Data fitting and error analysis

The purposes of this session are to introduce you to your lab-mates and your lab demonstrators, introduce you to the physicalab software that you will use to collect, analyze, and plot data, review essential data analysis concepts, and introduce you to error-analysis calculations. All of this is intended to make your life during subsequent physics lab experiments smoother and more satisfying.

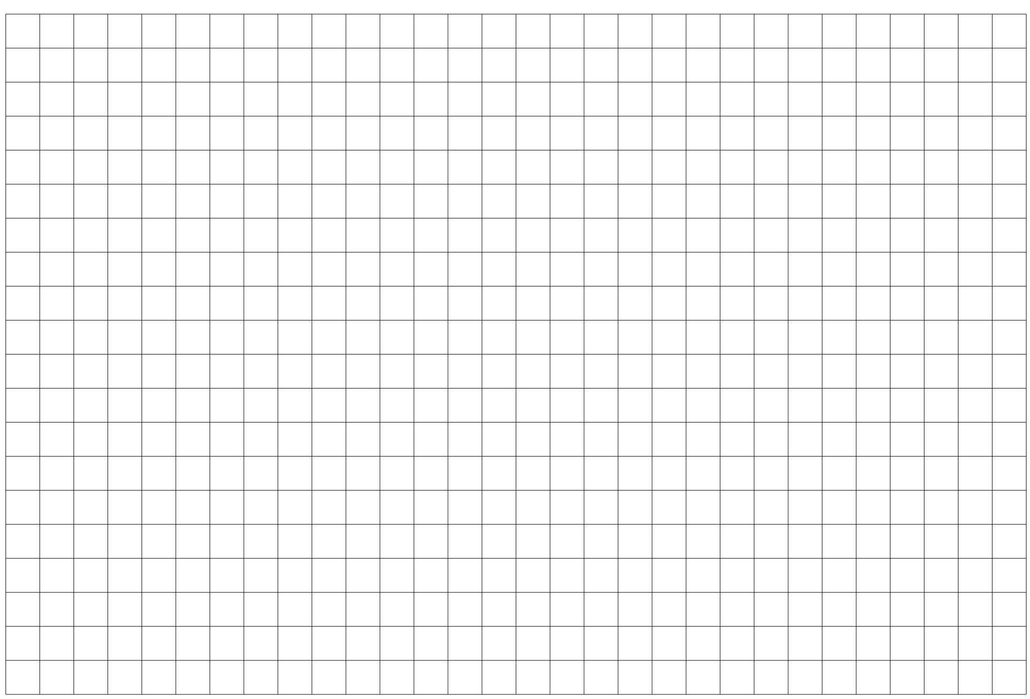
The Cartesian coordinate system for plotting data

The Cartesian coordinate system specifies each point in a two-dimensional grid by a unique pair of coordinates, x and y. The horizontal x-axis and the vertical y-axis cross at the point $(0, 0)$, known as the origin of the coordinate system.

By marking the x and y axes at regular intervals determined by the data to be plotted, the plane is divided into a scaled array of intersecting lines, as shown in the grid below.

To plot a point (x, y), move horizontally along the x-axis to position x and then vertically parallel to the y-axis to position y.

Cartesian coordinate grid



Why do curve fitting?

Suppose that you perform an experiment and collect some data. How would you analyse this set of data? Analysis here refers to the process of extracting useful conclusions from the data. For example, you might expect that the measured variables are related in some specific way; how would you decide whether the data supports the hypothesized relationship?

One can review the data and make a qualitative judgement regarding the trend that the data follows. Then, having this initial sense of how the data behaves, the method of curve fitting, or regression analysis, can be applied to analyse this data more precisely.

Curve fitting attempts to quantify the behaviour of a set of data points by determining a function that represents this behaviour.

The fitting process uses a trial function that includes one or more variable parameters, called fitting parameters. The fitting parameters are adjusted by the fitting algorithm to yield a curve of best fit. The curve of best fit does not necessarily pass through any of the data points, although it might do so. Instead, the curve of best fit is placed so that a certain kind of average distance from the curve to the points is minimized. (You might like to think about what kind of average might be helpful.)

The failure of a fitted curve to pass through the data points is typically due to uncertainties associated with the values of these data points; e.g., due to the limited precision of a measuring instrument.

This mismatch would also occur if the function being fitted to the data did not correctly model the data being fitted. Trying to fit the equation of a straight line to a periodic data set of pendulum oscillations would yield meaningless results.

Once the data is properly fitted, the fitting formula can be used to evaluate various properties inherent in the data. For example, fitting a sine function to a series of points that describe the periodic change in distance with time of a pendulum bob allows the fitting algorithm to precisely determine the period and amplitude of the pendulum oscillation.

Graphing and fitting to a linear data set

A linear data set consists of a series of coordinate points (x, y) where the relationship between the x and y cordinates is always such that a change Δy in the variable y is directly proportional to a change Δx in the variable x.

This relationship can be represented by a function $y = m * x + b$. This is the equation of a straight line with constant slope $m = \Delta y/\Delta x$. The y-intercept b is the value of y when $x = 0$.

Plotting and fitting a linear data set by hand

Here are some (x, y) coordinate points that follow a linear trend:

$$(0.2, 2.4)\ ,\ (1.1, 3.6)\ ,\ (2.2, 4.4)\ ,\ (3.1, 5.7)\ ,\ (4.1, 6.6)\ ,\ (4.9, 7.3)\ ,\ (6.0, 8.4)$$

Proceed as follows to plot these points on a Cartesian grid, then sketch an estimated line of best fit by hand, and finally determine the slope and y-intercept of your estimated line of best fit.

1. Review the data set and determine the x-coordinate minimum and maximum values. Subtract the minimum value from the maximum value, then divide this difference by the number of horizontal grid divisions. Round up the result, if necessary, to get a nice step increment that you can apply to the horizontal axis. (i.e., $x_{max} - x_{min} = 6.0 - 0.2 = 5.8$, $5.8/30 = 0.193$, so use 0.2 for the step size)

   Label the bottom of the grid with these incremental step values, starting from the left side, so that all the x values fit on the grid. (i.e., $x_{min} = 0.2$ so start with 0.2, then 0.4, 0.6, etc.)

2. Repeat the above procedure using the y-coordinates, dividing by the number of vertical grid divisions and labelling the left edge of the grid. Data graphs need not include the origin $(0, 0)$.

3. With the grid now scaled, plot each of the data points onto the grid. Carefully estimate the position of points that do not lie exactly on the intersection of two grid lines.

   You will note that the points appear to follow a linear behaviour; they appear to lie on a straight line. This observation gives you the hint that if you were going to try and fit some function to this data set, then the a straight line might be a good choice.

4. Now use a straight edge to draw a straight line that you think best estimates your data points, extending it past the left and right edges of the grid.

$\boxed{?}$ How did you choose where to place your *line of best fit*?

5. Estimate the coordinates of these two edge points (x_1, y_1) on the left edge and (x_2, y_2) on the right edge, then determine the slope and y-intercept of your line of best fit, as described above. This choice of points will typically yield the least relative error.

$$
\begin{aligned}
slope:\ m &= \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} &= \ldots\ldots\ldots\ldots\ldots\ldots &= \ldots\ldots\ldots\ldots\ldots\ldots \\
y - intercept:\ b &= y_1 - mx_1 \equiv y_2 - mx_2 &= \ldots\ldots\ldots\ldots\ldots\ldots &= \ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}
$$

Plotting and fitting a linear data set using Physicalab

Now that you have estimated a line of best fit by hand, use the Physicalab software to determine a more precise line of best fit for the same set of data.

- In Physicalab, click **File**, then **Load linear data** to place some points in the data window.

- Select **fit to: y=** and enter **A*x+B** in the fitting equation box then click **Draw** to generate a graph of the data with a straight line fitted to it.

  The slope **A** and y-intercept **B** are the *fitting parameters* adjusted by the fitting algorithm to yield the equation of the line of best fit that is drawn over your data points. The resulting values of the fitting parameters appear below the graph.

  Note that the points do not lie exactly on the line. This total difference between the y-values of the points and the y-values predicted by the fitted line is used by the fitting program to determine the uncertainty in the values of the fit parameters.

  The uncertainty, or error, in **A** is denoted by δ**A** and the error in **B** is denoted by δ**B**. These results are typically displayed in the form **A**$\pm\delta$**A**, **B**$\pm\delta$**B**. The smaller the error values, the better the fit of the equation to the data set; if all the points were to lie exactly on the line, then these uncertainties would be zero.

? Record below the resulting values from the Physicalab fit of the linear data set. How do these results compare with your fit estimates for the slope and the y-intercept?

$$\begin{aligned} slope:\ \mathbf{A} &= \ldots\ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots\ldots \\ y - intercept:\ \mathbf{B} &= \ldots\ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}$$

Graphing and fitting to a periodic data set

A data set that is periodic has y values that repeat regularly after an interval in x. Many such periodic data sets can be fitted to a sine function, or sine wave. The basic sine function has formula $y = \sin(x)$, and a general sine function has formula $y = A\sin(Bx + C) + D$. Do you recall from high school how changing the values of A, B, C, and D influence the graph?

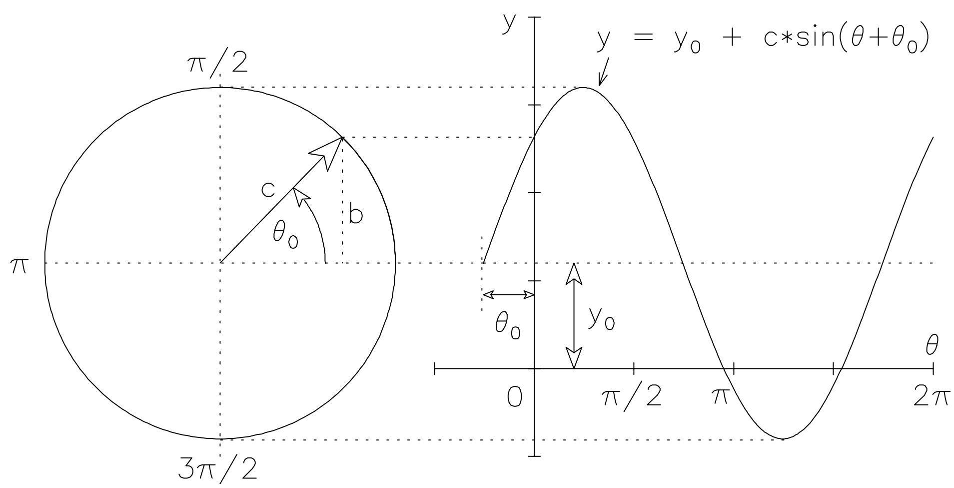


Figure 1: Projection of a circular motion to an xy- plane to generate a sine wave

Consider the radius vector of magnitude c that describes the circumference of a circle, as shown in Figure 1. Recall that a circle of radius r has a circumference of $2\pi r$. If we sweep the radius vector along this circumference by an arc distance equal to the circle radius, then the angle θ is increased by one radian. If we increase θ at a constant rate ω from 0 to 2π radians and plot the magnitude of the vertical line segment $b = c\ \sin\theta$ as a function of θ, a sine wave of amplitude c and period of 2π radians is generated.

You can see this in Figure 1 if you imagine a spotlight shining on the radius vector from the left so that the radius vector casts a shadow on a screen located towards the right (say, on the y-axis). As the radius vector rotates in a circle, the shadow of its tip just oscillates up and down on the screen. The location of the shadow of the tip of the radius vector is plotted on the sine curve at the right of the figure as time passes.

The variable $\omega = \Delta\theta/\Delta t$, represents the angular velocity of the radius vector as it rotates in a circle on the left diagram of Figure 1. This same quantity determines how quickly the sine wave evolves in time. For motions that are periodic, but are not circular, the quantity ω is called the angular frequency. An example of such a motion is the back-and-forth motion of the shadow of the tip of the radius vector.

If the sine wave advances by one cycle then the angle θ changes by $\Delta\theta = 2\pi$ radians. The time needed for the sine wave to advance by one cycle is called the period of the motion. Thus, the period T is related to the angular frequency ω by

$$\omega = \frac{2\pi}{T} \qquad \text{which is equivalent to} \qquad T = \frac{2\pi}{\omega}$$

Relative to some arbitrary coordinate system, in this case the xy-axes shown, the sine wave is shifted by an *offset distance* of y_0 from the horizontal axis and by a *phase angle* of θ_0 from the vertical axis. In this (θ, y) coordinate system, the sine wave is represented by the formula $y = c\sin(\theta + \theta_0) + y_0$. Expressing the formula using ω we get

$$y = c\sin(\omega t + \theta_0) + y_0 \tag{1}$$

Physicalab uses the same format, but different symbols, as follows:

$$y = A\sin(Bx + C) + D$$

By referencing Figure 1 you will be able to provide graphical interpretations for the Physicalab fitting parameters A, B, C, and D.

Fitting to a sine wave using Physicalab

- Click **File**, then **Load sinusoidal data**. This is some sample data obtained from the pendulum experiment (Experiment 1) that you will perform later in the course.
- Click **Draw** to generate a graph of the data. Always check that the data points follow the general shape of the function that you are going to fit to the data. In this case, the set of points should resemble a nice smooth sine wave, without spikes, stray points or flat spots.
- Select **fit to: y=** and enter **A*sin(B*x+C)+D** in the fitting equation box. Comparing this equation with Equation 1, we see that **x** represents the independent variable (typically in units of time), **A** is the amplitude of the sine wave, **C** is the initial phase angle (in radians) of the wave when **x** = 0, and **D** is the offset distance of the wave measured from the x-axis, so that when **A** = 0, $y =$ **D**.

  The fit parameter **B** (in radians/s) is the rate of change in angle with time, so that **B*x** is an angle in radians. If this angle is advanced by 2π radians, so that **B*x** $= 2\pi$, then the time changes by an amount **x** $= T$, where T represents the period of the sine wave in seconds. Then **B** $= 2\pi/T$.
- Look at your graph and enter some reasonable *approximate* values for the fitting parameters. You can get an initial guess for **B** by estimating the time **x** between two adjacent minima, or one period, of the sine wave and approximate the value of $2\pi = 6.2831\ldots$ with the number 6.

$$\begin{aligned} amplitude: &\quad \mathbf{A} \approx \ldots\ldots\ldots\ldots\ldots\ldots \\ angular\ velocity: &\quad \mathbf{B} \approx \ldots\ldots\ldots\ldots\ldots\ldots \\ initial\ phase\ angle: &\quad \mathbf{C} \approx \ldots\ldots\ldots\ldots\ldots\ldots \\ offset: &\quad \mathbf{D} \approx \ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}$$

- Click **Draw**. If you get an error message the initial guesses for the fitting parameters may be too distant from the required values for the fitting program to properly converge. Try again.

Introduction to the analysis of uncertainty (error analysis)

Beware! In general conversation, the term **error** usually refers to some sort of **mistake**.

In scientific discussions the term **error** refers specifically to the **uncertainty** δX in the **magnitude** of a measured quantity X. A measured quantity X without an associated error δX is meaningless, as there is no way to establish how reliable the value of X is. A proper result is expressed as a pair of numbers $X \pm \delta X$.

A sample data set

Table 1 contains a sample data set obtained from the Pendulum experiment. In the experiment, the length s of a string supporting a ball of diameter d that acts as the pendulum bob is adjusted and the length s is measured and recorded. The pendulum length L is the string length plus the radius of the ball: $L = s + \frac{1}{2}d$. (Note that the radius of the ball is half of its diameter.)

The pendulum ball is set swinging and the motion is recorded and fitted to a sine function. The fit parameter $\mathbf{B}$ is obtained from the fit and the acceleration due to gravity g is obtained from $g = \mathbf{B}^2 L$.

There were five trials done using a ball of mass m_1 and a single trial using a ball of mass m_2. The balls have a different diameter d. We will use this data to determine by different methods values for g and their associated errors δg.

Run, i	mass	m (kg)	d (m)	s (m)	L (m)	B (rad/s)	g_i (m/s^2)
1	m_1	0.0225	0.02540	0.300	0.3127	5.59641±0.00267	9.79370
2	m_1	0.0225	0.02540	0.450	0.4627	4.60396±0.00224	9.80760
3	m_1	0.0225	0.02540	0.600	0.6127	4.00688±0.00163	9.83695
4	m_1	0.0225	0.02540	0.750	0.7627	3.58703±0.00182	9.81350
5	m_1	0.0225	0.02540	0.900	0.9127	3.27880±0.00229	9.81201
1	m_2	0.0095	0.01904	0.500	0.5095	4.38192±0.00240	9.78344

Table 1: Table of experimental results

Determining the uncertainty in a single measurement

Because there was only one trial using the ball of mass m_2, the only option is to use error propagation rules to determine error estimates for L and g.

First, we need to determine the magnitude of the uncertainties in the measured values of the string length s and the diameter of the ball d.

The **measurement errors** in s and d, represented by δs and δd respectively, are determined from the **precision** of the scales of the measuring instruments. This error is expressed as ± one-half of the smallest increment, or **resolution**, of the scale used to make the measurement.

The scale used to set the string length s had a resolution of 0.001 m, while the micrometer used to measure the ball diameter d had a scale increment, of 0.00001 m. The errors are:

$$\delta s = \pm \ldots\ldots\ldots\ldots\ldots \qquad\qquad \delta d = \pm \ldots\ldots\ldots\ldots\ldots$$

The proper way to show a calculation is in three steps: first display the relevant equation, then replace the variables by their unrounded values and finally show the numerical result. Do not include units. The error equation for L is obtained from the error rules in the Appendix. Then for m_2,

$$L = s + \tfrac{1}{2}d = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\delta L = \sqrt{(\delta s)^2 + \left(\tfrac{1}{2}\delta d\right)^2} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Once these two values are obtained, the final result for $L \pm \delta L$, is obtained as follows:

1. round the error term, in this case δL, to one significant digit. Round up this digit by adding one to it if the digit that followed it was greater than four, i.e. $0.0149 \rightarrow 0.01$ and $0.0150 \rightarrow 0.02$;
2. round the result, in this case L, to the same precision (or decimal place) as the error term, rounding up this digit if the digit that followed it was greater than four.

   For example, if $\delta X = 0.014$ and $X = 4.354$ then $X \pm \delta X = 4.35 \pm 0.01$. If $\delta X = 0.015$ and $X = 4.355$ then $X \pm \delta X = 4.36 \pm 0.02$.

Finally, use the format below to show the properly rounded pair of values, with the appropriate units:

$$L = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$$

To get a final result for g using the ball of mass m_2, the error equation for $g = \mathbf{B}^2 L$ is needed. This time it is a good idea to perform a change of variables to determine δg.

Note that the error equation is a product of two terms, $\mathbf{B}^2$ and L. Let $A = \mathbf{B}^2$ then $g = AL$. Using the power rule and the product rule respectively:

$$F = X^2 \longrightarrow \frac{\delta F}{F} = 2\frac{\delta X}{X}, \qquad\qquad F = XY \longrightarrow \frac{\delta F}{F} = \sqrt{\left(\frac{\delta X}{X}\right)^2 + \left(\frac{\delta Y}{Y}\right)^2} \tag{2}$$

we replace the variables and substite the $\delta A/A$ term in the g error equation:

$$A = \mathbf{B}^2 \longrightarrow \frac{\delta A}{A} = 2\frac{\delta \mathbf{B}}{\mathbf{B}}, \qquad\qquad g = AL \longrightarrow \frac{\delta g}{g} = \sqrt{\left(\frac{\delta A}{A}\right)^2 + \left(\frac{\delta L}{L}\right)^2} = \sqrt{\left(2\frac{\delta \mathbf{B}}{\mathbf{B}}\right)^2 + \left(\frac{\delta L}{L}\right)^2} \tag{3}$$

$$g = \mathbf{B}^2 L = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\delta g = g\sqrt{\left(\tfrac{2\delta \mathbf{B}}{\mathbf{B}}\right)^2 + \left(\tfrac{\delta L}{L}\right)^2} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$g = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$$

Determining the uncertainty from a series of measurements

Note: the symbol Δ is traditionally used to represent a difference of two values, as with $\Delta x = x_2 - x_1$, whereas the symbols δ and σ represent the uncretainty associated with a result.

In Table 1, there are five trials ($i = 1, \ldots, N = 5$) using the large ball of mass m_1. These five results for g are *expected to have the same value*. In this case, you can invoke the theory of statistics to evaluate a sample average $\langle g \rangle$, or mean value, of the five trials as well as the standard deviation of the sample $\sigma(g)$ that gives a measure of the scattering of the trials around $\langle g \rangle$.

The sample average of N values g_i is given by the sum of the samples divided by the number of samples:

$$\langle g \rangle = \frac{1}{N} \sum_{1}^{N} g_i = \frac{g_1 + g_2 + \ldots + g_N}{N} \tag{4}$$

To get a feel for what is involved, let's perform a manual standard deviation calculation.

i	g_i	$\Delta g_i = g_i - \langle g \rangle$	$(\Delta g_i)^2$
1			
2			
3			
4			
5			
$\langle g \rangle =$		*variance* $=$ $\sigma(g) = \sqrt{variance} =$	

Table 2: Calculation template for g and $\sigma(g)$.

- To begin, we use Equation 4 to calculate $\langle g \rangle$.
- Then for each g_i we calculate the difference Δg_i and $(\Delta g_i)^2$.
- The variance of the sample is a sum of the $(\Delta g_i)^2$ terms, this time divided by $N - 1$:

$$variance = \sigma(g)^2 = \frac{1}{N-1} \sum_{1}^{N} (\Delta g_i)^2 = \frac{(\Delta g_1)^2 + (\Delta g_2)^2 + \ldots + (\Delta g_N)^2}{N-1}.$$

- Finally, we determine the standard deviation $\sigma(g)$ from the variance in the sample of g values:

$$g = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots$$

- You can also enter the five g values as a column in Physicalab, then from the **Edit** menu select **Insert X Index** to add a column of index values. Check **bellcurve** to view your data as a distribution.

Compare the mean and standard deviation values from the graph with your results from Table 2. They should be the same.

. . . .

- To view a distribution of g values with a larger number of samples, click **File** and **Load distribution data**. Check the **bargraph** box to display the number of samples in each bin as a series of vertical bars. See how the graph changes as you partition the data into a different number of bins.

$$g = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$$

How does this result for a larger data sample compare with the result for a sample of five g values?

. . . .

(TA INITIALS)

FIRST NAME (PRINT) LAST NAME (PRINT) BROCK ID (AB17CD) (LAB DATE)

Experiment 1

Archimedes' principle

In this experiment you will determine the unknown ratio of copper (Cu) to aluminum (Al) in a simulated "alloy" of the two metals . The experimental apparatus consists of a precise digital weight scale, a volumetric flask, a pipette, distilled water, a long bar of Cu, a long bar of Al, and a simulated Al/Cu "alloy" made up of two short bars of Al and Cu.

For each of the three metals (Al, Cu and the Al/Cu "alloy") you will need to perform three separate measurements, as summarized in Fig. 1.1:

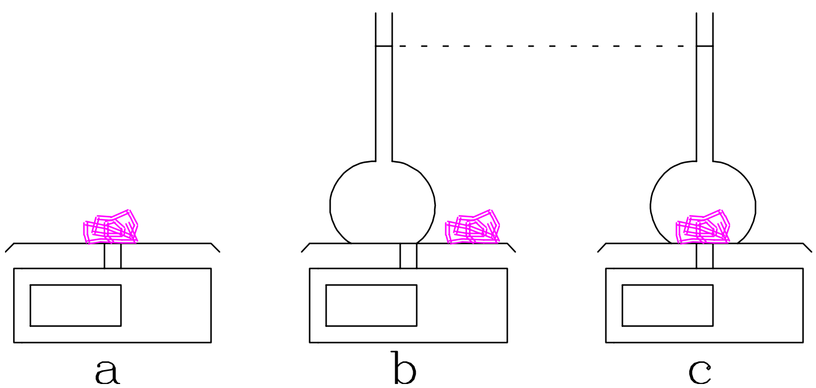


Figure 1.1: The three steps must be repeated for each metal

(**a**) weight $w_{\mathbf{a}}$ of the piece of metal;

(**b**) weight $w_{\mathbf{b}}$ of the piece of metal and of the volumetric task filled with distilled water to the exact mark on the neck of the flask;

(**c**) weight $w_{\mathbf{c}}$ of the piece of metal *submerged in* the volumetric task filled with distilled water to the exact same mark.

You will need to withdraw some water from the volumetric flask between steps (**b**) and (**c**). This is best achieved by simply pouring off some water and then adding the required amount back, drop-by-drop from the pipette when the level gets close to the target mark.

These weight measurements rely heavily on your experimental technique. To avoid introducing errors, be careful not to have any stray water droplets on the piece of metal, on the body of the flask or on the scale platform itself.

To get a feeling for how these experimental errors affect your results, you will repeat steps (**a**)–(**c**) several times and perform a statistical analysis of the data to determine the sample average $\langle w \rangle$ and the standard deviation $\sigma(w)$, which is a measure of how closely your data is distributed around $\langle w \rangle$.

Part 1: Aluminum bar

$\boxed{?}$ A major source of random error in this experiment is the precision with which you can reproduce the exact same water level time after time. How much does the water level rise after a drop is added?

- Starting with the aluminum bar, fill in Table 1.1. Proceed column by column.
- Close any open Physicalab windows, then start a new Physicalab session by clicking on the desktop icon and login with your Brock student ID. You will be emailing yourself all the graphs that you create for later inclusion in your lab report.

Al	1	2	3	4	5	6	7	8	9	$\langle w \rangle$	$\sigma(w)$
Metal bar, $w_{\mathbf{a}}$											
Metal & flask, $w_{\mathbf{b}}$											
Metal in flask, $w_{\mathbf{c}}$											

Table 1.1: Experimental data for aluminum bar

- In Physicalab, click `File, New` to clear the data entry window, then enter in a column the data points from step (**a**) for the aluminum metal bar. Click `Edit, Insert X index` to insert a column of indices to your data points, then select `scatter plot` and click **Draw** to view the variation in your data.
- Select `bellcurve` and click `bargraph` to view the distribution of your data. Physicalab has calculated the average $\langle w \rangle$ and standard deviation $\sigma(w)$ of your data set and these values are shown in the graph window as $\langle w \rangle \pm \sigma(w)$. Enter these two values in the $\langle w \rangle$ and $\sigma(w)$ columns of Table 1.1.
- Repeat the above steps for the `Metal & flask` data set and the `Metal in flask` data set.
- You can now determine the specific gravity S_{Al} of aluminum with the aid of Equation 1.4 and your results from Table 1.1:

$$S_{Al} \;=\; \ldots\ldots\ldots\ldots\ldots\ldots\ldots \;=\; \ldots\ldots\ldots\ldots\ldots\ldots\ldots \;=\; \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\Delta(S_{Al}) \;=\; \ldots\ldots\ldots\ldots\ldots\ldots\ldots \;=\; \ldots\ldots\ldots\ldots\ldots\ldots\ldots \;=\; \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Part 2: Copper bar

- Proceed to acquire data and fill in Table 1.2 for the copper bar.

Cu	1	2	3	4	5	6	7	8	9	$\langle w \rangle$	$\sigma(w)$
Metal bar, $w_{\mathbf{a}}$											
Metal & flask, $w_{\mathbf{b}}$											
Metal in flask, $w_{\mathbf{c}}$											

Table 1.2: Experimental data for copper bar

S_{Cu} = ... = ... = ...

$\Delta(S_{Cu})$ = ... = ... = ...

Part 3: Alloy bar

- Complete Table 1.3 for the Al/Cu alloy bar composed of the two short Al, Cu bars.

Alloy	1	2	3	4	5	6	7	8	9	$\langle w \rangle$	$\sigma(w)$
Metal bar, $w_{\mathbf{a}}$											
Metal & flask, $w_{\mathbf{b}}$											
Metal in flask, $w_{\mathbf{c}}$											

Table 1.3: Experimental data for Al/Cu alloy bar

S_{alloy} = ... = ... = ...

$\Delta(S_{alloy})$ = ... = ... = ...

Part 4: Results

- Measure the weight m_{Al} of the short bar of aluminum and the weight m_{Cu} of the short bar of copper that make up the alloy bar. Include the error in these measurements. These are the theoretical values that will be used to compare to your experimental results based on the previously calculated specific gravities of the various materials.

m_{Al} = ... $\pm$ .... m_{Cu} = ... $\pm$ ....

- From these measurements, determine the theoretical mass ratio of your "alloy" and the magnitude of uncertainty, or error, in this result:

$$\frac{m_{\text{Al}}}{m_{\text{Cu}}} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\Delta\left(\frac{m_{\text{Al}}}{m_{\text{Cu}}}\right) = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

- Use Equation 1.5 to calculate the experimental mass ratio and the associated error for your "alloy":

$$\frac{m_{\text{Al}}}{m_{\text{Cu}}} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\Delta\left(\frac{m_{\text{Al}}}{m_{\text{Cu}}}\right) = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

! Important! Be sure to have this printout signed and dated by a TA before you leave at the end of the lab session. All your work needs to be kept for review by the instructor, if so requested.

Lab report

Go to the "Lab Documents" web page to access the online lab report template for this experiment. Complete the template as instructed and submit it to Turnitin before the lab report submission deadline, late in the evening six days following your scheduled lab session. Do not wait until the last minute. Turnitin will not accept overdue submissions. Unsubmitted lab reports are assigned a grade of zero.

Archimedes prelab preparation

Worksheets, videos and all other lab-related content is located at:

`http://www.physics.brocku.ca/Courses/1P92/lab-manual`

- For a very entertaining and informative cartoon introduction to this lab experiment, and the famous story of Archimedes:

  `http://archimedespalimpsest.org/images/kaltoon/`

- To get a more quantitative feel for the concepts underlying this experiment, try the following simulation:

  `https://phet.colorado.edu/en/simulation/buoyancy`

  The section labelled "Intro" provides a basic introduction and the section labelled "Buoyancy Playground" allows you a lot more freedom to play.

- Devise an experiment to test the hypothesis that when a solid object is immersed in water, the volume of the water displaced by the object is equal to the volume of the object.

- Use your cell phone to make a short movie of your experiment.

- Open a word processor document and outline the steps that you intend to take to test your hypothesis. This document will be submitted to Turnitin.

  Your outline should itemize the steps in a sequence typical of the Scientific Method:

  1. **Hypothesis:** State the purpose of your experiment and the result that you expect to get.
  2. **Experiment:** Describe the experimental setup that you used and the procedural steps that you took to test your hypothesis. For example, you might have used an object of known volume or an aggregate of small particles, such as sand or rice, that will conform to the shape of a container and allow you to determine the volume.
  3. **Conclusion:** Summarize the outcome of your experiment: did your observations confirm or contradict your hypothesis?
  4. **Publish your results:** Upload the movie to your Facebook page or another social media site and include the link to the file as part of your Turnitin document.

     Login to Turnitin and submit your file to the Archimedes prelab assignment before the "Due" time and date shown. Do not wait until the last minute to submit your report. Turnitin will not accept submissions after the set due date/time. Note that overdue prelab reports are assigned a grade of zero.

- [ ] Print a copy of the experimental procedures from the Laboratory manual to bring to your scheduled lab session. The data, observations and notes entered on these pages will be needed when you write your lab report. Compile these printouts to create a lab book for the course.

CONGRATULATIONS! YOU ARE NOW READY TO PROCEED WITH THE EXPERIMENT!

Archimedes' principle

Everybody knows the story's punch line: A man is so excited by the idea that came to him in a bathtub that he runs naked to the emperor's palace, shouting "Eureka!" But what exactly was the idea that made Archimedes forget the dress code?

Survey your friends who are not taking this physics course, and most are likely to respond with some description of *buoyancy*: A body immersed in a fluid experiences a buoyant force equal to the weight of the displaced fluid.

That's important enough, and Archimedes did state a very concise formulation of the buoyancy principle. However, boats floated long before Archimedes came along. Why would a better formulation of an old idea excite him so?

The full story of "Eureka!" is somewhat more subtle. Archimedes figured out a way to *use* the buoyant force to solve a very important practical problem of catching the crooks who were defrauding the treasury by passing off gold-silver alloy coins as pure gold ones. An alloy is a material composed of two or more different metals.

Before you continue reading the next paragraph, spend a few minutes trying to think of a solution to this challenge. It's not an easy one!

Archimedes' solution (which brought him both the satisfaction of resolving a intellectual challenge and a considerable monetary reward) could be implemented quickly and easily and required only the simplest of tools: A balance scale, weights made of pure silver and pure gold, and a tub of water.

First, you had to use the weights made of gold balance out the unknown material. Then you would submerge both sides of the balance in water. If the two arms remained balanced, then the unknown material was also gold since the same mass of the material displaced the same volume of water on both sides and thus both sides experienced the same buoyant force equal to the weight of that water.

If, however, the material was not really gold, its density was slightly different from that of the pure gold, and the same mass would displace a different volume of water. The buoyant force would be slightly different on the two sides of the balance scale, and the submerged balance would tilt.

In fact, by replacing the pure gold weights with a mix of gold and silver weights and adjusting their ratio until the balance scale remained level in and out of water, one could measure the exact make-up of the alloy — and catch the crooks!

Density of materials

The *density* ρ of a material is defined as the ratio of its mass m to its volume V,

$$\rho = \frac{m}{V} \tag{1.1}$$

and has units such as kg/m^3 or g/cm^3. The density of an alloy can be determined by dividing the sum of the component masses by the sum of the component volumes. For an alloy of aluminum (Al) and copper (Cu), the alloy density is given by:

$$\rho_{\text{alloy}} = \frac{m_{\text{Al}} + m_{\text{Cu}}}{V_{\text{Al}} + V_{\text{Cu}}}. \tag{1.2}$$

Specific gravity of materials

The *specific gravity* $\mathcal{S}_x$ of a material x is defined as the dimensionless (unitless) ratio of the density of the material ρ_x to the density of water $\rho_{\mathrm{H_2O}}$:

$$\mathcal{S}_x = \frac{\rho_x}{\rho_{\mathrm{H_2O}}} \tag{1.3}$$

Suppose that you have a solid object of mass m_x and volume V_x that weighs $w_{\mathbf{a}} = m_x g$ at the earth's surface and a container full of water so that their combined weight is $w_{\mathbf{b}}$. If you submerge the object in the container, some of the water will spill out of the container, decreasing the combined weight of object and container to $w_{\mathbf{c}}$.

The volume of the spilled water $V_{\mathrm{H_2O}}$ is equal to the volume of the object: $V_{\mathrm{H_2O}} = V_x$. The weight of the spilled water, or the apparent weight loss of the object when submerged in water, is $m_{\mathrm{H_2O}}\, g = w_{\mathbf{b}} - w_{\mathbf{c}}$.

If the numerator and denominator of Equation 1.3 are multiplied by $V g$, the numerator becomes the weight of the object and the denominator becomes the weight of the water displaced. This provides us with a practical expression for the object's specific gravity:

$$\mathcal{S}_x = \frac{m_x g}{m_{\mathrm{H_2O}}\, g} = \frac{w_{\mathbf{a}}}{w_{\mathbf{b}} - w_{\mathbf{c}}} = \frac{\text{weight of object in air}}{\text{apparent weight loss when submerged in water}} \tag{1.4}$$

Determining the alloy mass ratio from specific gravity results

To compare a theoretical result, the mass ratio of the two metals, to that obtained from the experimentally determined specific gravities of the component materials, Equation 1.2 can be re-arranged as follows:

$$\begin{aligned}
m_{\mathrm{Al}} + m_{\mathrm{Cu}} &= \rho_{\mathrm{alloy}}\, (V_{\mathrm{Al}} + V_{\mathrm{Cu}}) \\
&= \rho_{\mathrm{alloy}} \left(\frac{m_{\mathrm{Al}}}{\rho_{\mathrm{Al}}} + \frac{m_{\mathrm{Cu}}}{\rho_{\mathrm{Cu}}} \right) \\
&= \mathcal{S}_{\mathrm{alloy}}\, \rho_{\mathrm{H_2O}} \left(\frac{m_{\mathrm{Al}}}{\mathcal{S}_{\mathrm{Al}}\, \rho_{\mathrm{H_2O}}} + \frac{m_{\mathrm{Cu}}}{\mathcal{S}_{\mathrm{Cu}}\, \rho_{\mathrm{H_2O}}} \right) \\
&= m_{\mathrm{Al}} \left(\frac{\mathcal{S}_{\mathrm{alloy}}}{\mathcal{S}_{\mathrm{Al}}} \right) + m_{\mathrm{Cu}} \left(\frac{\mathcal{S}_{\mathrm{alloy}}}{\mathcal{S}_{\mathrm{Cu}}} \right)
\end{aligned}$$

and is finally expressed in terms of the mass ratio:

$$\frac{m_{\mathrm{Al}}}{m_{\mathrm{Cu}}} = -\frac{1 - \mathcal{S}_{\mathrm{alloy}}/\mathcal{S}_{\mathrm{Cu}}}{1 - \mathcal{S}_{\mathrm{alloy}}/\mathcal{S}_{\mathrm{Al}}}. \tag{1.5}$$

The corresponding error equation is:

$$\delta \left(\frac{m_{\mathrm{Al}}}{m_{\mathrm{Cu}}} \right) = \left(\frac{m_{\mathrm{Al}}}{m_{\mathrm{Cu}}} \right) \sqrt{2 \left(\frac{\delta \mathcal{S}_{\mathrm{alloy}}}{\mathcal{S}_{\mathrm{alloy}}} \right)^2 + \left(\frac{\delta \mathcal{S}_{\mathrm{Al}}}{\mathcal{S}_{\mathrm{Al}}} \right)^2 + \left(\frac{\delta \mathcal{S}_{\mathrm{Cu}}}{\mathcal{S}_{\mathrm{Cu}}} \right)^2} \tag{1.6}$$

(TA INITIALS)

FIRST NAME (PRINT) LAST NAME (PRINT) BROCK ID (AB17CD) (LAB DATE)

Experiment 2

Calorimetry and heat capacity

Liquid nitrogen and safety

In this experiment, you will use liquid nitrogen as a handy source of a large temperature difference. The temperature of liquid nitrogen is 77 K or −196°C, more than 200°C below room temperature. To avoid injury, open-toed footwear will not be permitted in the laboratory, and be sure to:

Always wear gloves and goggles when handling liquid nitrogen!

Procedure and analysis

A computer-connected precise digital weight scale will allow you to monitor the weight of a Styrofoam cup containing liquid nitrogen. As heat is slowly transferred from the room-temperature environment, or as we add a known amount of room-temperature material into the cup, the added heat will turn some of this liquid nitrogen into gas. As a result, the mass reported by the scale will decrease as a function of time. As you analyze the nature of this time dependence, you should be able to extract several relevant physical quantities with considerable precision.

- Turn on the digital weight scale and wait for the display to read zero. Place the empty Styrofoam cup onto the scale. Note the reported mass, how quickly the scale reading settles to a constant value, the precision of the scale. Note what effect vibrations due to your footsteps or leaning on the table have on the readings, and plan the rest of your experiment accordingly.

$$m_{\text{cup}} = \ldots\ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots\ldots$$

- Fill the Styrofoam cup to about two thirds with liquid nitrogen, then place it on the scale. Close any running Physicalab software, then restart the program and enter your email address. Set the input channel to `Adam`, select `scatter plot`.

- Choose to collect a data point every 5 seconds for at least 20 minutes, then press **Get data** to start the data acquisition. You can click **Draw** at anytime to graph the data set acquired so far.

- Try a simple linear fit, `A*x+B`, and note the resulting χ^2 misfit value. Over short time intervals, a linear function might be appropriate, but as the level of liquid nitrogen in the cup changes, so does the slope of the curve $m(x)$, where x represents time. Label the axes and include the fit equation as part of the title, then press the **Send to:** button to email every member of the group a copy of the graph for inclusion in the lab report.

? What do you note when comparing the curve from fitting equation to the points from your data set?

- Now try an exponential fit, `A*exp(-x/B)` and note again the χ^2 misfit. Your plot should look similar to the one shown in Fig. 2.1.

? Compare once again the curve from the fitting equation to the trend in your data set. What changes do you note from the previous graph?

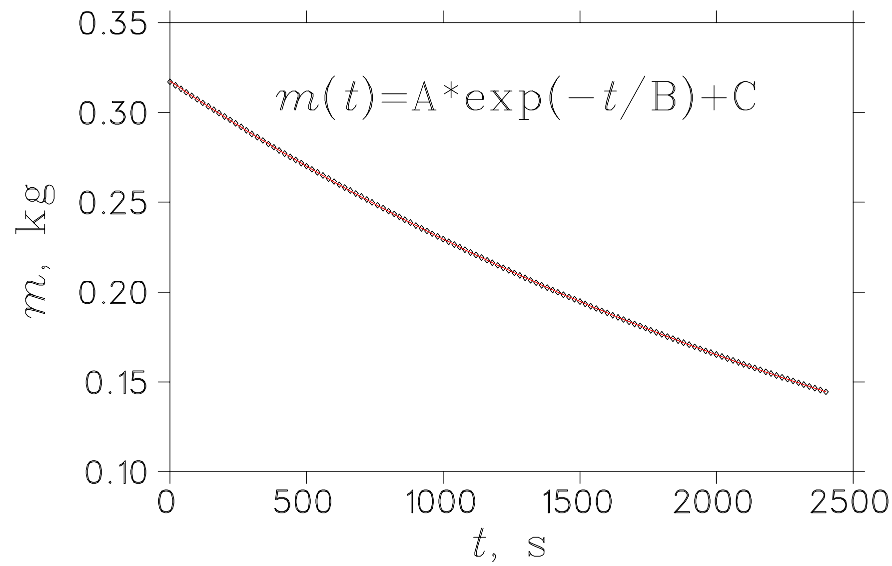


Figure 2.1: Mass of a cup of liquid nitrogen decreases with time due to boil-off

? What is the dimension and the physical meaning of the fit parameter `B` in the exponential fit?

? What value does this fitting equation converge to as time progresses toward infinity? Is this result consistent with what you would expect?

- If the liquid-nitrogen level falls below approximately one-third-full, the exponential function may become insufficient to describe the time dependence you observe, and an additional constant offset parameter may need to be added to the exponential function. Try fitting your data set to `A*exp(-x/B)+C` and make a note of the reported χ^2 values.

? Evaluate the result from this fit. What changes do you note from the previous graph?

? What is the dimension and the physical meaning of the fit parameter `C`?

? What value does this fitting equation converge to as time passes indefinitely (i.e., $t \to \infty$)?

? Is the result from the fit consistent with your expectation? Why might this be so?

- Use paper towels to wipe down the frost and accumulated condensation from the Styrofoam cup and the digital scale plate.

- Determine the weight of the provided brass block with the highest possible precision.

$$m_{\text{brass}} = \ldots\ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots\ldots$$

- Use the room thermostat thermometer to determine the "room temperature" which is the initial temperature of the brass block. Handle the block by the attached string to prevent it from absorbing your body heat.

$$T_{\text{room}} = \ldots\ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots\ldots$$

- Refill the Styrofoam cup to two-thirds-full and place *it and the brass block* on the weight scale then start your data acquisition run. Get a point every 5 seconds over approximately 15–20 mins. Five minutes should be enough to establish the initial shape of the mass-as-a-function-of-time curve.

- After five minutes, transfer the brass block into the Styrofoam cup, leaving the string to dangle outside the cup. Be careful to prevent splashing and avoid touching the surface of the liquid nitrogen with anything but the brass block itself. You can expect a significant cloud of cold gas to be generated when the brass is rapidly cooled by the liquid nitrogen to 77 K.

  Try to perform this transfer in between the data points to avoid picking up erratic readings; this is not essential, however.

- Continue the data acquisition and carefully monitor the process to note the rapid boil-off as the brass plug reaches the temperature of the liquid nitrogen, after which the boiling stops. Acquire data for another five minutes, to establish the trailing slope of the curve.

  `Note:` At the end of the run the brass block is at 77 K and is **EXTREMELY DANGEROUS**. Carefully transfer the brass block into the sink and run tap water over it until all traces of ice are gone. Wipe dry and return it to your station.

$\boxed{?}$ What do you suppose might be the cause of this rapid boil-off as the plug reaches 77 K?

- Establish through a visual inspection of the graph the two time points T_1 and T_2 that correspond to the beginning and the end of the rapid boil-off interval.

- Fit your data to `(A*exp(-x/B)+C)*(x<`T_1`)+((A-D)*exp(-x/B)+C)*(x>`T_2`)` using the values of T_1 and T_2 determined above. To get a valid χ^2 value, the constraint equation $(\mathtt{x} < T_1) + (\mathtt{x} > T_2)$ must be entered in the constraint box. Examine the quality of the fit, and note the χ^2 misfit value.

- Repeat several times, changing the values of T_1 and T_2 slightly to move away from the edges of the region of the rapid boil-off. Ideally, the value of `D` reported by the fit should not depend on the precise choice of T_1 and T_2.

T_1, s	T_2, s	$\mathtt{D} \pm \mathtt{dD}$	χ^2
	Best D value: (least χ^2)		

$\mathtt{D} = \ldots\ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots\ldots$

The fit parameter `D` represents the "extra" liquid nitrogen boiled off to cool down the brass block from T_{room} to 77 K. It takes 208 kJ to boil off 1 kg of liquid nitrogen. Your task is to calculate the *average* specific heat of brass, c_{brass} *i.e.* the amount of heat that is required to raise the temperature of 1 kg of brass by 1 K. Proceed as follows:

- The temperature change experienced by the brass plug in going from room temperature to that of the liquid nitrogen is:

  $dT = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$

- The error in this temperature change $\Delta(dT)$, assuming that there is no error associated with the boiling temperature of the liquid nitrogen, is given by:

  $\Delta(dT) = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$

- Convert `D` into the total amount of heat discharged by the brass plug dQ

  $dQ = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$

- The error in this heat change, $\Delta(dQ)$, is given by:

  $\Delta(dQ) = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$

- Calculate the heat capacity of brass c_{brass} and the associated error using $dQ = c_{\text{brass}} m_{\text{brass}} dT$. This result is only an average value because the heat capacity is not constant in brass between 77 K and the room temperature.

$$c_{\text{brass}} = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\Delta(c_{\text{brass}}) = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$c_{\text{brass}} = \ldots\ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots\ldots$$

- Convert your result to the units of the accepted value, $c_{\text{brass}} = 0.093 \pm 0.002\,\frac{\text{cal}}{\text{g}\,^\circ\text{C}}$ at 20°C, so that a direct comparison can be made:

$$c_{\text{brass}} = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\Delta(c_{\text{brass}}) = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$c_{\text{brass}} = \ldots\ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots\ldots$$

! Important! Be sure to have this printout signed and dated by a TA before you leave at the end of the lab session. All your work needs to be kept for review by the instructor, if so requested.

Lab report

Go to the "Lab Documents" web page to access the online lab report template for this experiment. Complete the template as instructed and submit it to Turnitin before the lab report submission deadline, late in the evening six days following your scheduled lab session. Do not wait until the last minute. Turnitin will not accept overdue submissions. Unsubmitted lab reports are assigned a grade of zero.

Heat Capacity prelab preparation

Worksheets, videos and all other lab-related content is located at:

`http://www.physics.brocku.ca/Courses/1P92/lab-manual`

- Read the Theory section for this experiment.
- Use the Internet or some other source to research the following topics. Here are some links:

  `https://www.youtube.com/watch?v=yhNHJ7WdT8A`

  `https://www.youtube.com/watch?v=67ijwSZ3gnQ`

  then in a separate file that you will upload to Turnitin, provide in your own words a description of the following physical principles. Support your description with a couple of examples and consider how the concept might be part of your experiment:

  1. Heat transfer by conduction:
  2. Heat transfer by convection:
  3. Heat transfer by radiation:
  4. Specific heat capacity:
  5. Newton's law of cooling:
  6. The Leidenfrost effect:

- Read through the rest of the lab instructions for this experiment.
- Login to Turnitin and submit your file in your prelab assignment before the "Due" time and date shown. Do not wait until the last minute to submit your report. Turnitin will not accept submissions after the set due date/time. Note that overdue prelab reports are assigned a grade of zero.

☐ Print a copy of this experiment to bring to your scheduled lab session. The data, observations and notes entered on these pages will be needed when you write your lab report. Compile these printouts to create a lab book for the course.

CONGRATULATIONS! YOU ARE NOW READY TO PROCEED WITH THE EXPERIMENT!

Heat capacity

Heat is the flow of thermal energy, and the *specific heat capacity* of an object is a measure of the amount of thermal energy that must flow into it to increase its temperature by one degree per unit mass. Heat capacity depends on the material the body is made of, and measuring it can help to identify the material.

Thermal energy naturally flows from hotter to colder bodies, and in general it is not easy to stop the flow of heat. There are several ways in which heat can be transported. One is direct contact of two bodies; the rate of direct heat *conduction* varies a great deal for different materials. Also, the rate of heat flow is roughly proportional to the temperature difference between the two bodies: The greater the difference, the faster the heat flow. For example, in very hot weather, as the temperature difference between a car's radiator and the air flowing past it is decreased, the effectiveness of cooling is diminished.

When one of the bodies is a gas or a liquid that flows away from another body after making thermal contact, heat is said to be transported by *convection*. When the differences in temperature are great, *radiative* heat transfer becomes important; all objects emit thermal radiation, and the amount of this radiated energy grows very rapidly with temperature.

Even if the temperature is constant, heat energy may flow in and out of a system, if an internal rearrangement of atoms is taking place, such as a change of state from liquid to gas (evaporation) or from solid to liquid (melting). This so-called *latent heat* of evaporation or of melting again depends a great deal on the material in question, and can be used to identify the material.

Liquid nitrogen can be used as a handy source of a large temperature difference since the gas becomes liquid at 77 K or $-196°$C, more than 200°C below room temperature. When liquid nitrogen comes into contact with room-temperature objects, a small amount of it evaporates very quickly and forms a thin layer of nitrogen gas. The formation of this gas layer is known as the **Leidenfrost Effect**.

Since heat conduction in a gas is much slower than in a solid or a liquid, this gas acts as a barrier to heat conduction, and it allows the remaining nitrogen to remain in liquid form. One could spill a small amount of liquid nitrogen directly onto skin, and live to tell the tale. But if a drop of liquid nitrogen is trapped in folds of the skin, or if the skin comes into contact with a metal or glass container holding the liquid nitrogen, a serious injury will result.

(TA INITIALS)

FIRST NAME (PRINT) LAST NAME (PRINT) BROCK ID (AB17CD) (LAB DATE)

Experiment 3

Resistance

The output terminals of the power supply constitute an *open circuit*. The test circuit, or load, of effective resistance R is connected across the power supply to establish a *closed circuit* through which a current I can flow. This load may be one or more resistors in parallel or series, a diode, light bulb, motor or any other electrical component or system.

To monitor the amount of current flowing in this closed circuit at a set voltage V, the power supply includes a current meter (ammeter) connected in *series* between the voltage source and the resistive load. Refer to Figure 3.1 for the schematic diagram for the circuit.

The digital meters have a measurement error of $\pm$ 0.5% plus one least significant digit (LSD) on the display.

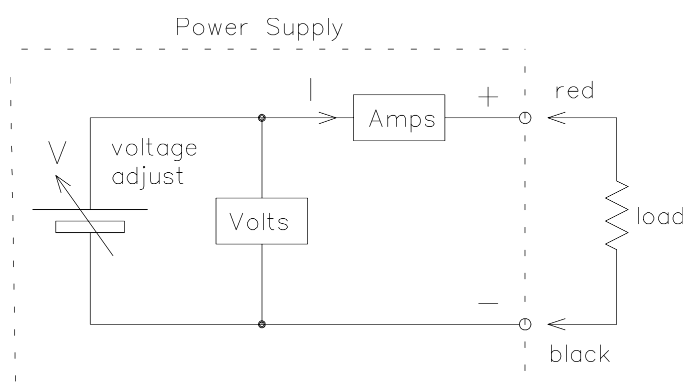


Figure 3.1: Experimental setup

Part 1: Single resistors

In this exercise, you will determine R for a resistor from the slope of a line of best fit through a series of (I, V) data points. You will then compare the result with the nominal resistance of the component.

- Your workstation is equipped with either a single HP power supply unit (PSU) or a double MPJA PSU that can be shared by two workstations. With the PSU turned off, rotate the **Voltage** adjust knob(s) fully *counterclockwise* to set the output voltage to 0 V.

1. For the HP PSU, check that the **Range** is *pressed*, then turn on the power supply. Press and hold the **CC set** button and use the curent adjust knob to set a maximum current to 2.0 A. This will limit the power supply current in case of a short circuit.
2. For the MPJA PSU, check that the **Tracking** buttons are set to **INDEP** and that the **Current** knobs are turned fully clockwise to set a current limit to to 3.0 A, then turn on the PSU.

- Connect the circuit as shown in Figure 3.1 with resistor R_1 as the load resistor. Resistors R_1 and R_2 are similar, with a nominal resistance of 40 Ω and an error, or tolerance, of ± 5%.
- With the voltage adjust knob, set the output voltage from 1 V to 10 V in nominal increments of ≈1 V and at each step record in Table 3.1 the magnitude of the current flowing through the resistor.

? With $V \approx 10$ V and a current of about 0.25 A flowing through the resistor, occasionally touch the resistor. What do you note?

? Does the current value change if this voltage is applied to the resistor for an extended period of time? Why might be the cause of this change? How should the current change?

V (V)	1									10
$\delta(V)$ (V)										
I (A)										
$\delta(I)$ (A)										

Table 3.1: Experimental results for resistor R_1

- Perform a sample calculation of $\delta(V)$ and $\delta(I)$, the errors in V and I, at $V = 5$ V and enter these results for all the values of V in Table 3.1.

$$\delta(V) = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\delta(I) = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

- Close any open Physicalab programs, then start a new PhysicaLab session and enter the emails of the group members. Enter the data pairs and their associated errors $(I, V, \delta(V), \delta(I))$ in the data window. Select **scatter plot**. Click **Draw** to generate a graph of your data. The graph should approximate a straight line. Select **fit to: y=** and enter **A*x+B** in the fitting equation box. Click **Draw** to perform a linear fit on your data. Label the axes and enter your name and a description of the data as part of the graph title. Click **Send to** to email yourself a copy of your graph for later inclusion in your lab report.
- Enter below the experimental values for R_1 and $\delta(R)$ from the slope of the graph. Also include for comparison the nominal value R_1 and tolerance $\delta(R)$ of resistor R_1.

$$(\text{slope})R_1 = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots\Omega$$

$$(\text{nominal})R_1 = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots\Omega$$

- Repeat the above steps by connecting the circuit shown in Figure 3.1 using the resistor R_2 as the load resistance. Enter your findings in Table 3.2.

V (V)	1									10
$\delta(V)$ (V)										
I (A)										
$\delta(I)$ (A)										

Table 3.2: Experimental results for resistor R_2

- Summarize below the values of R_2 from the slope of the graph and the the value displayed on the resistor R_2.

$$(\text{slope})R_2 = \ldots\ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots\ldots \Omega$$

$$(\text{nominal})R_2 = \ldots\ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots\ldots \Omega$$

Part 2: Resistors in series

- Replace the single resistor used in Part 1 by the two resistors connected in series as shown in Figure 3.5. Set $V \approx 5$ V and measure the corresponding value of I.

$V = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$ V $\qquad$ $I = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$ A

- Use Equation 3.1 to determine the experimental effective resistance R_s. Apply the proper error propagation rule to evaluate the error $\delta(R_s)$ of the two resistors in series.

$$R_s = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\delta(R_s) = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$R_s(\mathrm{O}hm's\ law) = \ldots\ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots\ldots \Omega$$

- Use the nominal component values for R_1 and R_2 and Equation 3.2 to calculate the theoretical effective resistance R_s and error $\delta(R_s)$ of the two resistors in series:

$$R_s = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\delta(R_s) = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$R_s(\text{Series Law}) = \ldots\ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots\ldots \Omega$$

Part 3: Resistors in parallel

- Connect the two resistors in parallel. Set $V \approx 5$ V and measure the corresponding value of I. Calculate the experimental effective resistance R_p and error $\delta(R_p)$ of the two resistors in parallel.

$$V = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots \text{ V} \qquad\qquad I = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots \text{ A}$$

- Use Equation 3.1 to determine the experimental effective resistance R_p and the error $\delta(R_p)$ of the two resistors in parallel.

$$R_p = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\delta(R_p) = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$R_p(Ohm's\ law) = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots\Omega$$

- Use the nominal component values and Equation 3.4 to calculate the theoretical resistance R_p and the error $\delta(R_p)$ of the two resistors in parallel:

$$R_p = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\delta(R_p) = R_p^2 \sqrt{\left(\frac{\delta R_1}{R_1^2}\right)^2 + \left(\frac{\delta R_2}{R_2^2}\right)^2} = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$R_p(\text{parallel law}) = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots\Omega$$

Part 4: IV characteristics of a diode

As mentioned in the Theory, many electrical devices do not obey Ohm's law. A diode is a semiconducting device whose resistance not only depends on the voltage V applied across its terminals, but also on the polarity, or direction of the applied voltage.

The diode has a positive and a negative terminal. The negative terminal is identified by a band at that end of the diode body. A diode is:

- **forward biased** when the band end is connected to the negative (-) terminal of the power supply;
- **reverse biased** when the band end is connected to the positive (+) terminal of the power supply.

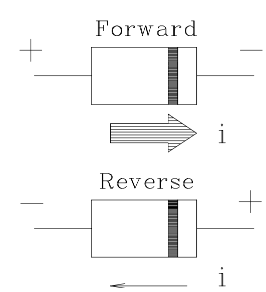


Figure 3.2: Current flow in a diode, showing the polarity of the applied voltage

A diode behaves like a voltage-controlled on/off switch.

For all applied negative voltages and positive voltages that are less than the diode turn-on voltage V_{on}, the diode is in an *off* state. The diode resistance is very large and the current flow is approximately zero.

Above V_{on}, the diode current increases exponentially with increasing voltage as the diode resistance drops to nearly zero. In this *on* state, an excessive current will flow through the diode, destroying it, unless you **always have a current-limiting resistor placed in series with a diode!**

- Set $V = 0$ V. Connect a resistor in series with the diode and the PSU so that the diode is *forward biased*. Measure the current in the circuit over a range of voltages from 1 V to 10 V in nominal steps of ≈1 V. Present your results in Table 3.3.

V (V)	1									10
$\delta(V)$ (V)										
I (A)										
$\delta(I)$ (A)										

Table 3.3: Experimental results for forward biased diode

V (V)	-1									-10
$\delta(V)$ (V)										
I (A)										
$\delta(I)$ (A)										

Table 3.4: Experimental results for reverse biased diode

? Is it difficult to read the current as the voltage is set to +10 V? Why? What would you think is happening? Does this effect eventually disappear? Why?

- Rearrange the diode so that it is connected in the *reverse biased* direction. Repeat the series of measurements and enter your data in Table 3.4.
- On the same graph, plot I as a function of $-10 \leq V \leq 10$ using the two data sets from Tables 3.3 and 3.4. Check **Line between points** to display the trend of your data points.
- To determine the turn-on voltage of the diode, fit a straight line **y=A*x+B** to the positive voltage points (V, I). Check the **Constrain to:** box and enter **x>1** to fit only this range of data. The turn-on voltage V_{on} is given approximately by the X-intercept of the line, at $I = 0$. Properly label and title the graph, then email a copy.

? Is a straight line the proper function to fit your data to? Why? What does the slope of the straight line represent? What are the units of this quantity?

Part 5: Resistance characteristics of a heated filament

In this exercise you are going to explore the temperature dependence of a resistor. The resistor in this case is the tungsten filament of a low voltage light bulb. As the voltage applied across the filament is increased, the current causes the filament to increase in temperature and eventually begin to glow.

NOTE: applying more than 5 V to the light bulb will destroy the Tungsten filament!

V (V)									5
I (A)									

Table 3.5: Experimental results for tungsten filament

- Connect the light bulb to the PSU. Measure I for a range of voltages from 0 V to -5 V and enter the results in Table 3.5. Estimate the voltage when the filament begins to glow.

$$V_{on1} = \ldots\ldots\ldots\text{V}, \qquad V_{on2} = \ldots\ldots\ldots\text{V}, \qquad V_{on3} = \ldots\ldots\ldots\text{V}, \qquad \langle V_{on}\rangle = \ldots\ldots\ldots\text{V}$$

- Plot the $(V, \langle I\rangle)$ results for the tungsten filament. Also include a point at (0,0). Connect the points with line segments as before, then save a copy of the graph.

$\boxed{?}$ As you analyse the graph, what do you note in the behaviour of the V, I curve in the region where the light bulb begins to glow? How would you explain this feature of the graph?

! Important! Be sure to have this printout signed and dated by a TA before you leave at the end of the lab session. All your work needs to be kept for review by the instructor, if so requested.

Lab report

Go to the "Lab Documents" web page to access the online lab report template for this experiment. Complete the template as instructed and submit it to Turnitin before the lab report submission deadline, late in the evening six days following your scheduled lab session. Do not wait until the last minute. Turnitin will not accept overdue submissions. Unsubmitted lab reports are assigned a grade of zero.

Resistance prelab preparation

Worksheets, videos and all other lab-related content is located at:

`http://www.physics.brocku.ca/Courses/1P92/lab-manual`

The following simulation allows you to explore the relationship between the variables in Ohm's Law:

`https://phet.colorado.edu/en/simulation/ohms-law`

You can construct your own electrical circuits with the PhET electric circuit simulator and monitor their electrical behaviour:

`https://phet.colorado.edu/sims/html/circuit-construction-kit-dc-virtual-lab/latest/circuit-construction-kit-dc-virtual-lab_en.html`

- To get a feel for how including more resistors in a series and parallel circuit changes the current flow in a circuit, begin by simulating a circuit with a single resistor and record the current flow for a set voltage.

[?] If you were to add a second resistor in series with the first resistor, how would you expect the current in the circuit to change? Why?

- Keeping the voltage fixed, add a second simulated resistor in series with the first resistor.
- Use Equation 3.2 for two resistors in series and your two resistor values form the simulation to calculate the effective resistance of the two resistors in series resistors in series, then
- use Ohm's Law to calculate the expected circuit current at your fixed voltage.

[?] Did the current change as predicted by Ohm's Law?

- Try adding a third resistor in series. Does the current change as expected?
- Use Equation 3.3 to calculate an equivalent resistance for the three resistors in series.

[?] How does your result for the circuit current compare with the result from the simulation?

- Now, repeat the above exploration for a circuit with two and three resistors in parallel, using Equations 3.4 and 3.5.
- Read through the rest of the lab instructions for this experiment in this document.
- Login to Turnitin and submit your file in your prelab assignment before the "Due" time and date shown. Do not wait until the last minute to submit your report. Turnitin will not accept submissions after the set due date/time. Note that overdue prelab reports are assigned a grade of zero.

[ ] Print a copy of this experiment to bring to your scheduled lab session. The data, observations and notes entered on these pages will be needed when you write your lab report. Compile these printouts to create a lab book for the course.

CONGRATULATIONS! YOU ARE NOW READY TO PROCEED WITH THE EXPERIMENT!

Resistance

When electrons, or other electric charge carriers (e.g. ions in a solution), are forced to move through a medium by an applied electric field (which can also be described by a potential difference (voltage), V), their motion is in most cases retarded by scattering from imperfections (impurities) and vibrating atoms in the medium. This resistance to the movement of charge is defined as

$$R = \frac{V}{I}$$

where V is the voltage, or *potential difference*, applied across the material and I is the current, or *rate of the movement of electric charge* (electrons) in the material. The resistance R of a medium (resistor) is dependent on its chemical properties, geometry, external magnetic field, temperature (the magnitude of atomic vibrations increases with temperature), etc.

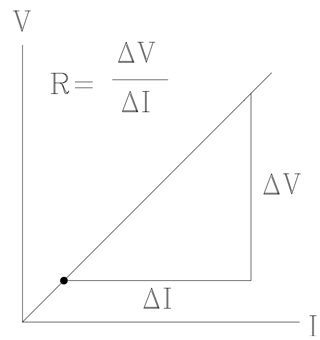


Figure 3.3: IV relationship for Ohmic resistor

The value of resistance may also depend on the *magnitude* and *polarity* of the voltage V applied across its terminals, as is observed with a device made of *semiconducting* material. Semiconductor resistance decreases with temperature, an effect known as thermal runaway.

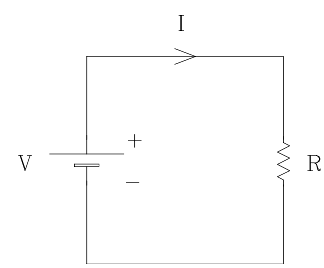


Figure 3.4: Basic resistor circuit

A resistor that is independent of the voltage applied across it is called an Ohmic resistor after George Simon Ohm (1787–1854) who described mathematically the electrical characteristics of such a device. Ohm's law states that the electric current I that flows in a conductor is *proportional* to the potential difference V between the ends of the conductor, and is inversely proportional to its resistance R.

$$I = \frac{V}{R} \tag{3.1}$$

The unit for resistance is the *ohm* (Ω), and is derived from the units of voltage and current:

$$1 \text{ ohm} = \frac{1 \text{ volt}}{1 \text{ ampere}}.$$

For an Ohmic resistor, a graph of V vs. I is a straight line, with the slope equal to the resistance R, as shown in Figure 3.3. By varying the voltage across a resistor and recording the current in each case, a graph of V vs. I can be plotted, and from that graph, the resistance of an unknown resistor can be established. A schematic representation of the simplest electric circuit is given in Figure 3.4.

Ohmic resistors are used primarily to *limit the current flow* in an electric circuit. Several methods are used in their construction. For example, some resistors consist of a fine wire wound on an insulating core. The ones that you will use are formed from various carbon compounds.

Kirchhoff's laws

The behaviour of any electric circuit can be examined with the aid of two rules developed by Gustav Kirchhoff (1824–1887). These rules arise from the application of fundamental physical laws to electric circuits, and have been verified by numerous experiments.

Kirchhoff's voltage law, or loop rule, states that the total work done on an electron by the voltage sources in a circuit equals the total work extracted from the electron while traversing the circuit. In following any such closed circuit loop, the gains in potential energy will be equal to the losses, so that $\sum \Delta V = 0$. This is the principle of conservation of energy.

A junction is a point in a circuit where a number of wires are connected together. *Kirchhoff's current law*, or junction rule, states that the total electric current entering a junction, or node, equals the total electric current leaving the junction, $\sum I = 0$. In effect, it states that no electrons are created or destroyed. This is the principle of conservation of electric charge.

Effective resistance of resistors in series

The effective resistance for R_1 and R_2 connected in series is represented by R_S. Applying Kirchhoff's voltage law (starting at O, traversing the loop clockwise) to the closed circuit loop in Figure 3.5 yields:

$$\begin{aligned} V - IR_1 - IR_2 &= 0 \\ V &= IR_1 + IR_2 \\ \frac{V}{I} &= R_S \;=\; R_1 + R_2. \end{aligned}$$

Therefore, for two resistors connected in series,

$$R_S = R_1 + R_2. \qquad (3.2)$$

and for any number N of resistors in series

$$R_S = R_1 + R_2 + \cdots + R_N = \sum_{i=1}^{N} R_i \qquad (3.3)$$

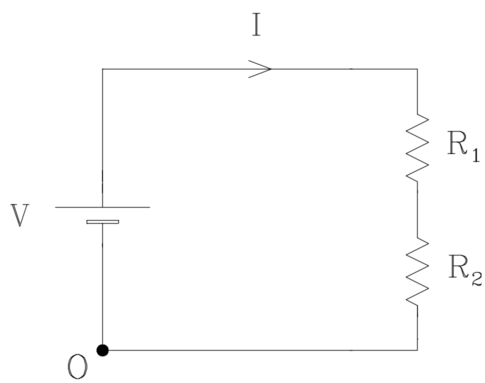


Figure 3.5: Resistors in series

Effective resistance of resistors in parallel

The effective resistance R_P of resistors R_1 and R_2 in Figure 3.6 can be determined by noting that the voltage V is the same across both resistors and applying Kirchhoff's current law at junction O:

$$I - I_1 - I_2 = 0$$

Then, for two resistors connected in parallel:

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}. \qquad (3.4)$$

and for any number N of resistors in parallel

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} = \sum_{i=1}^{N} \frac{1}{R_i}. \qquad (3.5)$$

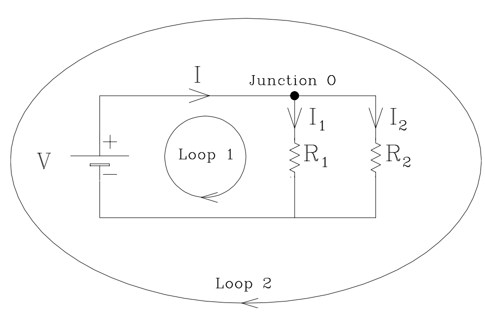


Figure 3.6: Resistors in parallel

(TA INITIALS)

FIRST NAME (PRINT) LAST NAME (PRINT) BROCK ID (AB17CD) (LAB DATE)

Experiment 4

Refraction of light

Part 1: Refraction into a denser medium

You will verify Snell's Law using a semi-circular plastic prism as the second medium, as shown in Figure 4.1. Arrange the prism so that it is *concentric* with the paper protractor, with the flat surface lined up with the 90° line so that the normal (0°) is perpendicular to the flat face of the prism.

Since the incident beam of light has a finite width, the same edge of the beam should be used to set the incident angle i and to measure the refracted angle r. Make sure that this edge of the beam passes through the centre point of the protractor, otherwise your angle measurements will be incorrect.

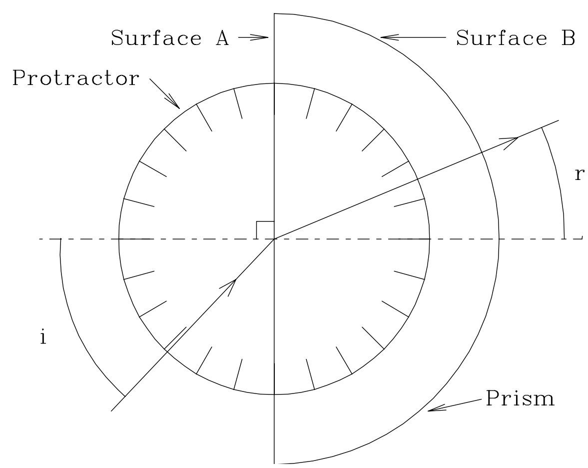


Figure 4.1: Refraction into a denser medium

? Does the refracted beam display any notable dispersion? Should the beam exhibit this dispersion? On what line of reasoning do you base your conclusion?

- Vary the incident angles i by rotating the prism/protractor combination, in 10° increments from 0° to 80°, and measure the corresponding refracted angle r values. Enter your results in Table 4.1.

? What is the resolution of the protractor scale? How well can you estimate a value for an angle with this scale? Did you express the above measurements to this precision?

- Calculate the values of $\sin i$ for the incident angles i and $\sin r$ for the refracted angles r. Enter the results in Table 4.1.

Equation 4.2 can be rearranged to give $\sin i = (n_{II}/n_I)\sin r$. This is the equation of a line with slope (n_{II}/n_I). Because in our case the incident medium is air, with index of refraction $n_I \approx 1$, a plot of $\sin i$ as a function of $\sin r$ will yield a line of slope n_{II}.

- Close any running Physicalab windows, then start a new session by clicking on the Desktop icon and login with your Brock student ID. You will email yourself all the graphs for later inclusion in your lab report.

- Enter the pairs of values $(\sin r, \sin i)$ in the Physicalab data window. Select **scatter plot**. Click **Draw** to generate a graph of your data. The displayed data should approximate a straight line.
- Select **fit to: y=** and enter **A*x+B** in the fitting equation box. Click **Draw** to fit a straight line to your data. Label the axes and title the graph with your name and a description of the data being graphed. Click **Send to:** to save a copy of the graph.
- Record the values for the slope and the standard deviation of the slope obtained from the graph.

$$n_{II} = \ldots\ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots\ldots$$

i°	0	10	20	30	40	50	60	70	80
r°									
$\sin i$									
$\sin r$									

Table 4.1: Exprimental results for Part 1

Part 2: Refraction from a denser medium

The refractive index n for the plastic was determined in Part 1 using light travelling from air to plastic.

? Should n for the plastic prism be the same if it is measured using a light ray passing from plastic to air? On what line of reasoning do you base your conclusion?

- To test your hypothesis, place the light source on the opposite side of the prism, as shown in Figure 4.2. Aim the beam of light so that it passes through the curved prism face (surface B).

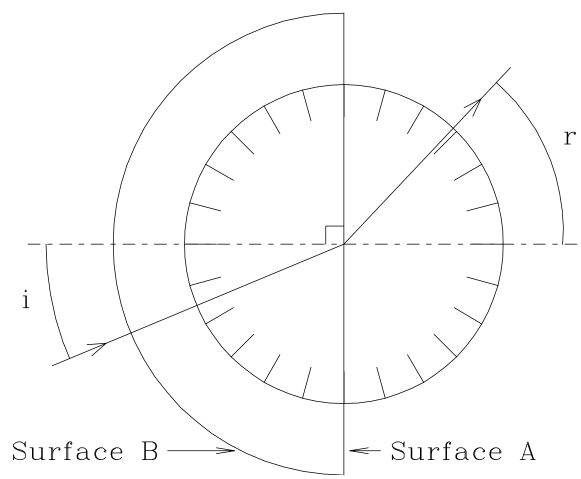


Figure 4.2: Refraction from a denser medium

Make sure that you use the side of the beam that is closer to the normal as your reference edge and that this edge goes through the centre point of the protractor. The angle of incidence i is now measured inside the prism and angle r is outside.

- Vary the angle i of the incident beam in 5° increments from 0° to 40° and measure the corresponding values of r.

- Calculate the values of $\sin i$ for the incident angles i and $\sin r$ for the refracted angles r. Enter your results in Table 4.2.

Remember that now the index of the refracted beam n_{II} is that of air, hence $N_{II} \approx 1$. With this in mind, we can conclude that the calculated value of the slope from the graph will be the inverse of the refractive index n_I of the incident medium.

- Use the Physicalab software to graph the pairs of values ($\sin r$, $\sin i$).
- Summarize below the values for the slope and the standard deviation of the slope displayed in the fitting parameter window, and from these determine a value and error for the index of refraction of the prism n_I.

$$\textit{slope} = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$$

$$n_I = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\Delta(n_I) = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$n_I = \ldots\ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots\ldots$$

i°	0	5	10	15	20	25	30	35	40
r°									
$\sin i$									
$\sin r$									

Table 4.2: Exprimental results for Part 2

Part 3: Critical angle for total internal reflection

For light incident on a boundary from a denser medium, Snell's Law indicates that there is a certain angle of incidence i for which the reflected angle will be $r = 90°$. This angle of incidence is known as the *critical angle* θ_c. If $i > \theta_c$, there will be no refracted beam and the incident ray will undergo total internal reflection from the boundary, back into the denser medium. For this special case, Snell's Law may be written as:

$$n = \frac{1}{\sin \theta_c}. \tag{4.1}$$

Using the experimental setup of Part 2:

- Observe the refracted beam and adjust the angle i of the incident beam to set the angle of refraction r at 90°so that the refracted beam disappears as in Figure 4.3.

$\boxed{?}$ How well were you able to estimate a critical angle measurement?

- Repeat this measurement of θ_c several times to fill Table 4.3.

$\boxed{?}$ What is the point of repeating the same measurement so many times?

trial	1	2	3	4	5	6	7	8	9	$\langle\theta\rangle_c$	$\sigma(\theta_c)$
θ_c											

Table 4.3: Critical angle results for Part 3

Close any open Physicalab windows, then start a new Physicalab session by clicking on the desktop icon and login with your Brock student ID. You will be emailing yourself all the graphs that you create for later inclusion in your lab report.

- In Physicalab, click `File, New` to clear the data entry window, then enter in a column the critical angle values. Click `Options, Insert X index` to insert a column of indices to your data points. Select `bellcurve`, click `bargraph` then **Draw** to view the distribution of your data. Click **Send to:** to email yourself a copy of the graph.
- Physicalab has calculated the average $\langle\theta\rangle_c$ and standard deviation $\sigma(\theta_c)$ of your data set and these values are shown in the graph window as $\langle\theta\rangle_c \pm \sigma(\theta_c)$. Enter these two values in the $\langle\theta\rangle_c$ and $\sigma(\theta_c)$ columns of Table 4.3.

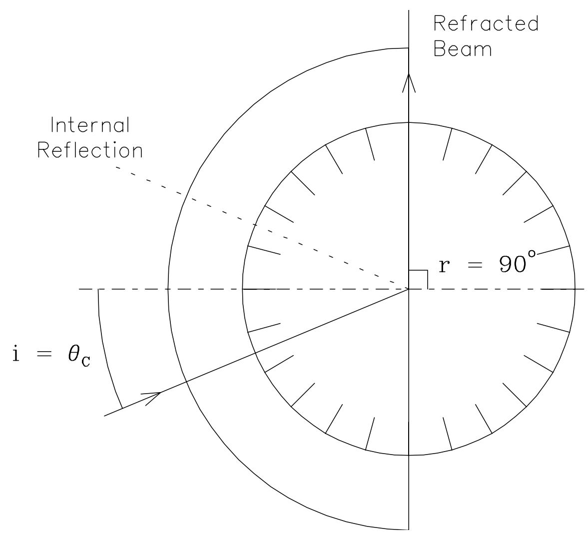


Figure 4.3: Critical angle setup for Part 3

- From these results *converted to radians*, calculate a value of n for the prism from Equation 4.1 and the error $\Delta(n)$ using the appropriate error propagation relation, then report properly rounded final results for θ_c and n:

n = ... = ... = ...

$\Delta(n)$ = ... = ... = ...

θ_c = ... ± ....

n = ... ± ....

Part 4: Measurements of Index of Refraction of water

- Fill the hollow hemispherical lens with water and then determine its index of refraction. Decide which measurements you will make and then neatly record the measurements and their errors. Then calculate the index of refraction from your measurements and include an appropriate error.

Part 5: Measurements of Focal Lengths of Lenses

- Determine the focal length of the converging lens. Decide which measurements you will make and then neatly record the measurements and their errors. Then determine the focal length and include an appropriate error.
- Determine the focal length of the diverging lens. Decide which measurements you will make and then neatly record the measurements and their errors. Then determine the focal length and include an appropriate error.

(!) Important! Be sure to have this printout signed and dated by a TA before you leave at the end of the lab session. All your work needs to be kept for review by the instructor, if so requested.

Lab report

Go to the "Lab Documents" web page to access the online lab report template for this experiment. Complete the template as instructed and submit it to Turnitin before the lab report submission deadline, late in the evening six days following your scheduled lab session. Do not wait until the last minute. Turnitin will not accept overdue submissions. Unsubmitted lab reports are assigned a grade of zero.

Refraction prelab preparation

Worksheets, videos and all other lab-related content is located at:

`http://www.physics.brocku.ca/Courses/1P92/lab-manual`

- As an introduction to refraction, watch the video at:

  `https://www.youtube.com/watch?v=95V-QJYZ2Dw`.

- To get a feel for how changing the indexes of refraction change the path of refracted light, play with the "Intro" section of the following simulation:

  `https://phet.colorado.edu/en/simulation/bending-light`

  Move the sliders to change the indexes of refraction of the two media, and observe what happens to the refracted ray. (Also notice what happens to the reflected ray.)

- Now move to the "Prisms" section of this simulation. Select the triangular prism and drag it into the path of the light. Grab the "knob" at one corner of the prism and rotate the prism, noticing the change in direction of the emerging ray. There will be a critical angle for which the incident light reflects at the "second" interface instead of passing through. Play with this to get a feel for when this happens.

- To get a feel for other concepts underlying this experiment, play with the following simulation:

  `https://phet.colorado.edu/en/simulation/geometric-optics`

  Move the lens around and observe what happens. You can also make use of the sliders to change the curvature of the lens and its index of refraction; observe what happens when you do this. You can also click on the "Virtual Image" check-box to show a virtual image when one is present.

- In Part 4 of the Experiment you are asked to determine the index of refraction of a lens. How will you do this? Which measurements will you make? How many measurements will you make? Which calculations will be needed? How will you determine the index of refraction using your measurements? How will you minimize error? Write a paragraph to summarize your method and to explain the answers to the questions asked here.

- In Part 5 of the Experiment you are asked to determine the focal length of a lens. How will you do this? Which measurements will you make? How many measurements will you make? Which calculations will be needed? How will you determine the focal length using your measurements? How will you minimize error? Write a paragraph to summarize your method and to explain the answers to the questions asked here.

- Read through the rest of the lab instructions for this experiment in this document.

- Login to Turnitin and submit your file in your prelab assignment before the "Due" time and date shown. Do not wait until the last minute to submit your report. Turnitin will not accept submissions after the set due date/time. Note that overdue prelab reports are assigned a grade of zero.

☐ Print a copy of this experiment to bring to your scheduled lab session. The data, observations and notes entered on these pages will be needed when you write your lab report. Compile these printouts to create a lab book for the course.

CONGRATULATIONS! YOU ARE NOW READY TO PROCEED WITH THE EXPERIMENT!

Refraction

The phenomenon of refraction can be explained geometrically with the aid of Figure 4.4. A beam of light incident on a boundary surface is composed of wavefronts that are perpendicular to the direction of propagation of the beam. This beam will propagate more slowly through a dense medium than it does through air.

If the incident beam is not normal to the boundary surface, one edge of the wavefront will enter the denser material first and be slowed down. This effect will propagate across the wavefront, changing the direction of the refracted beam relative to the incident beam.

This change in direction is always toward the normal to the boundary surface when the light beam crosses into a denser medium. It will be the opposite for a beam crossing into a less dense medium.

The mathematical relationship between the incident angle i and the refracted angle r of the light beam is given by Snell's Law:

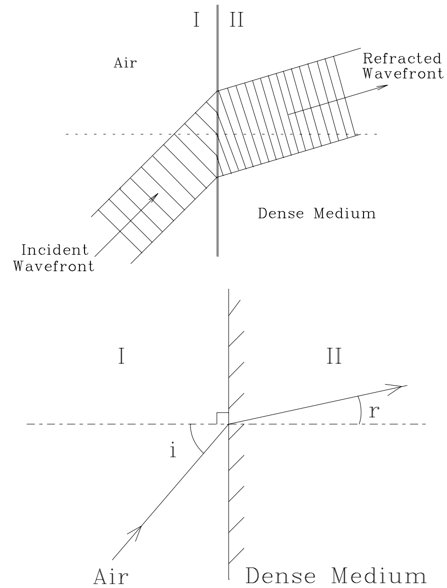


Figure 4.4: Geometry of refraction

$$\frac{\sin i}{\sin r} = \frac{n_{\mathrm{II}}}{n_{\mathrm{I}}} \tag{4.2}$$

The angles i and r are measured from the normal (i.e., perpendicular) direction to the boundary. The numbers n_I and n_{II} are characteristic of each medium, and are called the refractive indices. For a vacuum, $n = 1$, the refractive index of air is approximately 1, and *all* other materials have $n > 1$.

The refractive index is also a function of the wavelength of the light, and therefore light rays of different colour have different angles of refraction r for identical angles of incidence i. This effect, called the *dispersion of light*, is observed when white light from a source such as sunlight or a light bulb crosses a boundary. Light from a source of monochromatic light, such as a laser, consists of a single wavelength and therefore will not be dispersed.

Note that when a ray travels from a medium to a more dense medium (e.g. from air to plastic), it always refracts *towards* the normal ($r \leq i$). Conversely, when a ray travels from a medium to a less dense medium (e.g. plastic to air), it always refracts *away* from the normal ($r \geq i$).

(TA INITIALS)

FIRST NAME (PRINT) LAST NAME (PRINT) BROCK ID (AB17CD) (LAB DATE)

Experiment 5

Diffraction of light by a grating

Part 1: Determining the spacing of a diffraction grating

In this part of the experiment you will determine the spacing d of lines on a glass grating by passing a laser beam through it and examining the diffraction pattern projected on a screen.

The laser beam is parallel and monochromatic, with wavelength

$$\lambda \pm \Delta\lambda = 632.8 \pm 0.5 \text{ nm}.$$

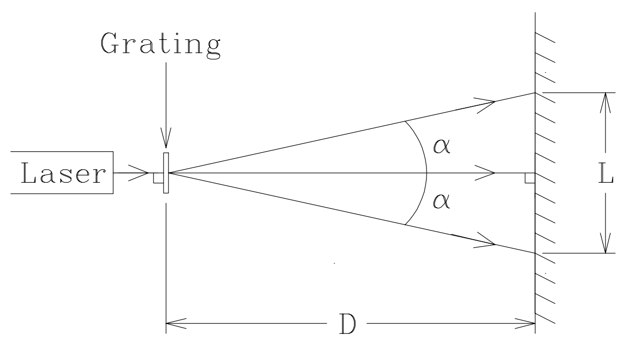


Figure 5.1: Experimental setup for Part 1.

- Check that the screen and grating surface are perpendicular to the incident beam, and measure the distance D between the grating and the screen, as shown in Figure 5.1.

  $D =$ . . . . ± . . . . .m

$\boxed{?}$ What do you note in the diffraction pattern as you slowly rotate the grating? Based on your observation, if the grating was set to some angle θ from being perpendicular to the beam, how would you adjust your L values to account for this offset?

- Mount a sheet of graph paper on the screen and carefully mark the series of interference maxima. Identify the straight path $m = 0$ maximum. Measure the distance L *between the centres* of pairs of spots of order m, ($+m$ to $-m$) for $m = \pm 1$ to $m = \pm 10$. Record your results in Table 5.1.
- With L and D measured, the relationship between these variables and the angle α in Figure 5.1 is given by

$$\tan(\alpha) = L/2D. \tag{5.1}$$

  If $L/2 \ll D$ then $\alpha \approx 0$ and $\sin\alpha \approx \tan\alpha$. Equating these two terms in Equations 5.1 and 5.4 yields

$$d = 2m\lambda D/L. \tag{5.2}$$

- Calculate d for the ten measurements of L, then calculate an average value $\langle d \rangle$ and standard deviation $\sigma(d)$ of d and enter the results in Table 5.1.

m	L (m)	d (m)	$(d - \langle d \rangle)$ (m)	$(d - \langle d \rangle)^2$ (m)
± 1				
± 2				
± 3				
± 4				
± 5				
± 6				
± 7				
± 8				
± 9				
± 10				
		$\langle d \rangle =$	$variance =$	
		$\sigma(d) = \sqrt{variance}$		

Table 5.1: Calculation of $\langle x \rangle$ and $\sigma(x)$ for d. Here, variance$(x) = \sigma^2(x) = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \langle x \rangle)^2$.

- Close any running Physicalab programs, then start a new Physicalab session and enter your email address. Enter the d values as coordinates (m, d). Select **bellcurve** and **bargraph**, then click **draw** to display a distribution of your n values with the average $\langle d \rangle$ and standard deviation $\sigma(d)$ of the sample. Click **Send to:** to email yourself a copy of the graph.
- Verify that the results from the distribution are identical to those from Table 5.1. If they are not, you need to review your calculations. Report below the grating spacing for this grating:

$$d = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots \text{m}$$

Part 2: Determining the wavelengths of the Balmer spectrum of H_2

The light source is a hydrogen discharge tube. This source illuminates a slit at one end of the collimator tube, and a lens at the other end makes a parallel beam of the light passing through the slit. The beam is diffracted by the grating and collected by the front lens of the telescope, which focuses the light on a set of cross-hairs.

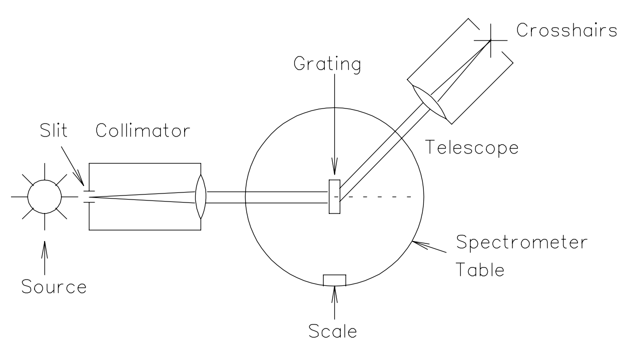


Figure 5.2: Experimental setup for part 2

If necessary, the image sharpness can be improved by rotating focusing knobs on the collimator and telescope. The slit width is adjusted with a screw on the collimator. The orientation of the crosshairs is set by rotating the telescope eyepiece. They should be oriented diagonally, like an X.

The telescope can rotate around the grating, its angular position with respect to an *arbitrary* zero given by an angular scale on the base of the instrument.

Note: To extend the life of the H2 discharge tube, keep it from overheating; turn it on for 30 seconds or less to adjust the crosshairs, then turn it off for at least 30 seconds while you read the Vernier scale.

Proceed as follows, remembering never to touch the grating as it is easily damaged.

- Switch on the H2 discharge tube. The lamp should emit a bright pink light. If the light looks grey, the tube is worn out. See the instructor.
- Check that the telescope locking screw located at the centre of the telescope base is loose, then slowly rotate the telescope assembly until you see a sharp pink image of the slit, Gently tighten the screw to lock the telescope in place.
- Turn the fine-adjust knob located on the right side of the telescope base to *place the centre of the crosshairs* on an edge of the slit image.
- Turn off the H2 discharge tube.

? Does it matter which edge of the image is used for reference? How does this choice affect the remainder of your experiment?

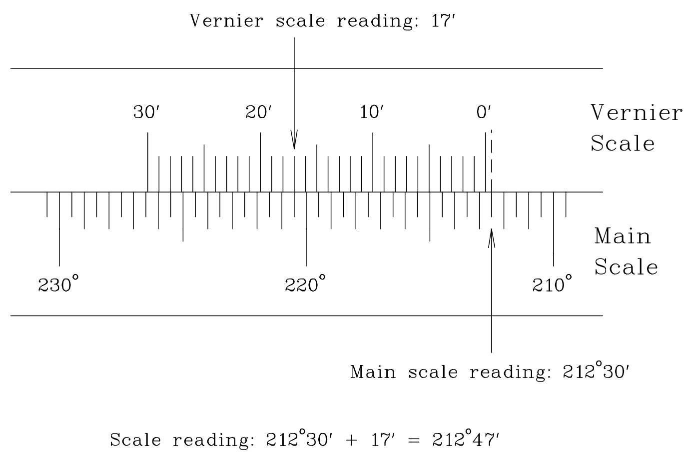


Figure 5.3: Example of Angular Vernier Scale Reading — 212°47′

- Read the position of the telescope from the angular scale on the base. This can be done to a precision of $1'$ ($\pm$ one minute, $60' = 1°$). To read the scale:
  1. Locate the "0" line on the vernier scale, and note which main scale division it is immediately after; e.g. $212°30'$ on the main scale in Figure 5.3. Note that the numbers on the main and vernier scales increase from right to left, and not from left to right as you are used to reading.
  2. Scan along the line where the main and vernier scales meet, and note which *one* vernier scale division is directly in line with a main scale division, e.g. $17'$ on the vernier scale in Figure 5.3.
  3. Add the main and vernier scale readings to obtain the angular scale reading, e.g. $212°30' + 17' = 212°47'$ in Figure 5.3.
- Enter your measurement in Table 5.2.

- Turn on the H2 lamp, then unlock the telescope and slowly rotate it to the right until the first violet slit image is in the field of view. Lock the telescope, and use the fine-adjust knob until the centre of the crosshairs is again situated *on the same edge* of the slit image as was used before.
- Turn off the H2 lamp, then read the position indicated on the angular scale. This is Violet(θ_{+1}), the angle of diffraction of the violet spectral line with $m = +1$.
- Turn on the H2 lamp, then unlock the telescope, rotate it to the left of center until the first violet image is seen again. Lock the telescope and centre the crosshairs again *on the same edge* of the slit image as was used before.
- Turn off the H2 lamp, then determine Violet(θ_{-1}), the angle of diffraction with $m = -1$.
- Repeat the above steps for the blue and violet spectral lines to complete the first line of Table 5.2.
- Convert your data values from degrees and minutes to decimal degrees, recalling that $1^\circ = 60'$, and enter these in the second row of Table 5.2.

	Pink (θ_0)	Violet (θ_{+1})	Violet (θ_{-1})	Blue (θ_{+1})	Blue (θ_{-1})	Red (θ_{+1})	Red (θ_{-1})
$^\circ$, $'$							
$^\circ$							

Table 5.2: Measurements for the spectral lines of H_2 in degrees and minutes, and also in decimal degrees

The diffraction angle $\alpha_{\pm 1}$ for a particular colour is the measured angle of deviation $\theta_{\pm 1}$ of that colour from the light's direct path reference angle θ_0. Calculating the difference between the angular positions of the pink and coloured lines will give you $\alpha_{\pm 1}$, i.e.

$$\alpha_{\pm 1} = |\theta_0 - \theta_{\pm 1}| \tag{5.3}$$

- Calculate from the data in Table 5.2 the values of the diffraction angle $\alpha_{\pm 1}$ for the three lines of the H_2 spectrum. There will be two results for each colour, $\alpha_{\pm 1}$, one for each side of the reference angle θ_0. Calculate the average $\langle \alpha \rangle = \frac{1}{2}|\alpha_{+1} + \alpha_{-1}|$ of these two angles then estimate the error with $\Delta(\alpha) = \frac{1}{2}|\alpha_{+1} - \alpha_{-1}|$.

line	α_{+1}	α_{-1}	$\langle \alpha \rangle$	$\Delta(\alpha)$
Violet (α_V)				
Blue (α_B)				
Red (α_R)				

Table 5.3: Calculated diffraction angles for the spectral lines of H_2

The diffraction grating used in the spectrometer is made with a line density of

$$N \pm \Delta(N) = 600 \pm 1 \text{ lines/mm}.$$

This is *not* the same value as the grating spacing in Part 1. The line density and grating spacing are related by $d = 1/N$. The distance d between the lines and error $\Delta(d)$ for the grating used in this spectrometer is

$$d = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots \text{ mm}$$

$$\Delta(d) = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots \text{ mm}$$

- Calculate a wavelength $\lambda(\alpha)$ and the associated error $\Delta(\lambda(\alpha))$ for the violet, blue and red spectral line of H_2. Angle errors *must* be expressed in radians. Record this data in Table 5.4.

$$\lambda_V = \frac{d}{m} sin(\alpha_V) = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\lambda_B = = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\lambda_R = = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\Delta(\lambda_V) = \lambda_V \sqrt{\left(\frac{\Delta(d)}{d}\right)^2 + \left(\frac{\cos(\alpha_V)\Delta(\alpha_V)}{sin(\alpha_V)}\right)^2} = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\Delta(\lambda_B) = = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\Delta(\lambda_R) = = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

- Use the Balmer Equation 5.5 to calculate the theoretical wavelengths $\lambda(Balmer)$ of the violet, blue and red lines of the H_2 spectrum. Append this data to Table 5.4.

$$\lambda_V(Balmer) = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\lambda_B(Balmer) = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\lambda_R(Balmer) = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$$

line	*transition*	$\lambda(\alpha)$	$\Delta(\lambda(\alpha))$	$\lambda(Balmer)$
violet				
blue				
red				

Table 5.4: Calculated $\lambda(\alpha)$ from angles $\langle\alpha\rangle$ and $\lambda(Balmer)$ from Equation 5.5

(!) Important! Be sure to have this printout signed and dated by a TA before you leave at the end of the lab session. All your work needs to be kept for review by the instructor, if so requested.

Lab report

Go to the “Lab Documents” web page to access the online lab report template for this experiment. Complete the template as instructed and submit it to Turnitin before the lab report submission deadline, late in the evening six days following your scheduled lab session. Do not wait until the last minute. Turnitin will not accept overdue submissions. Unsubmitted lab reports are assigned a grade of zero.

Diffraction prelab preparation

Worksheets, videos and all other lab-related content is located at:

`http://www.physics.brocku.ca/Courses/1P92/lab-manual`

Perform the following tasks as indicated. Then answer the following questions and submit them in your Diffraction Prelab assignment, due at Turnitin the day before you perform your experiment in the lab. Turnitin will not accept submissions after the due date. Unsubmitted prelab reports are assigned a grade of zero.

The Diffraction Prelab assignment template is found at the "Lab Documents" page at the course website:

`http://www.physics.brocku.ca/Courses/1P92_DAgostino/lab-manual/`

- To get a feel for the Bohr model of the atom, play with the following simulation:

  `https://phet.colorado.edu/en/simulation/hydrogen-atom`

  Once you have entered the simulation, first click on "Experiment" in the top left corner to see photons streaming up the screen, with an occasional one scattering from the interior square containing the question mark. (I guess at this point we're not supposed to know what is going on in there, although we can guess that there are hydrogen atoms in there.) Also click on "Show spectrometer" to see which wavelengths of light are emitted. It will take some time for this diagram to be filled in.

  After some time, click on "Prediction" in the upper left corner, and then click on "Bohr" in the left margin to play with the Bohr model, as this is the most relevant model to study to prepare for this experiment. Again, the spectrometer diagram should fill in. Also notice the various n-values changing in the bottom-right corner of the hydrogen box. You might like to see these in a different way by clicking on "Show electron energy level diagram" at the top right of the hydrogen box.

  After playing with this simulation for some time you may get the impression that only certain specific wavelengths of light are emitted by a hydrogen atom.

- To get a feel for diffraction in general, play with the following simulation:

  `https://phet.colorado.edu/en/simulation/wave-interference`

  Once you have entered the simulation, first click (twice) on the icon to the far right labelled "Slits." Click on the green button on the far left of the screen to simulate waves experiencing diffraction after passing through a single slit. See what happens when you adjust the slit width using the slider. Then use the drop-down menu to change from "One Slit" to "Two Slits." Now there are two sliders to play with; try adjusting both of them and see what happens.

- To get a feel for how a diffraction grating works, play with the following simulation:

  `https://terpconnect.umd.edu/t̃oh/models/DiffractionGrating.html`

  Click on either Excel or Calc (the latter if you use Open Office) to download the simulation for the fixed grating. Play with the sliders and change the diffraction order and the line density. Start with an incidence angle of zero degrees and a diffraction order of 1. Once you have played with this, change the diffraction order and play further. After a bit of play you might get a better feel for how diffraction gratings work and what you can expect in the lab.

  Are the colours of the diffracted beams reasonable?

There are versions of the simulation here that work for white incident light and versions that work for monochromatic incident light. Try both and compare!

If you are interested in specific values of diffracted wavelengths, scroll down in the spreadsheet to see the data.

- Read through the rest of the lab instructions for this experiment in this document.
- Login to Turnitin and submit your file in your prelab assignment before the "Due" time and date shown. Do not wait until the last minute to submit your report. Turnitin will not accept submissions after the set due date/time. Note that overdue prelab reports are assigned a grade of zero.

☐ Print a copy of this experiment to bring to your scheduled lab session. The data, observations and notes entered on these pages will be needed when you write your lab report. Compile these printouts to create a lab book for the course.

CONGRATULATIONS! YOU ARE NOW READY TO PROCEED WITH THE EXPERIMENT!

Diffraction gratings

The optical diffraction grating is a glass or plastic plate with many fine, parallel grooves spaced the same distance d from each other on its surface. According to Huygens' principle, when monochromatic light from a distant source or laser hits the grating, each groove behaves as a source of spherical wavelets that re-radiate from the grating in phase with the incident wave.

The superposition of these secondary waves will contribute to either increase or decrease the brightness of ths diffracted beam at a given point in space, resulting in a pattern of bright and dark regions in space. This pattern can be viewed and analysed by placing a screen parallel to and far away from the grating somewhere in the path of the diffracted beam. Then,

1. If the incident light beam is a parallel beam and is incident at a right angle to the grating, a diffracted beam will be offset from its parallel neighbour by a distance $d \sin \alpha$, as shown in Figure 5.4.

2. When this distance is equal to an integer number m of wavelengths λ of the incident light, the two beams are in phase and will undergo constructive interference, which can be observed as a series of bright regions on the screen. These interference maxima are given by

$$d \sin \alpha = m\lambda, \qquad m = 0, \pm 1, \pm 2, \ldots \tag{5.4}$$

   where λ is the wavelength of the incident light, d is the distance between adjacent lines on the grating, and m is an integer called the order number. The zero-order beam $m = 0$ is a continuation of the incident beam (i.e. $\alpha = 0$).

3. When the path difference between adjacent beams is $(m + 1/2)\lambda$, then destructive interference will result in dark regions, or interference minima, on the screen.

4. There are two first order beams, $m = \pm 1$, at angles given by $\sin \alpha = \pm \lambda/d$, two second order beams $m = \pm 2$, at $\sin \alpha = \pm 2\lambda/d$, etc. Hence the measurement of the angle α, together with the order number m, gives the ratio λ/d, and if either λ or d is known, the other can be calculated.

To diffract X-rays, electrons, or neutron matter waves, one needs a diffraction grating for which the line spacing d is comparable to the wavelength of the waves. It turns out that crystal materials have interatomic spacings comparable with the λ of X-rays. X-ray diffraction is now a standard way of determining the atomic arrangements in a crystal.

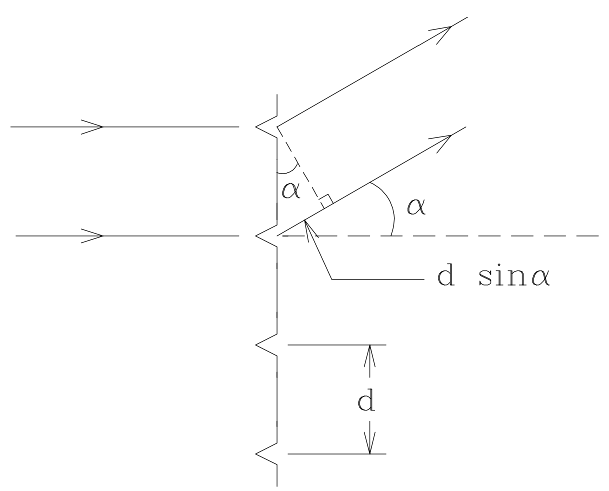


Figure 5.4: Diffraction by a grating

Note that if $\lambda > d$ one doesn't get diffraction maxima of order $m \geq 1$ and only the zero-order beam ($m = 0$) will be visible at the centre of the diffraction pattern. On the other hand, if λ is much less than d, then the corresponding angles become much too small and the diffraction pattern will not be resolved, appearing again as a bright spot on the screen.

Thus, diffraction effects are observable when $\lambda < d$ and λ is not too small a fraction of d.

To experimentally determine a value for the grating distance d of a diffraction grating a monochromatic light source of a known wavelength λ is used as the incident beam. The beam is diffracted from the grating and generates an interference pattern on a screen a distance D from the grating, as shown in Figure 5.1.

On the screen the distance L between pairs of bright spots from the diffraction of order $\pm m$ can be measured and Equation 5.4 can be used to calculate the grating distance d. Note that this equation assumes that the beams from adjacent grating slits are parallel, that is, that $D \gg L$.

Energy levels of hydrogen atoms

A hydrogen atom consists of a positively charged proton making up the atomic nucleus and a negatively charged electron orbiting this nucleus. Quantum theory predicts that the electron may only find itself in one of a number of discrete orbits, or energy levels, around the nucleus, labelled by $n = 1, 2, 3, \ldots$.

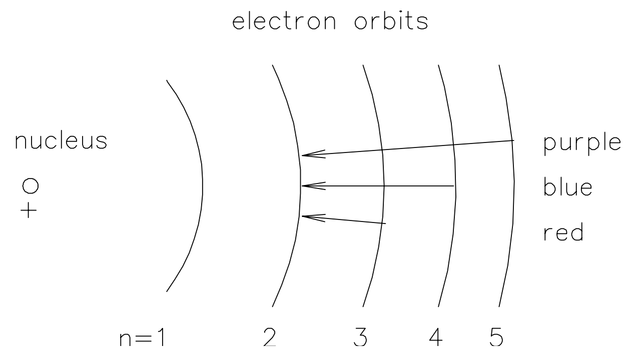


Figure 5.5: Electronic energy transitions of H_2

When energy of some sort is incident on a hydrogen atom, its one electron may absorb some of this energy. When this happens, the electron makes a transition from an orbit n_1 to an orbit n_2 where $n_2 > n_1$. The electron eventually returns to a lower orbit, releasing this surplus energy in the form of a photon of one of several specific wavelengths λ. Some of these transitions radiate photons that have the wavelength of visible light and can thus be measured with a grating spectrometer.

Johann J. Balmer (1825–1898) discovered that for a hydrogen atom the series of energy transition wavelengths from an initial energy level $n_i > 2$ to the final energy level $n_f = 2$, now known as the Balmer series, are approximately given by:

$$\lambda = \left[R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \right]^{-1} \tag{5.5}$$

$$= \left[R \left(\frac{1}{2^2} - \frac{1}{n_i^2} \right) \right]^{-1} \tag{5.6}$$

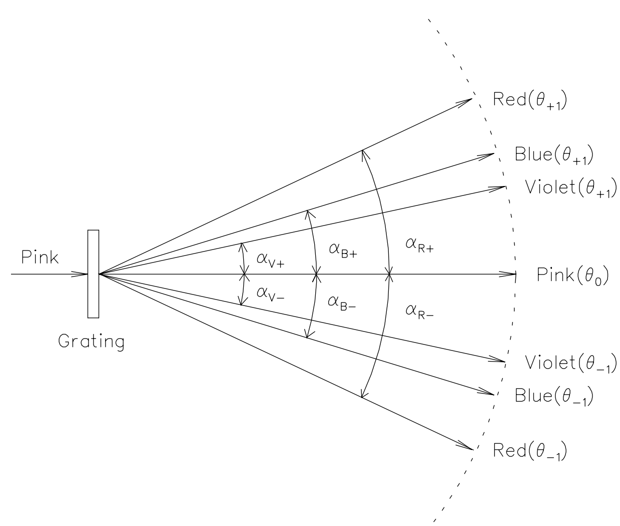


Figure 5.6: Diffraction of the H_2 spectrum

This equation is a special case of the more general Rydberg formula that accounts for all transitions from n_i to n_f. For example, the Lyman series outlines the transitions from an initial energy level $n_i > 1$ to a final energy level $n_f = 1$ that are not in the visible region of the spectrum.

The value of the Rydberg constant R is determined by fitting this empirical equation to experimental data and is equal to $R = 1.097 \times 10^7$ m^{-1}.

The electron energy level transitions that produce wavelengths in the visible region are from an initial orbit $n = 3$, $n = 4$, or $n = 5$ to a final orbit $n = 2$, as shown in Figure 5.5. These transitions will be visible as red ($n = 3 \rightarrow 2$), blue ($n = 4 \rightarrow 2$), and violet ($n = 5 \rightarrow 2$) lines in the spectrum of molecular hydrogen (H_2) from an H_2 discharge tube. The combination of these three colors, as in the unscattered beam, is observed as a pink line.

The spectrometer, shown in Figure 5.2, is an optical instrument that uses a prism or a diffraction grating to disperse an incident light beam of interest into component wavelengths. The grating used in this spectrometer is chosen to optimally view the first order ($m = \pm 1$) diffraction pattern of the incident beam.

When light from the H_2 discharge lamp is viewed through a spectrometer, the three coloured lines scatter at different angles symmetrically about the direct path of the incident light, as in Figure 5.6.

Appendix A

Review of math basics

Fractions

$$\frac{a}{c}+\frac{b}{d}=\frac{ad+bc}{cd}\ ; \qquad \text{If}\quad \frac{a}{c}=\frac{b}{d}\ ,\quad \text{then}\quad ad=cb \quad \text{and} \quad \frac{ad}{bc}=1.$$

Quadratic equations

Squaring a binomial: $(a+b)^2=a^2+2ab+b^2$
Difference of squares: $a^2-b^2=(a+b)(a-b)$

The two roots of a quadratic equation $ax^2+bx+c=0$ are given by $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$.

Exponentiation

$$(a^x)(a^y)=a^{(x+y)}\ , \qquad \frac{a^x}{a^y}=a^{x-y}\ , \qquad a^{1/x}=\sqrt[x]{a}\ , \qquad a^{-x}=\frac{1}{a^x}\ , \qquad (a^x)^y=a^{(xy)}$$

Logarithms

Given that $a^x=N$, then the logarithm to the *base* a of a number N is given by $\log_a N=x$.

For the decimal number system where the base of 10 applies, $\log_{10} N\equiv\log N$ and

$$\begin{aligned}
\log 1 &= 0 \quad (10^0=1)\\
\log 10 &= 1 \quad (10^1=10)\\
\log 1000 &= 3 \quad (10^3=1000)
\end{aligned}$$

Addition and subtraction of logarithms

Given a and b where $a, b > 0$: The log of the product of two numbers is equal to the sum of the individual logarithms, and the log of the quotient of two numbers is equal to the difference between the individual logarithms .

$$\begin{aligned}\log(ab) &= \log a + \log b \\ \log\left(\frac{a}{b}\right) &= \log a - \log b\end{aligned}$$

The following relation holds true for all logarithms:

$$\log a^n = n \log a$$

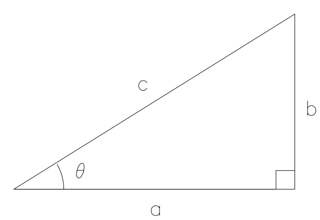
Natural logarithms

It is not necessary to use a whole number for the logarithmic base. A system based on "e" is often used. Logarithms using this base $\log_e$ are written as "ln", pronounced "lawn", and are referred to as *natural logarithms.* This particular base is used because many natural processes are readily expressed as functions of natural logarithms, i.e. as powers of e. The number e is the sum of the infinite series (with $0! \equiv 1$):

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots = 2.71828\ldots$$

Trigonometry

Pythagoras' Theorem states that for a right-angled triangle $c^2 = a^2 + b^2$. Defining a trigonometric identity as the ratio of two sides of the triangle, there will be six possible combinations:



$$\begin{aligned}\sin\theta &= \frac{b}{c} & \cos\theta &= \frac{a}{c} & \tan\theta &= \frac{b}{a} = \frac{\sin\theta}{\cos\theta} \\ \csc\theta &= \frac{c}{b} & \sec\theta &= \frac{c}{a} & \cot\theta &= \frac{a}{b} = \frac{\cos\theta}{\sin\theta}\end{aligned}$$

$$\begin{aligned}\sin(\theta \pm \phi) &= \sin\theta\ \cos\phi \pm \cos\theta \sin\phi & \sin 2\theta &= 2\sin\theta\ \cos\theta & 180^\circ &= \pi \text{ radians} = 3.15159\ldots \\ \cos(\theta \pm \phi) &= \cos\theta\ \cos\phi \mp \sin\theta \sin\phi & \cos 2\theta &= 1 - 2\sin^2\theta & 1 \text{ radian} &= 57.296\ldots^\circ \\ \tan(\theta \pm \phi) &= \frac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta\ \tan\phi} & \tan 2\theta &= \frac{2\tan\theta}{1 - \tan^2\theta} & \sin^2\theta + \cos^2\theta &= 1\end{aligned}$$

To determine what angle a ratio of sides represents, calculate the inverse of the trig identity:

$$if \quad \sin\theta = \frac{b}{c}, \quad then \quad \theta = \arcsin\left(\frac{b}{c}\right)$$

For *any* triangle with angles A, B, C respectively opposite the sides a, b, c:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}\ , \ (\textit{sine law}) \qquad c^2 = a^2 + b^2 - 2ac\ \cos C.\ (\textit{cosine law})$$

The sinusoidal waveform

Consider the radius vector that describes the circumference of a circle, as shown in Figure A.1 If we increase θ at a constant rate from 0 to 2π radians and plot the magnitude of the line segment $b = c\ \sin\theta$ as a function of θ, a sine wave of *amplitude* c and *period* of 2π radians is generated.

Relative to some arbitrary coordinate system, in this case the X-Y axis shown, the *origin* of this sine wave is located at a *offset distance* y_0 from the horizontal axis and at a *phase angle* of θ_0 from the vertical axis.

The sine wave referenced from this (θ, y) coordinate system is given by the equation

$$y = y_0 + c\sin(\theta + \theta_0)$$

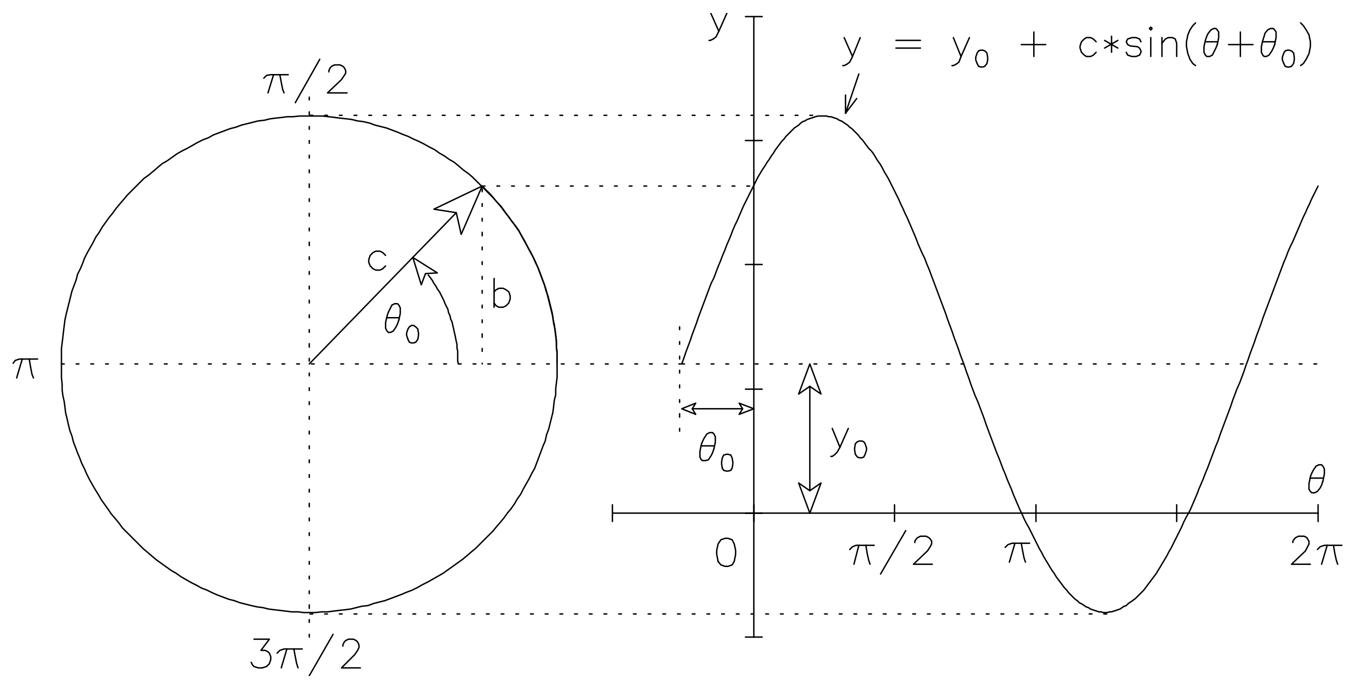


Figure A.1: Projection of a circular motion to an X-Y plane to generate a sine wave

Appendix B

Error propagation rules

- The *Absolute Error* of a quantity Z is given by δZ, always ≥ 0.
- The *Relative Error* of a quantity Z is given by $\frac{\delta Z}{Z}$, always ≥ 0.
- If a constant k has no error associated with it: constant factors out of relative error

$$Z = kA \qquad \delta Z = k\delta A \;\; and \;\; \frac{\delta Z}{Z} = \frac{\delta A}{A}$$

- Addition and subtraction of independent variables: note that error terms *always* add

$$Z = kA \pm B \pm \ldots \qquad \delta(Z) = \sqrt{(k\delta A)^2 + (\delta B)^2 + \ldots}$$

- Multiplication and division of independent variables: constants factor out of relative errors

$$Z = \frac{kA \times B \times \ldots}{C \times D \times \ldots} \qquad \frac{\delta Z}{Z} = \sqrt{\left(\frac{\delta A}{A}\right)^2 + \left(\frac{\delta B}{B}\right)^2 + \left(\frac{\delta C}{C}\right)^2 + \left(\frac{\delta D}{D}\right)^2 \ldots}$$

- Functions of one variable: if the quantity A is measured with uncertainty δA and is then used to compute $F(A)$, then the uncertainty δF in the value of $F(A)$ is given by

$$\delta F = \left(\frac{dF}{dA}\right)\delta A$$

Function $F(A)$	**Derivative,** $\frac{dF}{dA}$	**Error equation**
A^n	nA^{n-1}	$\frac{\delta F}{F} = \lvert n\rvert \frac{\delta A}{A}$
$\log_e A$	A^{-1}	$\delta F = \frac{\delta A}{A}$
$\exp(A)$	$\exp(A)$	$\frac{\delta F}{F} = \delta A$
$\sin(A)$	$\cos(A)$	$\delta F = \lvert\cos(A)\rvert\delta A$
$\cos(A)$	$-\sin(A)$	$\delta F = \lvert -\sin(A)\rvert\delta A$
$\tan(A)$	$\sec(A)^2$	$\delta F = \sec(A)^2\delta A$

All trigonometric functions and the errors in the angle variables are evaluated in radians

How to derive an error equation

Let's use the change of variable method to determine the error equation for the following expression:

$$y = \frac{M}{m}\sqrt{0.5\ kx\,(1-\sin\theta)} \tag{B.1}$$

- Begin by rewriting Equation B.1 as a product of terms:

$$\begin{aligned} y &= M * m^{-1} * [\,0.5 * k * x * (1-\sin\theta)]^{\ 1/2} && \text{(B.2)} \\ &= M * m^{-1} * 0.5^{1/2} * k^{1/2} * x^{1/2} * (1-\sin\theta)^{1/2} && \text{(B.3)} \end{aligned}$$

- Assign to each term in Equation B.3 a new variable name $A, B, C, \ldots$, then express v in terms of these new variables,

$$y = A * B * C * D * E * F \tag{B.4}$$

- With $\delta(y)$ representing the error or uncertainty in the magnitude of y, the error expression for y is easily obtained by applying Rule 4 to the product of terms Equation B.4:

$$\frac{\delta(y)}{y} = \sqrt{\left(\frac{\delta(A)}{A}\right)^2 + \left(\frac{\delta(B)}{B}\right)^2 + \left(\frac{\delta(C)}{C}\right)^2 + \left(\frac{\delta(D)}{D}\right)^2 + \left(\frac{\delta(E)}{E}\right)^2 + \left(\frac{\delta(F)}{F}\right)^2} \tag{B.5}$$

- Select from the table of error rules an appropriate error expression for each of these new variables as shown below. Note that F requires further simplification since there are two terms under the square root, so we equate these to a variable G:

$$\begin{aligned}
&A = M, && \delta(A) = \delta(M) \\
&B = m^{-1}, && \frac{\delta(B)}{B} = |-1|\ \frac{\delta(m)}{m} = \frac{\delta(m)}{m} \\
&C = 0.5^{1/2}, && \frac{\delta(C)}{C} = |\tfrac{1}{2}|\ \frac{\delta(0.5)}{|0.5|} = 0 \\
&D = k^{1/2}, && \frac{\delta(D)}{D} = |\tfrac{1}{2}|\ \frac{\delta(k)}{k} = \frac{\delta(k)}{2k} \\
&E = x^{1/2}, && \frac{\delta(E)}{E} = |\tfrac{1}{2}|\ \frac{\delta(x)}{x} = \frac{\delta(x)}{2x} \\
&F = G^{1/2}, && \frac{\delta(F)}{F} = |\tfrac{1}{2}|\ \frac{\delta(G)}{G} = \frac{\delta(G)}{2G} \\
&G = 1-\sin\theta, && \delta(G) = \sqrt{(\delta(1))^2 + (\delta(\sin\theta))^2} = \cos\theta\delta\theta
\end{aligned}$$

- Finally, replace the error terms into the original error Equation B.5, simplify and solve for $\delta(y)$ by multiplying both sides of the equation with y:

$$\delta(y) = y\sqrt{\left(\frac{\delta M}{M}\right)^2 + \left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta k}{2k}\right)^2 + \left(\frac{\delta x}{2x}\right)^2 + \left(\frac{\cos\theta\delta\theta}{2-2\sin\theta}\right)^2} \tag{B.6}$$