Chapter 15: Travelling Waves and Sound

Wave and particle models of physical phenomena

In mechanics, we made frequent use of particle models of physical phenomena; in this course we'll make frequent use of wave models of physical phenomena.

We humans are fond of thinking visually, and we are fond of using models to help us understand complex phenomena. "Oh, it's similar to this," is the kind of thinking that helps us connect a new, poorly understood phenomenon to a more familiar well-understood phenomenon. It may be limited, but it has proven to be spectacularly effective so far.

Almost every physical phenomenon can be described by a particle model or a wave model, with a few dramatic exceptions, as we shall see at the end of the course when we discuss quantum mechanics.

Because we are rooted in our concrete experience of reality, we humans have tended to prefer mechanical models of our physical concepts. Classically, our habit has been to think of electrons as very small marbles, for example, and we tend to think of waves as being mechanical waves. We are used to seeing waves on the surfaces of bodies of water, and we have seen waves on guitar strings and whatnot, so we are used to thinking of waves in something. This makes it tough to understand more abstract waves, such as electromagnetic waves, which are NOT mechanical
waves. They are NOT waves in a material medium; instead, they are waves in an electromagnetic field.

Remember that there is a difference between reality and the physical models that are present in theoretical physics. Don't confuse the two. "The map is not the territory."

Having said this, I should also tell you that in the deepest understanding that we have in our most fundamental physical theories of reality (quantum field theories), fundamental reality is modelled by fields. A field is a continuous, non-material thing. There are many different types of fundamental physical fields; electron fields, various types of quark fields, gravitational fields, electromagnetic fields, etc. What we think of as particles are explained in quantum field theories as excitations of a field; and by excitation, what we mean is a certain type of wave. So our deepest understanding is that particles are a manifestation of certain types of waves; in modern physics, fields and waves are fundamental, particles are not.

Besides this classification of waves into mechanical waves, and others, we can also classify waves as travelling waves or standing waves. A wave such as a water wave in the ocean that results from dropping a stone into the water is called a travelling wave because it has a source and then it moves away from the source "endlessly." (In reality, the wave amplitude dies down because of friction.) A standing wave is one that is trapped between boundaries, such as a wave on a guitar string. We'll study standing waves in detail in the
following chapter (Chapter 16); this chapter is all about travelling waves.

**Types of waves**

For the rest of the chapter we'll talk about travelling waves only, not standing waves. Among travelling waves, there are two types: transverse and longitudinal. The classic example of a transverse wave is a wave on a string; in a transverse wave, the displacement of particles in the material medium is perpendicular to the direction of motion of the wave. (Electromagnetic waves are transverse, even though there is no medium; in this case, it is the electric and magnetic field vectors that oscillate in directions that are perpendicular to the direction of motion of the wave.)
In a longitudinal wave, the displacement of the particles in the medium is along the line of motion of the wave. The classic example of a longitudinal wave is a sound wave in air.
Microscopic perspective on transverse waves on a string:
In simple situations, wave speed depends only on properties of the medium.

In more complicated situations, wave speed depends on the frequency of the wave as well; this is called dispersion. An example of dispersion is the separation of white light into colours upon passing through a prism.

We'll leave a general discussion of dispersion to a higher-level course; in this course we'll assume that we are dealing with simple situations where the wave speed depends only on properties of the medium, and not on the frequency of the wave.

Note that the speed of a wave in a medium depends on how "springy" the medium is (i.e., the "stiffness constant" of the medium), and also on how dense the medium is. Greater springiness means greater restoring forces, and therefore greater wave speeds; greater density means slower accelerations (due to Newton's...
second law) and therefore slower wave speeds.

speed of a transverse wave on a string:

\[
\nu = \sqrt{\frac{T}{\mu}}
\]

where \( T \) is the tension in the string, and \( \mu \) is the \textbf{linear} density of the string.

speed of a longitudinal sound wave in a gas

\[
\nu = \sqrt{\frac{\gamma k_B T}{m}}
\]

where \( \gamma \) depends on properties of the gas, \( k_B = \text{Boltzmann constant} \), \( T = \text{temperature} \), \( m = \text{mass of molecule of gas} \).

Example

The speed of a wave on a string is 140 m/s and its tension is 20 N. Determine the linear density of the string.

Solution:

\[
\nu = 140 \text{ m/s} \quad \nu = \sqrt{\frac{T}{\mu}}
\]

\[
T = 20 \text{ N} \quad \nu = \sqrt{\frac{T}{\mu}}
\]

\[
m = ?
\]
Example

If the tension in a string is doubled, how does the wave speed change?

Solution:

\[ \mu = \frac{T}{u^2} \]

\[ \mu = \frac{20 \text{ kg} \cdot \text{m} \cdot \text{s}^{-2}}{140^2 \text{ m}^2 \cdot \text{s}^{-2}} \]

\[ \mu = 1.02 \times 10^{-3} \frac{\text{kg}}{\text{m}} \]
Conclusion: When you double the tension in the string, the wave speed increases by a factor of the square root of 2.

Example

You hear a clap of thunder 5 s after you see the flash of lightning that caused the thunder. How far away was the lightning strike?

Solution: Light travels almost instantly from the lightning strike to the observer; the time needed is so
small that we can safely ignore it. (The speed of light is 300,000,000 m/s.) The sound that the lightning makes takes time to reach the observer, and the time $t$ needed is related to the distance $x$ between the lightning strike and the observer by the speed $v$ of sound:

$$x = v \cdot t$$

$$= \left(340 \frac{m}{s}\right)(5s)$$

$$= 1700 m$$

$$x = 1.7 \text{ km}$$

Thus, the lightning strike is nearly 2 km away from the observer.

This problem suggests a good rule of thumb for determining the distance to a lightning strike in km: Measure the time interval in seconds between seeing the lightning and hearing the thunder and then divide this time interval by 3.

**Graphical descriptions of waves**

One way to visualize the motion of a wave is to use a sequence of "snapshot" graphs, as follows. Not all waves are sinusoidal, but we study these types of waves at first because they are relatively simple, as they can be produced by a source that undergoes simple harmonic motion.
Note that the previous two graphs are "snapshot" graphs, not position-time graphs (the latter are also called "history" graphs in the textbook). In snapshot graphs we speak of wavelength, not period; the units of period are time units, whereas the units of wavelength are distance units.

Here is a history graph; it's worth comparing it to the two previous graphs and understanding how the two are related.
The period $T$ is the *time* between successive crests.

The motion of the point on the string is SHM.

The following sequence of history graphs illustrates their use in portraying a travelling wave.
This crest is moving to the right.

During a time interval of exactly one period, the crest has moved forward exactly one wavelength.
Here are some formulas for a sinusoidal transverse travelling wave. The variable $y$ represents the transverse displacement (i.e., the displacement of the medium perpendicular to the wave motion), the variable $x$ represents the displacement of the wave in the direction of its motion, and $t$ represents time.

$$y(x, t) = A \cos \left( 2\pi \left( \frac{x}{\lambda} - \frac{t}{T} \right) \right)$$ \hspace{1cm} (15.7)

Displacement of a traveling wave moving to the right with amplitude $A$, wavelength $\lambda$, and period $T$

$$y(x, t) = A \cos \left( 2\pi \left( \frac{x}{\lambda} + \frac{t}{T} \right) \right)$$ \hspace{1cm} (15.8)

Displacement of a traveling wave moving to the left

An alternative way to write these two equations is

$$y = A \cos (kx - \omega t) \hspace{1cm} (\text{to the right})$$

and

$$y = A \cos (kx + \omega t) \hspace{1cm} (\text{to the left})$$

where

$$k = \frac{2\pi}{\lambda} \hspace{1cm} (\text{"wave number"})$$

and

$$\omega = \frac{2\pi}{T} \hspace{1cm} (\text{"angular frequency"})$$

There is a fundamental relationship between wavelength, frequency, and wave speed for a sinusoidal wave; here's how to derive it: Focus your attention on a position of the
wave at a certain time that has a certain value for the argument of the sinusoidal function. Let's use 0 as this value, for simplicity (any other value would work just as well). Now ask yourself how this place in the wave (that has a constant value of the argument) moves in time; here's the answer:

\[
y = A \cos \left(2\pi \left(\frac{x}{\lambda} - \frac{t}{T}\right)\right)
\]

Set \( \frac{x}{\lambda} - \frac{t}{T} = 0 \)

Then:

\[
\frac{x}{\lambda} = \frac{t}{T}
\]

\[
x = \left(\frac{\lambda}{T}\right) t \quad \text{Recall} \quad f = \frac{1}{T}
\]

This is a position-time function for a motion to the right with the quantity in parentheses representing the speed of the wave. (Repeat the calculation with the opposite sign in the starting formula, and you'll understand why it represents a wave moving to the left.) That is, the point on the wave that we are focussing our attention on moves to the right with speed

\[
\mathbf{v} = \frac{\lambda}{T}
\]

which is equivalent to

\[
\mathbf{v} = \lambda f
\]

But every point on the wave moves with the same speed, at least for the simple waves that we are dealing with in this chapter. Thus, the entire wave moves to the right
with the given speed.

Example

The $y$-displacement of a transverse wave traveling in the negative $x$-direction is $y = 4.3 \cos (2.4x + 30t)$, where $t$ is measured in seconds, $x$ is measured in metres, and $y$ is measured in centimetres. Determine the amplitude, frequency, wavelength, and speed of the wave.

Solution:

\[ y = A \cos \left( \frac{2\pi}{\lambda} \left( x + \frac{t}{T} \right) \right) \]

\[ y = 4.3 \cos \left( 2.4x + 30t \right) \]

By comparing coefficients we get:

\[ A = 4.3 \text{ cm} \]

\[ \frac{2\pi}{\lambda} = 2.4 \implies \lambda = \frac{2\pi}{2.4} \text{ m} \]

Example

In the deep ocean, a water wave with wavelength 83 m travels at a speed of 15 m/s. Suppose that a small boat is at a crest of this wave, 0.82 m above the equilibrium
level. Determine the vertical displacement of the boat above or below the equilibrium level 6.8 s later. (The boat simply moves up and down.)

Solution: Write a formula for the vertical displacement of the wave, and then substitute 6.8 s for \( t \).

Assuming that the water wave moves to the right (assuming that the water wave moves to the left leads to the same result; try it!), a generic formula is

\[
y = A \cos\left(\frac{2\pi}{\lambda} x - \frac{2\pi}{T} t\right)
\]

Given information:

\[
A = 0.82 \text{ m}, \quad \nu = 15 \text{ m/s}, \quad \lambda = 8.3 \text{ m}
\]

To determine the period \( T \), use the speed and the wavelength:

\[
\nu = \frac{\lambda}{T} \quad \rightarrow \quad T = \frac{\lambda}{\nu}
\]

\[
T = \frac{8.3 \text{ m}}{15 \text{ m/s}} = 0.553 \text{ s}
\]

Substituting the values for \( A \), \( T \), and \( \lambda \) into the formula for \( y \), we obtain

\[
y = 0.82 \cos\left(\frac{2\pi}{8.3} x - \frac{2\pi \cdot 15}{8.3} t\right)
\]
We are free to choose our coordinate system (i.e., the initial values of \( x \) and \( t \) for the boat) at our convenience, provided that it matches the initial conditions given in the statement of the problem.

A simple choice that works is to choose the horizontal position of the boat to be \( x = 0 \) and to choose the time at which the boat is initially at the crest of the wave to be \( t = 0 \). Substituting these values into the formula for \( y \), you will see that this is consistent with the given initial \( y \)-value of the boat, \( y = 0.82 \) m.

Substituting \( x = 0 \) and \( t = 6.8 \) s into the formula for \( y \), we obtain the vertical position of the boat after 6.8 s:

\[
y = 0.82 \cos \left( \frac{2\pi}{8} \left[ x - 15 t \right] \right)
\]

\[
y = 0.82 \cos \left( \frac{2\pi}{8} \left[ 0 - 15 \cdot 6.8 \right] \right)
\]

\[
y = 0.82 \cos \left( -7.72 \text{ rad} \right)
\]

\[
y = 0.108 \text{ m}
\]

\[
y = 10.8 \text{ cm}
\]

Thus, the vertical displacement of the boat after 6.8 s is 10.8 cm above the equilibrium position.
Note that the argument of the cosine function is in radians; make sure to adjust your calculator settings if necessary.

- some details on sound waves and light waves

  ultrasound imaging; shorter wavelength means sharper resolution of images
  
echolocation

  some typical sound frequencies that are audible for different animals:

**TABLE 15.2** Range of hearing for animals

<table>
<thead>
<tr>
<th>Animal</th>
<th>Range of hearing (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elephant</td>
<td>&lt;5–12,000</td>
</tr>
<tr>
<td>Owl</td>
<td>200–12,000</td>
</tr>
<tr>
<td>Human</td>
<td>20–20,000</td>
</tr>
<tr>
<td>Dog</td>
<td>30–45,000</td>
</tr>
<tr>
<td>Mouse</td>
<td>1000–90,000</td>
</tr>
<tr>
<td>Bat</td>
<td>2000–100,000</td>
</tr>
<tr>
<td>Porpoise</td>
<td>75–150,000</td>
</tr>
</tbody>
</table>

Typical light frequencies within the electromagnetic spectrum:
Energy and intensity of a wave

Recall that power is the rate at which energy is transferred, or transmitted, or transformed, etc.

When it comes to wave motion, we are also interested in how the energy of a wave is distributed in the wave; that is, how concentrated is the energy in the wave?

Often a wave is created by some sort of source, and then the wave spreads out. As the wave spreads, the power in the wave is also spread out; provided the power in the wave is evenly spread, the intensity of the wave is defined as the power density:

$$I = \frac{P}{A}$$

where $I$ is the intensity of the wave at the wave front of interest, $P$ is the power emitted by the source, and $A$ is the area (i.e., surface area) of the wave front of interest. A particularly simple situation occurs when the wave spreads out spherically from a point source. In this case the area of the wave front is the surface area of a sphere,
Notice the inverse-square dependence of the wave intensity as the distance of the wave front from the source increases. This formula has the same structure as Newton's law of gravity; this happens over and over again in physics, with diverse phenomena described by similar formulas.

Example

Sound is detected when a sound wave causes the eardrum to vibrate. Typically, the diameter of the eardrum is about 8.4 mm in humans. When someone speaks to you in a normal tone of voice, the sound intensity at your ear is about $1.0 \times 10^{-6} \text{ W/m}^2$.

Determine the amount of energy delivered to each eardrum each second.

Solution: First determine the area of each eardrum:

$$r = \frac{8.4 \text{ mm}}{2} = 4.2 \times 10^{-3} \text{ m}$$

$$A = \pi r^2 = \pi (4.2 \times 10^{-3} \text{ m})^2$$

Now determine the power delivered to an eardrum:

$$I = \frac{P}{A} \Rightarrow P = IA$$
Example

The intensity of electromagnetic waves from the Sun is 1.4 kW/m² just above the Earth's atmosphere. Eighty percent of this reaches the surface at noon on a clear summer day. Suppose you model your back as a 30 cm by 50 cm rectangle. Determine the solar energy incident on your back in 1 h.

Solution: The intensity of the Sun's electromagnetic waves at the Earth's surface is

\[
I = (0.8)(1.4 \ \text{kW/m}^2) = 1.12 \ \text{kW/m}^2
\]

Thus, the power incident on the back is

\[
P = (1.0 \times 10^{-6} \ \text{W/m}^2) \pi (4.2 \times 10^{-3} \text{m})^2
\]

\[
P = 5.54 \times 10^{-7} \ \text{W}
\]

Thus, the energy delivered to an eardrum in 1 s is

\[
E = P \Delta t
\]

\[
E = \left(5.54 \times 10^{-7} \ \frac{\text{J}}{\text{s}}\right)(1 \text{s})
\]

\[
E = 5.54 \times 10^{-7} \ \text{J}
\]
The decibel scale for the loudness of sound

Our ears respond to sound in a non-linear way; in fact the response is logarithmic. This allows the ears to safely respond to an extremely wide range of sound levels.

Review of logarithms

Imagine what it would be like if you had to work with and communicate numbers that had a very wide range of magnitudes, day in and day out, for many years. After some time, you would certainly take short cuts, right? For example, if you had to talk about numbers such as $10^2$, $10^{13}$, $10^{-5}$, $10^7$, and so on, eventually you would just communicate the exponents, right? The base 10 would be understood if you were dealing with such numbers day after day.
This is the idea behind logarithms, except that logarithms are more precise, as they also take into account any factors of the power of 10. Indeed, if you pay attention to the slogan "a logarithm is an exponent" it will help you understand them.

Example: Determine the logarithm of each number to base 10.
(a) $10^4$ (b) $2.3 \times 10^7$ (c) $4.9 \times 10^{-5}$

Solution:

(a) $\log_{10}(10^4) = 4$

(b) $\log_{10}(2.3 \times 10^7) = \log_{10}(2.3) + \log_{10}(10^7)$

\[
= 0.362 + 7
\]

\[
= 7.36
\]

(c) $\log_{10}(4.9 \times 10^{-5}) = \log_{10}(4.9) + \log_{10}(10^{-5})$

\[
= 0.69 + (-5)
\]

\[
= -4.31
\]

\[
\beta = (10 \text{ dB}) \log_{10}\left(\frac{I}{I_0}\right) \quad (15.14)
\]

Sound intensity level in decibels for a sound of intensity $I$
Example

Determine the sound intensity of a whisper at a distance of 2.0 m. Determine the corresponding sound intensity level.

Solution: $$I = \frac{P}{4\pi r^2}$$
solution: \[ \frac{1}{I_2} = \frac{4\pi r_2^2}{P} = \frac{P}{4\pi r_1^2} \]

\[ \frac{I_2}{I_1} = \frac{P}{4\pi r_2^2 \cdot \frac{4\pi r_1^2}{P}} = \frac{r_1^2}{r_2^2} = \left(\frac{r_1}{r_2}\right)^2 \]

\[ \frac{I_2}{I_1} = \left(\frac{1.0}{2.0}\right)^2 = \frac{1}{4} \]

\[ I_2 = \frac{1}{4} I_1 = \frac{1}{4} \left(1.0 \times 10^{-10} \text{ W/m}^2\right) \]

\[ I_2 = 2.5 \times 10^{-11} \text{ W/m}^2 \]

\[ \beta = (10 \text{ dB}) \log_{10} \left( \frac{I_2}{I_0} \right) \]

\[ \beta = (10 \text{ dB}) \log_{10} \left( \frac{2.5 \times 10^{-11}}{1.0 \times 10^{-12}} \right) \]

\[ \beta = (10 \text{ dB}) \log_{10} (25) \]

\[ \beta = 14 \text{ dB} \]
Example
(a) Determine the change in sound level if the intensity of a sound is doubled.
(b) Determine the change in intensity of a sound if the sound level increases by 15 dB.

Solution:

(a) \[ \beta = (10 \text{ dB}) \log_{10} \left( \frac{I}{I_0} \right) \]

\[ \beta_2 - \beta_1 = (10 \text{ dB}) \log_{10} \left( \frac{I_2}{I_0} \right) - (10 \text{ dB}) \log_{10} \left( \frac{I_1}{I_0} \right) \]

\[ = (10 \text{ dB}) \left[ \log_{10} \left( \frac{I_2}{I_0} \right) - \log_{10} \left( \frac{I_1}{I_0} \right) \right] \]

\[ = (10 \text{ dB}) \left[ \log_{10} \left( \frac{I_2}{I_0} \div \frac{I_1}{I_0} \right) \right] \]

\[ = (10 \text{ dB}) \log_{10} (2) \]

\[ = (10 \text{ dB}) (0.301) \]

\[ \beta_2 - \beta_1 = 3 \text{ dB} \]

(b) \[ \beta_2 - \beta_1 = (10 \text{ dB}) \log_{10} \left( \frac{I_2}{I_1} \right) \]

\[ 15 \text{ dB} = (10 \text{ dB}) \log_{10} \left( \frac{I_2}{I_1} \right) \]

\[ \frac{15 \text{ dB}}{10 \text{ dB}} = \log_{10} \left( \frac{I_2}{I_1} \right) \]
A gentleman in class on Friday tried to make a point, but I was unable to understand what his point was. After class we had a conversation, and I realized his point is similar to the point of the previous example.

His point can be expressed in two ways. First, an increase in sound level of 1 dB at high intensity means a much greater actual change in intensity than a 1 dB increase in sound level at low intensity. Similarly, an increase in intensity of $10^{-8}$ W/m$^2$ at low intensity produces a much greater increase in sound level than an increase in intensity of $10^{-8}$ W/m$^2$ at high intensity. His point is that the previous sentence explains that the ear is more sensitive at low intensity than at high intensity. This makes sense based on our experience hearing loud and quiet sounds, doesn't it?

The Doppler effect
If the source of a sound is in motion relative to an observer, then the frequency of the sound as heard by the observer is not the same as the frequency of the sound emitted by the source. This effect, named after Christian Doppler (1842), depends on whether the source and observer are moving towards each other or away from each other. Therefore, there is a distinct shift in the observed frequency when a source of sound passes an observer. You may have heard this shift when a train or ambulance passes you.

http://www.youtube.com/watch?v=imoxDcn2Sgo

The Wikipedia page on the Doppler effect has some nice simulations:
Here are the relevant formulas:

For a stationary observer and a moving source:

\[ f_+ = \frac{f_0}{1 - \frac{v_s}{v}} \]

Observed frequency of a wave of speed \( v \) emitted from a source approaching at speed \( v_s \)

\[ f_- = \frac{f_0}{1 + \frac{v_s}{v}} \]

Observed frequency of a wave of speed \( v \) emitted from a source receding at speed \( v_s \)

For a stationary source and a moving observer:

\[ f_+ = \left( 1 + \frac{v_o}{v} \right) f_0 \]

Doppler effect for an observer approaching a source

\[ f_- = \left( 1 - \frac{v_o}{v} \right) f_0 \]

Doppler effect for an observer receding from a source

All speeds are relative to the (assumed stationary) medium. These formulas can be summarized into one super-formula:

\[ f_\pm = \left( \frac{v \mp v_0}{v \mp v_s} \right) f_0 \]

\( f_\pm \rightarrow \) observed frequency

\( f_0 \rightarrow \) source frequency
\[ f_0 \rightarrow \text{source frequency} \]
\[ v \rightarrow \text{sound speed} \]
\[ v_o \rightarrow \text{observers} \]
\[ v_s \rightarrow \text{source speed} \]

- applications of the Doppler effect: radar guns, echolocation, etc.

\[ \Delta f = \pm 2f_0 \frac{v_o}{v} \]  \hspace{1cm} (15.18)

Frequency shift of waves reflected from an object moving at speed \( v_o \).

cosmological occurrence of Doppler effect

moving toward you: blueshift

at rest

moving away from you: redshift
shock waves and bow waves

... see photo and diagrams on wikipedia page cited above

Example

An opera singer in a convertible sings a note at 600 Hz while cruising down the highway at 90 km/h. Determine the frequency heard by a person standing beside the highway (a) in front of the car, and (b) behind the car. (Use 340 m/s for the speed of sound.)

Solution: First convert 90 km/h to m/s:

\[
\frac{90 \text{ km}}{\text{h}} = \frac{90000 \text{ m}}{3600 \text{ s}} = 25 \text{ m/s}
\]

(a) \( f_+ = \left( \frac{v}{v - v_s} \right) f_0 \)

\[
= \left( \frac{340 \text{ m/s}}{340 \text{ m/s} - 25 \text{ m/s}} \right)(600 \text{ Hz})
\]

\( f_+ = 648 \text{ Hz} \)

(b) \( f_+ = \left( \frac{v}{v + v_s} \right) f_0 \)

\[
= \left( \frac{340 \text{ m/s}}{340 \text{ m/s} + 25 \text{ m/s}} \right)(600 \text{ Hz})
\]
\[
\begin{align*}
\mathcal{S} & \approx \left\{ \left. \begin{array}{c}
340 \text{ m/s} + 25 \text{ m/s} \\
\end{array} \right/ \right. \\
\mathcal{S} & = 559 \text{ Hz}
\end{align*}
\]