enter for a the trials below, the simulation settings and your predicted speed of the ball:

trial	1	2	3	4	5
g					
mass of ball					
mass of box					
length of string					
maximum angle					
predicted speed					
correct speed					

   `.....copy this completed table to a separate file for upload to Turnitin.......`

3. Play with the following simulation, which may help you get a feel for how changing the values of the colliding masses and other initial parameters changes the final values of some of the final parameters.

   `http://physics.bu.edu/~duffy/HTML5/ballistic_pendulum.html`

4. Summarize in a few sentences the sequential energy conversion steps that begin with the compression of the spring and end with the elevation of the pendulum to a maximum height above it's initial equilibrium position. These steps are outlined in the Theory for this experiment.
5. Read through the rest of the lab instructions for this experiment in this document.
6. Login to Turnitin and submit your file in your prelab assignment before the "Due" time and date shown. Do not wait until the last minute to submit your report. Turnitin will not accept submissions after the set due date/time. Note that overdue prelab reports are assigned a grade of zero.

☐ Print a copy of this experiment to bring to your scheduled lab session. The data, observations and notes entered on these pages will be needed when you write your lab report. Compile these printouts to create a lab book for the course.

CONGRATULATIONS! YOU ARE NOW READY TO PROCEED WITH THE EXPERIMENT!

Theory

The ballistic pendulum is used to explore the transfer and conservation of energy and momentum in a collision of two objects. One of these objects is a small projectile of mass m that is projected at a certain speed v by a launcher. The second object is a pendulum that is initially stationary. As the projectile hits the pendulum, a re-distribution of energy and momentum takes place.

In a certain class of collisions, the projectile is captured by the pendulum. For such *inelastic collisions*, we can consider the pendulum and projectile to be one single object after the collision, and this one combined object carries all of the kinetic energy K_{after} and momentum P_{after} after the collision. If the mass of the pendulum is M then the total mass of the combined object after the collision is $M_T = M + m$. If the pendulum is stationary when the projectile hits, the pendulum contributes nothing to the total kinetic energy and momentum of the system before the collision. Thus the principle of conservation of momentum in this case yields (where v_T is the speed of the combined object immediately after the collision):

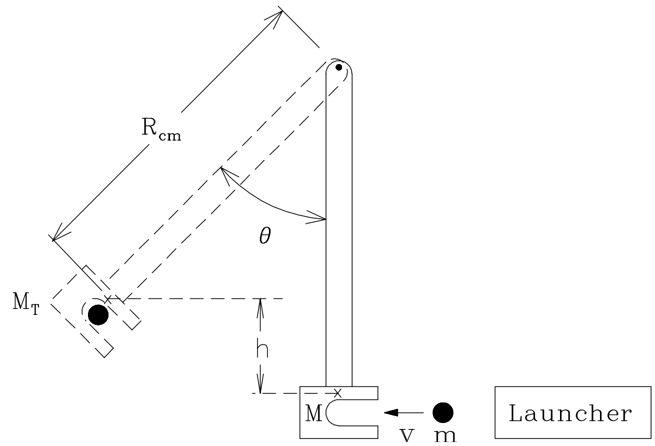


Figure 3.2: Ballistic Pendulum

$$\begin{aligned} P_{before} &= P_{after} \\ mv &= M_T v_T \\ mv &= (M+m)v_T \end{aligned} \tag{3.3}$$

Solving Equation 3.3 for v_T, we obtain

$$v_T = \frac{mv}{M_T} \tag{3.4}$$

Using the expression for the speed of the combined object immediately after the collision from the previous equation, we can show that the kinetic energy of the combined object immediately after the collision is *less* than the kinetic energy of the projectile just before the collision:

$$\begin{aligned} K_{after} &= \frac{1}{2}M_T v_T^2 \\ K_{after} &= \frac{1}{2}M_T\left(\frac{mv}{M_T}\right)^2 \\ K_{after} &= \frac{1}{2}M_T\left(\frac{m^2v^2}{M_T^2}\right) \\ K_{after} &= \frac{1}{2}\left(\frac{m^2v^2}{M_T}\right) \\ K_{after} &= \frac{m}{M_T}\left(\frac{1}{2}mv^2\right) \\ K_{after} &= \frac{m}{M_T}K_{before} \end{aligned}$$

Because the mass m of the projectile is less than the mass M_T of the combined object after the collision, it follows that

$$\frac{m}{M_T} < 1$$

and therefore

$$K_{after} < K_{before}$$

As the pendulum begins to swing after the collision, the kinetic energy of the combined object is gradually converted to gravitational potential energy as the pendulum rises. At the peak of its swing, the pendulum's mechanical energy is all in the form of gravitational potential energy, and its kinetic energy is zero. As the pendulum begins to move downwards after it reaches the peak of its motion, its gravitational potential energy is gradually converted to kinetic energy as the pendulum falls. At the lowest point of its motion, the pendulums's mechanical energy is all in the form of kinetic energy, and its graivational potential energy can be considered to be zero. Thus, we can write

$$E_{top} = M_T g h \tag{3.5}$$

and

$$E_{bottom} = \frac{1}{2} M_T v_T^2 \tag{3.6}$$

Assuming that mechanical energy is conserved during the initial swing of the pendulum, we can equate the expressions in Equations 3.5 and 3.6. While doing this, we can combine Equations 3.4 and 3.6 to eliminate v_T to obtain an expression that relates the initial speed v of the projectile to the final elevation h of the combined object, as follows.

$$\begin{aligned}
\frac{1}{2} M_T v_T^2 &= M_T g h \\
\frac{1}{2} M_T \left(\frac{mv}{M_T}\right)^2 &= M_T g h \\
\frac{1}{2} M_T \left(\frac{m^2 v^2}{M_T^2}\right) &= M_T g h \\
\frac{1}{2} \left(\frac{m^2 v^2}{M_T}\right) &= M_T g h \\
\left(\frac{m^2 v^2}{M_T}\right) &= 2 M_T g h \\
m^2 v^2 &= 2 M_T^2 g h \\
v^2 &= \left(\frac{M_T^2}{m^2}\right) 2 g h \\
v &= \frac{M_T}{m} \sqrt{2gh} \qquad (3.7)
\end{aligned}$$

Equation 3.7 can be expressed in terms of the maximum angle of the pendulum's motion, as follows. The length R_{cm} describes the radius of the arc from the pivot point to the centre of mass of combined rod, block, and block contents. With the vertical orientation of R_{cm} as the base of a right-angled triangle, h can be expressed in terms of the maximum angle θ of the swing: $R_{\mathrm{cm}} \cos\theta = (R_{\mathrm{cm}} - h)$. Solving for h, we obtain

$$h = R_{\mathrm{cm}} - R_{\mathrm{cm}} \cos\theta = R_{\mathrm{cm}} (1 - \cos\theta) \tag{3.8}$$

Inserting this expression for h into Equation 3.7, we obtain

$$v = \frac{M_T}{m}\sqrt{2gR_{cm}\left(1-\cos\theta\right)} \tag{3.9}$$

There is another energy conversion taking place, even before the collision. In the experiment, the launcher transfers some of its elastic potential energy into the projectile's initial kinetic energy. (Similarly, the force exerted by the spring on the projectile provides the projectile's initial momentum.) Applying the principle of conservation of energy to the transfer of energy from the spring to the projectile yields (where x is the maximum displacement of the spring from its equilibrium position and k is the stiffness constant of the spring)

$$\begin{aligned}
\text{elastic potential energy of spring} &= \text{initial kinetic energy of projectile} \\
\frac{1}{2}kx^2 &= \frac{1}{2}mv^2 \\
k &= \frac{mv^2}{x^2} \qquad (3.10)
\end{aligned}$$

Calculating v using Equation 3.7 and measuring the value of x allows us to use Equation 3.10 to determine the stiffness constant of the spring.

Note that there is an additional complication: The spring has a "pre-load" displacement. That is, when the spring is in its "relaxed" state, there is some compression in the spring. Think about how the previous analysis has to be modified to account for this pre-load compression.

(TA INITIALS)

FIRST NAME (PRINT) LAST NAME (PRINT) BROCK ID (AB17CD) (LAB DATE)

Experiment 4

Collisions and conservation laws

Procedure

The equipment consists of an air table that discharges streams of air from a series of closely spaced holes on its flat surface. Two plastic pucks float on this cushion of air with little friction. The pucks are launched and made to collide. An overhead camera records a movie of the collision.

A program called Tracker is then used to make a frame-by-frame analysis of the movie. For each frame, the software determines the position of the two pucks and records their coordinates in a table.

These dots map the trajectory of the two pucks in time, as shown in Figure 4.1. From this data, vectors of the velocities before and after the collision are determined for each puck.

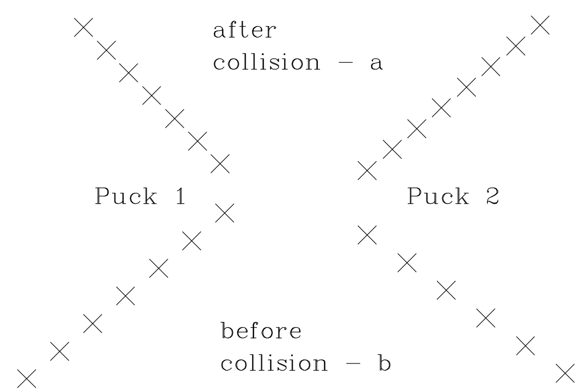


Figure 4.1: Trails left by moving pucks

To record a video of the collision:

Note: There may not be sufficient air tables for every student to use simultaneously. In this case, the equipment will have to be shared. Once a collision is recorded, the video analysis can be done on any of the other workstations in the lab room, making the air table available for the next user.

Every student is required to record and analyse their own collision video.

1. At an airtable workstation, login to Physicalab and select “USB camera” from the “Hardware” menu.

2. Click “Start video stream”; a window should open to display the video from the camera situated above the air table. Verify that the table is framed evenly within the displayed picture. You can adjust the brightness of the image with the scrollbox above the video frame.

3. Place the pucks on the air table and turn on the air until the pucks begin to float freely, then, if necessary, level the frame by adjusting the height of the two legs on the side until the pucks float in place.

4. Place the launchers in adjacent corners of the frame. Load the pucks and press the trigger to release them toward the centre of the air table, where they will collide. Do several practice runs.

   While *any* collision is in principle usable, *for ease of analysis* your collision should be fairly symmetric as in Figure 4.1 and take about 2–3 seconds to complete. Small initial velocities will cause the pucks to slow down due to friction and yield unsatisfactory results.

5. To record the collision, have an assistant press the "Capture video" button and then release the pucks. The collision process will be recorded at 30 frames per second for about 5 seconds.

6. Review your collision video by clicking on the "Review video" button. Repeat the above procedure until you get a nice collision to analyse. The white dots on the pucks should be fairly sharp as these are the objects that the software will track.

7. When done, logout and let the next student login to Physicalab to record their own video.

To track the puck trajectories:

To access your video, login to Physicalab at any workstation in the lab and select "USB camera" from the "Hardware" menu, then click "Analyse video" to start the Tracker software. Your video should appear there for analysis. If two video windows appear, click **View** and uncheck **Bottom View**. Stretch the Tracker window to maximize the size of the video image.

The actual centre of the puck is elevated from the air table surface, causing its position to be distorted in the video image as the puck moves around the field of view of the camera. To avoid this problem, you will use the two white dots on the flat surface of the puck to define a virtual centre-of-mass for each puck that will be tracked accurately. Here is a summary of the required steps:

1. Calibrate the video frame to the air table grid by clicking **Track, New, Calibration Tools, Calibration Points**. An xy-coordinate system will appear. The grid is divided into 5-cm squares.

   Shift-click on intersecting grid lines about 15 cm (3 squares or divisions) below and 15 cm left of the grid centre and **enter -0.15** for the x and y coordinates of **point 1**, then move 30 cm (6 divisions) up and 30 cm to the right of point 1, shift-click and **enter 0.15** for the x and y coordinates of **point 2**. The xy-axes should centre between these two reference points. You can review or change the coordinates of these points by clicking on them.

   The grid in the movie is now calibrated to the air table grid.

2. Use the scrollbar under the viewer window to advance the video frame-by-frame to the start of the collision sequence after the pucks have left the launchers and both white dots are visible. Right-click on the scroll pointer and select this frame as the start frame for the collision analysis.

3. Use the scrollbar to advance the movie to the the end of the collision before the pucks reach the edge of the air table. Right-click the pointer and select this frame as the end frame for the collision analysis.

   You can review this section of the video at various playback rates or step through frames. Adjust the start and end frames, if necessary, using the pointer as before.

4. Click the arrow to the left of the scrollbar to reset the video to the first frame of the selected collision sequence.

5. Referring to the puck on the left as Puck 1, click **Track, New, Point Mass**. A **Track Control** window opens with a tracking object called **mass A**. Hold down 'ctrl+shift' and click on the centre of one of the white dots to set it as the tracking object. You can zoom in with the mouse wheel to get a close-up view of the region of interest.

6. When the **Autotracker** window appears, click **Search**. The software will then proceed to track the object in every frame and generate a table of xy-coordinates as a function of time for that mass. When done, close the window.

7. Reset the video, then identify the other white dot on puck 1 as **mass B** and click **Search** to track it.

8. Repeat these steps to define and track the two white dots on puck 2 as **mass C** and **mass D**.

9. Reset the video, then click **Track, New, Center Of Mass** and check **mass A** and **mass B** to define for puck 1 a centre-of-mass point **cm A** using masses A and B. There will now be three traces visible. Click **View, Track Control** then double-click **mass A** and uncheck the **Visible** option to hide the track for mass A. Repeat for mass B, leaving only the centre-of-mass **cm A** trace visible.

10. Reset the video, then click **Track, New, Center Of Mass** and check **mass C** and **mass D** to define for puck 2 a centre-of-mass point **cm B**. Hide the mass traces.

$\boxed{?}$ Run the video. Were the pucks rotating, thus having angular momentum? Were the dots that will be used to define the before and after vectors evenly spaced and in a straight line?

Analysis of puck trajectories:

You will now use the centre-of-mass coordinate data to generate before and after collision vectors for the two pucks. Note that four velocity vectors ($\vec{v}_{1b}$, $\vec{v}_{2b}$, $\vec{v}_{1a}$, and $\vec{v}_{2a}$) are required.

The Tracker software is able to analyse your data in various convenient ways. For example, after a set of x and y coordinates have been recorded, Tracker can calculate and display for each time interval between two sequential dots, the components of the velocity of the puck in the x and y directions, v_x and v_y, as well as the speed v and direction angle θ_v.

Recall that a vector $\vec{v}$ confined to an xy-plane can be resolved into two orthogonal vectors, an x-component $\vec{v}_x$ with magnitude v_x and a y-component $\vec{v}_y$ with magnitude v_y.

The vector $\vec{v}$ has a magnitude $v = \sqrt{(v_x)^2 + (v_y)^2}$ and points in the xy plane at an angle of $\theta = \arctan(v_y/v_x)$ from the origin.

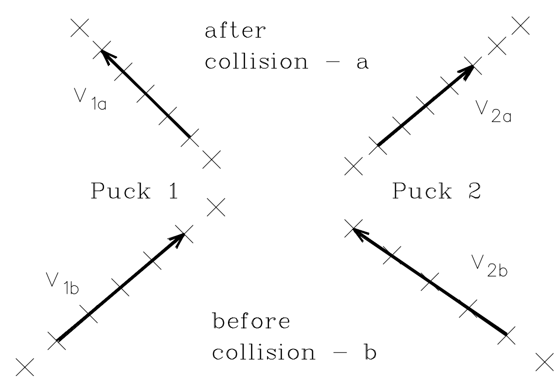


Figure 4.2: Collision vector labels

1. Starting with Puck 1, select **cm A** in the Tracker **Plot** and **Table** windows. The track plot should consist of two collinear series of dots. The series of dots from time $t = 0$ to the moment just before the track changes direction due to the collision are used to define the velocity vector before the collision for Puck 1, labelled v_{1b} in Figure 4.2. The series of dots that are made after the collision define the velocity vector after the collision for Puck 1, labelled v_{1a} in Figure 4.2.

2. Click on **Table** and in the window that appears uncheck x and y to hide the puck displacements, then check v_x, v_y to display columns of the puck component velocities and v, θ_v to display the speed and direction angle.

$\boxed{?}$ Review the speed and angle data. Are the speeds fairly constant or do they display a systematic decrease in time, suggesting that friction is significant?

3. Right-click on one of the table column labeled t and select **Analyse**. The **Data Tool** window appears. Note that angles here are given in radians, although Tracker may be set to display angles in degrees or radians.

   Note: If the velocity data in the Data Tool does not match the data in the Tracker table, right-click any tabs under **File**, select **Close all**, then **No** to save and finally, re-open the **Data Tool** window.

4. In the **Data Tool** click **Analyse** and verify that **Statistics** is checked, then highlight a range of velocities v_{1bx} and v_{1by} for the puck *before* the collision. In the **mean** row are displayed the average values of v_{1bx}, v_{1by}, v_{1b}, and θ_{1b} for the selected data.

   The **sd** row contains the standard deviations (errors), $\delta v_{1bx}, \delta v_{1by}, \delta v_{1b}$, and $\delta\theta_{1b}$. Enter these results in Table 4.1.

5. Highlight in the data table the range of velocities *after* the collision to get the mean and error values $v_{1ax}, v_{1ay}, v_{1a}, \theta_{1a}, \delta v_{1ax}, \delta v_{1ay}, \delta v_{1a}$, and $\delta\theta_{1a}$. Enter these results in Table 4.1.

6. Now repeat the preceding steps for Puck 2. Select **cm B** in the Tracker **Plot** and **Table** windows, then display vectors v_{2b} and v_{2a}.

7. Measure the masses m_1 and m_2 of pucks 1 and 2 using a digital scale and record these values to an accuracy of 0.01 g in Table 4.1. Note the scale measurement error. Be sure to match each mass with the corresponding puck trace, otherwise your Q value will not make sense.

Puck 1	Puck 2
$m_1 \pm \delta m =$	$m_2 \pm \delta m =$
$v_{1bx} \pm \delta v_{1bx} =$	$v_{2bx} \pm \delta v_{2bx} =$
$v_{1by} \pm \delta v_{1by} =$	$v_{2by} \pm \delta v_{2by} =$
$v_{1b} \pm \delta v_{1b} =$	$v_{2b} \pm \delta v_{2b} =$
$\theta_{1b} \pm \delta\theta_{1b} =$	$\theta_{2b} \pm \delta\theta_{2b} =$
$v_{1ax} \pm \delta v_{1ax} =$	$v_{2ax} \pm \delta v_{2ax} =$
$v_{1ay} \pm \delta v_{1ay} =$	$v_{2ay} \pm \delta v_{2ay} =$
$v_{1a} \pm \delta v_{1a} =$	$v_{2a} \pm \delta v_{2a} =$
$\theta_{1a} \pm \delta\theta_{1a} =$	$\theta_{2a} \pm \delta\theta_{2a} =$

Table 4.1: Results of collision analysis

The equation, derived in the Theory section of the lab and used to calculate the experimental mass ratio of the pucks is given by:

$$\frac{m_1}{m_2} = \frac{|\vec{v}_{2a} - \vec{v}_{2b}|}{|\vec{v}_{1a} - \vec{v}_{1b}|}.$$

The vectors $(\vec{v}_{2a} - \vec{v}_{2b})$ and $(\vec{v}_{1b} - \vec{v}_{1a})$ needed to solve this equation depend on the **magnitude and direction** of the individual vectors, hence a **vector subtraction** must be performed.

Since the magnitudes of the x and y components of these vectors are available in Table 4.1, the component vectors can be subtracted separately and then combined to give the magnitude and direction angle of the resultant difference vector.

- Evaluate the magnitudes and directions of these difference vectors, as follows:

$$v_{1x} = (v_{1bx} - v_{1ax}) \quad = \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad = \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

$$v_{1y} = (v_{1by} - v_{1ay}) \quad = \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad = \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

$$(v_{1b} - v_{1a}) = \sqrt{v_{1x}^2 + v_{1y}^2} \quad = \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad = \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\theta_1 = \arctan\left(\frac{v_{1y}}{v_{1x}}\right) \quad = \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad = \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

$$v_{2x} = (v_{2ax} - v_{2bx}) \quad = \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad = \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

$$v_{2y} = (v_{2ay} - v_{2by}) \quad = \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad = \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

$$(v_{2a} - v_{2b}) = \sqrt{v_{2x}^2 + v_{2y}^2} \quad = \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad = \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\theta_2 = \arctan\left(\frac{v_{2y}}{v_{2x}}\right) \quad = \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad = \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

- Since the change in momentum of one puck is equal and opposite to that of the other puck, then the two resultant vectors should then be nearly equal in magnitude since $m_1 \approx m_2$ but now *parallel.*

$\boxed{?}$ Compare the lengths and orientations (angle) of vectors $(\vec{v}_{2a} - \vec{v}_{2b})$ and $(\vec{v}_{1b} - \vec{v}_{1a})$. Does the result make sense in light of the preceding explanation?

Analysis:

- Determine a theoretical value (m_1/m_2) and error $\delta(m_1/m_2)$ for the mass ratio from the given values of m_1 and m_2.

$$\frac{m_1}{m_2} \quad = \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad = \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad = \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\delta\left(\frac{m_1}{m_2}\right) \quad = \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad = \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad = \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\text{(Theoretical)}\ \frac{m_1}{m_2} = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots$$

- Approximate the errors in $(v_{2a} - v_{2b})$ and $(v_{1b} - v_{1a})$ using $\delta v_{1b}, \delta v_{1a}, \delta v_{2b}$, and δv_{2a} from Table 4.1 and the appropriate error equation.

$\delta(v_{1b} - v_{1a})$ = ... = ... = ...

$\delta(v_{2a} - v_{2b})$ = ... = ... = ...

$\dfrac{m_1}{m_2}$ = ... = ... = ...

$\delta\left(\dfrac{m_1}{m_2}\right)$ = ... = ... = ...

(Experimental) $\dfrac{m_1}{m_2}$ = ... ± ....

- The quality factor for the collision is derived in the Theory section of this lab and is given by

$$Q = \frac{m_1\, v_{1a}^2 + m_2\, v_{2a}^2}{m_1\, v_{1b}^2 + m_2\, v_{2b}^2}.$$

Use it and the **theoretical** m_1 and m_2 values to determine Q and δQ.

When solving equations such as for Q and δQ, a good technique is to separate the equation into several terms and solve for each term separately. This avoids repetition and will expose possible calculation errors.

For example, there are four identical mv^2 terms, each of which appears several times in the Q and δQ equations. Evaluate each term only once, then compare the four results. You would expect them to be similar since the m and v values are also similar.

Likewise, the equations for each error term are identical in form and should also yield similar results. You can then confidently complete the calculation of Q and δQ.

$A = m_1 v_{1a}^2$ = ... = ...

$B = m_2 v_{2a}^2$ = ... = ...

$C = m_1 v_{1b}^2$ = ... = ...

$D = m_2 v_{2b}^2$ = ... = ...

$Q = \dfrac{A+B}{C+D}$ = ... = ...

$\delta A = A\sqrt{\left(\dfrac{\delta m_1}{m_1}\right)^2 + \left(2\dfrac{\delta v_{1a}}{v_{1a}}\right)^2}$ = ... = ...

$$\delta B = B\sqrt{\left(\frac{\delta m_2}{m_2}\right)^2 + \left(2\frac{\delta v_{2a}}{v_{2a}}\right)^2} \quad = \ldots\ldots\ldots\ldots\ldots\ldots \quad = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\delta C = C\sqrt{\left(\frac{\delta m_1}{m_1}\right)^2 + \left(2\frac{\delta v_{1b}}{v_{1b}}\right)^2} \quad = \ldots\ldots\ldots\ldots\ldots\ldots \quad = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\delta D = D\sqrt{\left(\frac{\delta m_2}{m_2}\right)^2 + \left(2\frac{\delta v_{2b}}{v_{2b}}\right)^2} \quad = \ldots\ldots\ldots\ldots\ldots\ldots \quad = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\delta Q = Q\sqrt{\frac{(\delta A)^2 + (\delta B)^2}{(A+B)^2} + \frac{(\delta C)^2 + (\delta D)^2}{(C+D)^2}} \quad = \ldots\ldots\ldots\ldots\ldots\ldots \quad = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$Q = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots$$

! Important! Be sure to have this printout signed and dated by a TA before you leave at the end of the lab session. All your work needs to be kept for review by the instructor, if so requested.

Lab report

Go to the “Lab Documents” web page to access the online lab report template for this experiment. Complete the template as instructed and submit it to Turnitin before the lab report submission deadline, late in the evening six days following your scheduled lab session. Do not wait until the last minute. Turnitin will not accept overdue submissions. Unsubmitted lab reports are assigned a grade of zero.

Collisions prelab preparation

The worksheets, videos and all other lab-related content is found at:

`http://www.physics.brocku.ca/Courses/1P91/lab-manual`

- Watch the **sample collision** video, similar to the video that you will recoed and analyse.
- Watch the **Tracker how-to** slideshow introduction to the use of the Tracker software.
- To get a feel for the concepts underlying this experiment, play with the following simulation:

  `https://phet.colorado.edu/en/simulation/legacy/collision-lab`

  The simulation at the "Introduction" tab gives a sense for basic one-dimensional collisions. Try setting both balls to the same mass. What do you note about the velocities and momenta before and after the collision? Try setting the ball 1 mass much larger than the other. What do you note?

  Click on the "Advanced" tab to play with the two-dimensional collisions simulator, which is most relevant for this experiment. Try different starting masses and velocities and see how changing the initial conditions changes the resulting velocities after the collision.

  Again, set both balls to the same mass, then set ball 2 velocity=0 and click on "Show paths". Try a few different collisions. What do you note about the angle between the trajectories of the balls after the collisions?
- Click the Theory link to review how the experiment relies on the following concepts:
  1. momentum
  2. kinetic energy
  3. elastic and inelastic collisions
  4. the principle of conservation of momentum
  5. the principle of conservation of energy
- Do your observations made while trying out the collisions simulation agree with the predictions outlined in the Theory for this experiment?
- Read through the rest of the lab instructions for this experiment in this document.

☐ Print a copy of this experiment to bring to your scheduled lab session. The data, observations and notes entered on these pages will be needed when you write your lab report. Compile these printouts to create a lab book for the course.

CONGRATULATIONS! YOU ARE NOW READY TO PROCEED WITH THE EXPERIMENT!

Conservation of momentum

The velocity $\vec{v}$ of a body of mass m is a vector quantity, symbolized an arrowed line segment.

- The magnitude of the vector represents the speed of the body, a scalar quantity.
- The orientation of the vector in space represents the direction of motion of the body.

If we consider a collision of two objects with masses m_1 and m_2, and with velocities $\vec{v}_{1b}$ and $\vec{v}_{2b}$ *before* the collision, and velocities $\vec{v}_{1a}$ and $\vec{v}_{2a}$ *after* the collision, we note that it is generally difficult, if not impossible, to *predict* these resulting velocities $\vec{v}_{1a}$ and $\vec{v}_{2a}$.

To accomplish this, one would need to have a complete knowledge of the physical characteristics of the objects (size, shape, etc.) and of the geometry of the interaction. However, it is practical to measure all of the relevant quantities and then check to see whether their values are reasonable.

The linear momentum $\vec{p}$ of an object is a vector equal to the product of its mass m (a scalar) and its velocity $\vec{v}$ (a vector). For two colliding objects, the law of conservation of linear momentum states that:

> *If the net external force acting on the colliding objects is zero, then the total momentum $\vec{p}_b$ of the colliding objects before the collision is equal to the total momentum $\vec{p}_a$ after the collision.*

A mathematical formulation of this law for a collision between two objects with masses m_1 and m_2 is a vector equation and can be expressed as follows:

$$\begin{aligned} \vec{p}_{1b} + \vec{p}_{2b} &= \vec{p}_{1a} + \vec{p}_{2a} \\ m_1\vec{v}_{1b} + m_2\vec{v}_{2b} &= m_1\vec{v}_{1a} + m_2\vec{v}_{2a}. \end{aligned} \tag{4.1}$$

Rearranging Equation 4.1 in terms of the changes in the momentum $\Delta\vec{p}_1$ and $\Delta\vec{p}_2$ of the two objects shows that the net change will be zero and that these vectors will be oriented anti-parallel to one another:

$$\vec{p}_{1a} - \vec{p}_{1b} + \vec{p}_{2a} - \vec{p}_{2b} = 0 \quad \rightarrow \quad (\vec{p}_{1a} - \vec{p}_{1b}) = -(\vec{p}_{2a} - \vec{p}_{2b}) \quad \rightarrow \quad \Delta\vec{p}_1 = -\Delta\vec{p}_2$$

Similarly, the changes in the velocity for the two objects will result in two anti-parallel vectors:

$$m_1(\vec{v}_{1a} - \vec{v}_{1b}) = -m_2(\vec{v}_{2a} - \vec{v}_{2b}) \quad \rightarrow \quad m_1\Delta\vec{v}_1 = -m_2\Delta\vec{v}_2 \tag{4.2}$$

Since the two vectors $\Delta\vec{v}_1$, $\Delta\vec{v}_2$ are aligned, the vector Equation 4.2 can be reduced to a scalar equation and expressed in terms of the magnitudes of the vector changes in velocities $|\vec{v}_{2a} - \vec{v}_{2b}|$ and $|\vec{v}_{1a} - \vec{v}_{1b}|$. That is, the magnitudes of the vectors on each side of Equation 4.2 are equal:

$$m_1|\Delta\vec{v}_1| = m_2|\Delta\vec{v}_2| \tag{4.3}$$

Rearranging Equation 4.3 as a ratio of masses m_1 and m_2, we obtain:

$$\frac{m_1}{m_2} = \frac{|\Delta\vec{v}_2|}{|\Delta\vec{v}_1|} = \frac{|\vec{v}_{2a} - \vec{v}_{2b}|}{|\vec{v}_{1a} - \vec{v}_{1b}|}. \tag{4.4}$$

Thus, measuring the masses of the two pucks and the velocities of the pucks before and after the collision to see if Equation 4.4 is satisified is an experimental test of the law of conservation of linear momentum.

Conservation of energy

It is also of interest to know whether kinetic energy $K = mv^2/2$ is conserved during a collision. The "quality factor" $Q = K_a/K_b$ is defined as the ratio of the total kinetic energy K_a after the collision to the total kinetic energy K_b before.

The kinetic energy, and therefore Q, are scalar quantities.

- In an *elastic* collision, the kinetic energy of the system is conserved, and so $Q = 1$ and

$$m_1\, v_{1b}^2 + m_2\, v_{2b}^2 = m_1\, v_{1a}^2 + m_2\, v_{2a}^2 \tag{4.5}$$

- in an *inelastic* collision some of the kinetic energy may be transformed into heat and sound energy during the collision, or there could be frictional forces acting on the masses during the interaction, and so $Q < 1$.

- In principle, in a *superelastic* collision, the total translational kinetic energy may even increase ($Q > 1$), for example if some *rotational* kinetic energy imparted onto the pucks before the collision transfers into translational kinetic energy, or if additional energy is released by the collision itself (*e.g.*, a collision of two spring-loaded mousetraps).

Note that Q depends on the square of the velocity and hence will be very sensitive to variations in v.

$$Q = \frac{K_a}{K_b} = \frac{\frac{1}{2}\, m_1\, v_{1a}^2 + \frac{1}{2}\, m_2\, v_{2a}^2}{\frac{1}{2}\, m_1\, v_{1b}^2 + \frac{1}{2}\, m_2\, v_{2b}^2} = \frac{m_1\, v_{1a}^2 + m_2\, v_{2a}^2}{m_1\, v_{1b}^2 + m_2\, v_{2b}^2} \tag{4.6}$$

Some interesting collisions trivia

In a one-dimensional elastic collision between two objects of masses m_1 and m_2, where the first object has a speed $v_{1b} = |\vec{v}_{1b}|$ and the second object is stationary so that $v_{2b} = 0$:

- if $m_1 = m_2$, then after the collision $v_{1a} = 0$ and $v_{2a} = v_{1b}$;
- if $m_1 \gg m_2$, then after the collision $v_{1a} \approx v_{1b}$ and $v_{2a} \approx 2v_{1b}$;

These results can be obtained from the conservation of energy and momentum Equations 4.5 and 4.1, setting $v_{2b} = 0$ then solving each for v_{1a} and equating the two equations to get

$$v_{2a} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1b} \qquad \text{and from this} \qquad v_{1a} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1b}. \tag{4.7}$$

When $m_1 = m_2$, then $v_{2a} = v_{1b}$ and when $m_1 \gg m_2$, then $v_{2a} \approx 2v_{1b}$, as expected.

Consider a two-dimensional elastic collision between two identical round objects of equal mass $m_1 = m_2$ that are not spinning. The first object has a velocity v_{1b} and the second object is stationary so that $v_{2b} = 0$. Then

- the angle between the velocity vectors $\vec{v}_{1a}$ and $\vec{v}_{2a}$ after the collisions is 90°.

This result can be proved as follows. Because the masses of the two objects are equal, we can set $m_1 = m_2$ in Equation 4.1 (momentum conservation) to obtain

$$\vec{v}_{1b} = \vec{v}_{1a} + \vec{v}_{2a}$$

This means that the three vectors in the previous equation form a triangle. One can see that the triangle is a right triangle by setting $m_1 = m_2$ in Equation 4.5 (conservation of kinetic energy for an elastic collision) to obtain

$$v_{1b}^2 = v_{1a}^2 + v_{2a}^2.$$

Because the side lengths of the triangle are related by the theorem of Pythagoras, it follows that the triangle is a right-angled triangle. Thus, the angle between the outgoing velocity vectors, $\vec{v}_{1a}$ and $\vec{v}_{2a}$, is 90°. This completes the argument.

Billiards enthusiasts will know that this is true based on their experience. Certainly the situation with billiard balls is more complicated, because a skilled practitioner can cause the cue ball to spin in various ways, but if the spin of the cue ball is minimal then the result is approximately true on the billiard table.

(TA INITIALS)

FIRST NAME (PRINT) LAST NAME (PRINT) BROCK ID (AB17CD) (LAB DATE)

Experiment 5

Standing waves

Hardware

The experimental setup consists of a string that is fixed at one end, extends horizontally, wraps a quarter-turn around a pulley, then continues vertically where its end is connected to a mass holder. The tension $F = mg$ applied to the string can be varied by placing different masses m on the mass holder.

A wave driver causes a mechanical vibrator attached to the string near the fixed end to oscillate the string in a vertical plane. The frequency f and amplitude of the sinusoidal vibration can be adjusted to form visible standing wave segments and to vary their number.

The necessary conditions for the production of standing waves on a stretched string fixed at both ends is that the length of the string be equal to a whole number of half wavelengths so that there can be a node at each fixed end of the string.

For this experiment, one fixed end is where the string rests on a pulley and the other is where the string attaches to the mechanical vibrator. The end attached to the pulley is a true node, but the end attached to the metal wand on the vibrator is not exactly a node since the wand vibrates up and down a little. Close examination shows that the true node is a little closer to the fixed end, so the effective string length will be a bit longer than measured. However, the difference will not be more than a few millimeters, so the error introduced is only a fraction of a percent.

Procedure

Carry out your procedure to collect data.

Data analysis

Use available tools that you think are appropriate to analyze your data.

Once you have analyzed the data, draw appropriate conclusions. How confident are you in your conclusions? Quantify your confidence.

(!) Important! Be sure to have this printout signed and dated by a TA before you leave at the end of the lab session. All your work needs to be kept for review by the instructor, if so requested.

Lab report

Complete a discussion of your results and submit it to Turnitin before the lab report submission deadline, late in the evening six days following your scheduled lab session. Do not wait until the last minute. Turnitin will not accept overdue submissions. Unsubmitted lab reports are assigned a grade of zero.

Standing waves prelab preparation

The worksheets, videos and all other lab-related content is found at:

`http://www.physics.brocku.ca/Courses/1P91/lab-manual`

1. Watch the video introduction that describes the equipment that is available for this experiment. The video introduction is found at the "Lab Documents" page at the course web site:
2. Play with the PhET wave motion simulation, found at the following link:

   `https://phet.colorado.edu/sims/html/wave-on-a-string/latest/wave-on-a-string_en.html`

3. In the PhET simulation, briefly explain how to change the number of nodes in the oscillating wave motion.
4. Search for information to learn more about the factors that influence the speed of a transverse wave on a string. Briefly summarize what you have learned.
5. Read the "Hardware" section about the equipment available to conduct this experiment.
6. After you have thought about the available equipment and materials, and you have considered what you have learned by reading the sources of information that you have found and by playing with the simulation linked above:

   design an experiment that will allow you to test the hypotheses that you have formulated about the factors that influence the speed of a transverse wave on a string. You will have to think about which kind of data you will collect, how you will collect the data, which steps you will take to minimize experimental error, and so on.

   Once you have given this some thought, write down your procedure, and produce tables in which you will collect your data.

CONGRATULATIONS! YOU ARE NOW READY TO PROCEED WITH THE EXPERIMENT!

(TA INITIALS)

FIRST NAME (PRINT) LAST NAME (PRINT) BROCK ID (AB17CD) (LAB DATE)

Experiment 6

Review of math basics

Fractions

$$\frac{a}{c}+\frac{b}{d}=\frac{ad+bc}{cd}\ ; \qquad \text{If} \quad \frac{a}{c}=\frac{b}{d}\ , \quad \text{then} \quad ad=cb \quad \text{and} \quad \frac{ad}{bc}=1.$$

Quadratic equations

Squaring a binomial: $(a+b)^2 = a^2 + 2ab + b^2$
Difference of squares: $a^2 - b^2 = (a+b)(a-b)$

The two roots of a quadratic equation $ax^2 + bx + c = 0$ are given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Exponentiation

$$(a^x)(a^y) = a^{(x+y)}\ , \qquad \frac{a^x}{a^y} = a^{x-y}\ , \qquad a^{1/x} = \sqrt[x]{a}\ , \qquad a^{-x} = \frac{1}{a^x}\ , \qquad (a^x)^y = a^{(xy)}$$

Logarithms

Given that $a^x = N$, then the logarithm to the *base* a of a number N is given by $\log_a N = x$.

For the decimal number system where the base of 10 applies, $\log_{10} N \equiv \log N$ and

$$\begin{aligned} \log 1 &= 0 \quad (10^0 = 1) \\ \log 10 &= 1 \quad (10^1 = 10) \\ \log 1000 &= 3 \quad (10^3 = 1000) \end{aligned}$$

Addition and subtraction of logarithms

Given a and b where $a, b > 0$: The log of the product of two numbers is equal to the sum of the individual logarithms, and the log of the quotient of two numbers is equal to the difference between the individual logarithms .

$$\begin{aligned} \log(ab) &= \log a + \log b \\ \log\left(\frac{a}{b}\right) &= \log a - \log b \end{aligned}$$

The following relation holds true for all logarithms:

$$\log a^n = n \log a$$

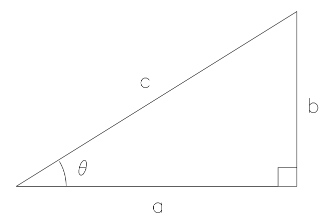
Natural logarithms

It is not necessary to use a whole number for the logarithmic base. A system based on "e" is often used. Logarithms using this base $\log_e$ are written as "ln", pronounced "lawn", and are referred to as *natural logarithms.* This particular base is used because many natural processes are readily expressed as functions of natural logarithms, i.e. as powers of e. The number e is the sum of the infinite series (with $0! \equiv 1$):

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots = 2.71828\ldots$$

Trigonometry

Pythagoras' Theorem states that for a right-angled triangle $c^2 = a^2 + b^2$. Defining a trigonometric identity as the ratio of two sides of the triangle, there will be six possible combinations:



$$\sin\theta = \frac{b}{c} \qquad \cos\theta = \frac{a}{c} \qquad \tan\theta = \frac{b}{a} = \frac{\sin\theta}{\cos\theta}$$

$$\csc\theta = \frac{c}{b} \qquad \sec\theta = \frac{c}{a} \qquad \cot\theta = \frac{a}{b} = \frac{\cos\theta}{\sin\theta}$$

$$\sin(\theta \pm \phi) = \sin\theta\ \cos\phi \pm \cos\theta \sin\phi \qquad \sin 2\theta = 2\sin\theta\ \cos\theta \qquad 180^\circ = \pi \text{ radians} = 3.15159\ldots$$

$$\cos(\theta \pm \phi) = \cos\theta\ \cos\phi \mp \sin\theta \sin\phi \qquad \cos 2\theta = 1 - 2\sin^2\theta \qquad 1 \text{ radian} = 57.296\ldots^\circ$$

$$\tan(\theta \pm \phi) = \frac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta\ \tan\phi} \qquad \tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta} \qquad \sin^2\theta + \cos^2\theta = 1$$

To determine what angle a ratio of sides represents, calculate the inverse of the trig identity:

$$if \quad \sin\theta = \frac{b}{c}, \quad then \quad \theta = \arcsin\left(\frac{b}{c}\right)$$

For *any* triangle with angles A, B, C respectively opposite the sides a, b, c:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}\ , \ (sine\ law) \qquad c^2 = a^2 + b^2 - 2ac\ \cos C.\ (cosine\ law)$$

The sinusoidal waveform

Consider the radius vector that describes the circumference of a circle, as shown in Figure 6.1 If we increase θ at a constant rate from 0 to 2π radians and plot the magnitude of the line segment $b = c\ \sin\theta$ as a function of θ, a sine wave of *amplitude* c and *period* of 2π radians is generated.

Relative to some arbitrary coordinate system, in this case the X-Y axis shown, the *origin* of this sine wave is located at a *offset distance* y_0 from the horizontal axis and at a *phase angle* of θ_0 from the vertical axis.

The sine wave referenced from this (θ, y) coordinate system is given by the equation

$$y = y_0 + c\sin(\theta + \theta_0)$$

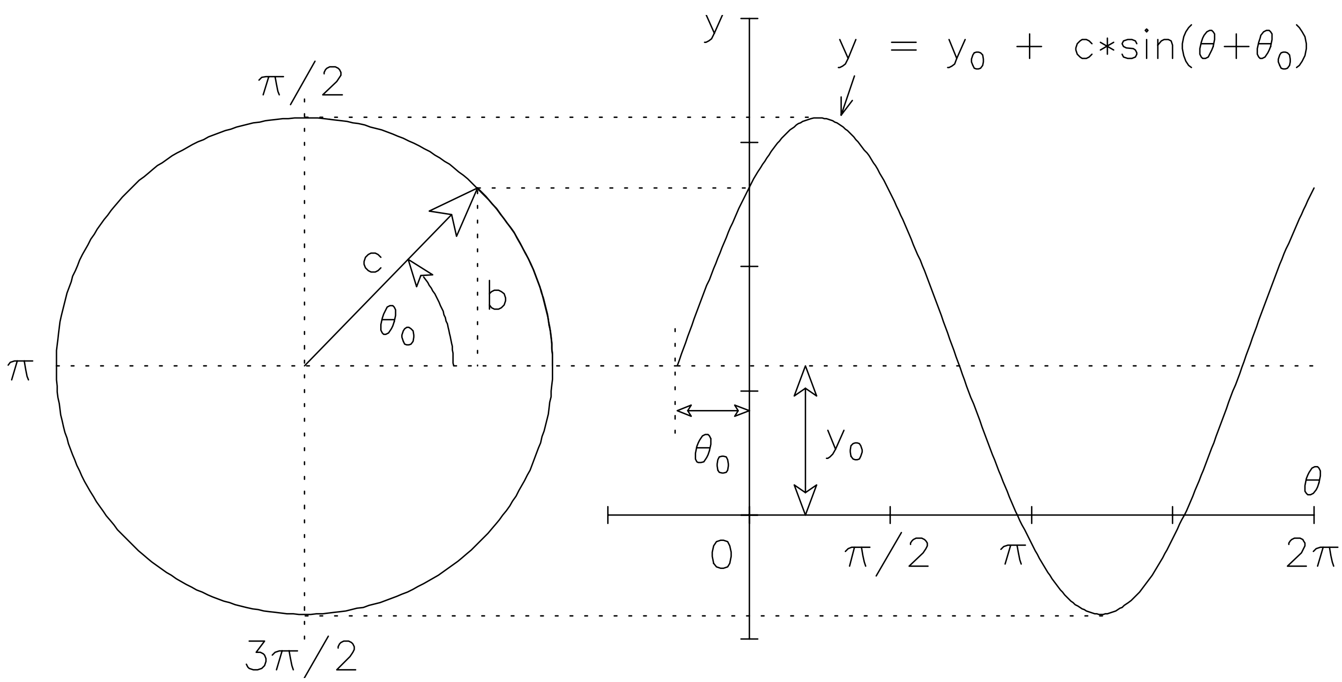


Figure 6.1: Projection of a circular motion to an X-Y plane to generate a sine wave

			(TA INITIALS)
FIRST NAME (PRINT)	LAST NAME (PRINT)	BROCK ID (AB17CD)	(LAB DATE)

Experiment 7

Error propagation rules

- The *Absolute Error* of a quantity Z is given by δZ, always ≥ 0.
- The *Relative Error* of a quantity Z is given by $\frac{\delta Z}{Z}$, always ≥ 0.
- If a constant k has no error associated with it: constant factors out of relative error

$$Z = kA \qquad \delta Z = k\delta A \;\; and \;\; \frac{\delta Z}{Z} = \frac{\delta A}{A}$$

- Addition and subtraction of independent variables: note that error terms *always* add

$$Z = kA \pm B \pm \ldots \qquad \delta(Z) = \sqrt{(k\delta A)^2 + (\delta B)^2 + \ldots}$$

- Multiplication and division of independent variables: constants factor out of relative errors

$$Z = \frac{kA \times B \times \ldots}{C \times D \times \ldots} \qquad \frac{\delta Z}{Z} = \sqrt{\left(\frac{\delta A}{A}\right)^2 + \left(\frac{\delta B}{B}\right)^2 + \left(\frac{\delta C}{C}\right)^2 + \left(\frac{\delta D}{D}\right)^2 \ldots}$$

- Functions of one variable: if the quantity A is measured with uncertainty δA and is then used to compute $F(A)$, then the uncertainty δF in the value of $F(A)$ is given by

$$\delta F = \left(\frac{dF}{dA}\right)\delta A$$

Function $F(A)$	**Derivative,** $\frac{dF}{dA}$	**Error equation**
A^n	nA^{n-1}	$\frac{\delta F}{F} = \lvert n\rvert\frac{\delta A}{A}$
$\log_e A$	A^{-1}	$\delta F = \frac{\delta A}{A}$
$\exp(A)$	$\exp(A)$	$\frac{\delta F}{F} = \delta A$
$\sin(A)$	$\cos(A)$	$\delta F = \lvert\cos(A)\rvert\delta A$
$\cos(A)$	$-\sin(A)$	$\delta F = \lvert -\sin(A)\rvert\delta A$
$\tan(A)$	$\sec(A)^2$	$\delta F = \sec(A)^2\delta A$

All trigonometric functions and the errors in the angle variables are evaluated in radians

How to derive an error equation

Let's use the change of variable method to determine the error equation for the following expression:

$$y = \frac{M}{m}\sqrt{0.5\ kx\,(1-\sin\theta)} \tag{7.1}$$

- Begin by rewriting Equation 7.1 as a product of terms:

$$\begin{aligned} y &= M \ * \ m^{-1} \ * \ [\,0.5 \ * \ k \ * \ x \ * \ (1-\sin\theta)]^{\ 1/2} && (7.2)\\ &= M \ * \ m^{-1} \ * \ 0.5^{1/2} \ * \ k^{1/2} \ * \ x^{1/2} \ * \ (1-\sin\theta)^{1/2} && (7.3)\end{aligned}$$

- Assign to each term in Equation 7.3 a new variable name $A, B, C, \ldots$, then express v in terms of these new variables,

$$y = A \ * \ B \ * \ C \ * \ D \ * \ E \ * \ F \tag{7.4}$$

- With $\delta(y)$ representing the error or uncertainty in the magnitude of y, the error expression for y is easily obtained by applying Rule 4 to the product of terms Equation 7.4:

$$\frac{\delta(y)}{y} = \sqrt{\left(\frac{\delta(A)}{A}\right)^2 + \left(\frac{\delta(B)}{B}\right)^2 + \left(\frac{\delta(C)}{C}\right)^2 + \left(\frac{\delta(D)}{D}\right)^2 + \left(\frac{\delta(E)}{E}\right)^2 + \left(\frac{\delta(F)}{F}\right)^2} \tag{7.5}$$

- Select from the table of error rules an appropriate error expression for each of these new variables as shown below. Note that F requires further simplification since there are two terms under the square root, so we equate these to a variable G:

$$\begin{aligned}
&A = M, && \delta(A) = \delta(M)\\
&B = m^{-1}, && \frac{\delta(B)}{B} = |-1|\ \frac{\delta(m)}{m} = \frac{\delta(m)}{m}\\
&C = 0.5^{1/2}, && \frac{\delta(C)}{C} = |\tfrac{1}{2}|\ \frac{\delta(0.5)}{|0.5|} = 0\\
&D = k^{1/2}, && \frac{\delta(D)}{D} = |\tfrac{1}{2}|\ \frac{\delta(k)}{k} = \frac{\delta(k)}{2k}\\
&E = x^{1/2}, && \frac{\delta(E)}{E} = |\tfrac{1}{2}|\ \frac{\delta(x)}{x} = \frac{\delta(x)}{2x}\\
&F = G^{1/2}, && \frac{\delta(F)}{F} = |\tfrac{1}{2}|\ \frac{\delta(G)}{G} = \frac{\delta(G)}{2G}\\
&G = 1-\sin\theta, && \delta(G) = \sqrt{(\delta(1))^2 + (\delta(\sin\theta))^2} = \cos\theta\delta\theta
\end{aligned}$$

- Finally, replace the error terms into the original error Equation 7.5, simplify and solve for $\delta(y)$ by multiplying both sides of the equation with y:

$$\delta(y) = y\sqrt{\left(\frac{\delta M}{M}\right)^2 + \left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta k}{2k}\right)^2 + \left(\frac{\delta x}{2x}\right)^2 + \left(\frac{\cos\theta\delta\theta}{2-2\sin\theta}\right)^2} \tag{7.6}$$