GRAVITATIONAL REDSHIFT

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INTRODUCTION AND THEORY

- Review of the Doppler Effect

$$f_2 = f_1(1 \pm \frac{v_r}{c})$$

- Gravitational Redshift
- Questions:
  - How much of a wavelength shift is from gravitational redshift? How much is from the Doppler Effect?
  - What influences how appreciable gravitational redshift is compared to the Doppler Effect?

DERIVATION OF THE GRAVITATIONAL REDSHIFT EQUATION

$$E = E_0 + \triangle GPE$$

$$E = m_o c^2 + \frac{-GMm_0}{r_2} - (-\frac{-GMm_0}{r_1})$$

$$r_1 \longrightarrow \infty$$

$$-\frac{-GMm_0}{r_1} \longrightarrow 0$$

$$E = m_o c^2 - \frac{GMm_0}{r_2}$$

$$E_0 = m_0 c^2 = hf_1$$

$$m_0 = \frac{hf_1}{c^2}$$

$$E = \frac{hf_1}{c^2} c^2 - \frac{GM \frac{hf_1}{c^2}}{r_2}$$

E = Total Energy
E_0 = Rest Energy
M = Mass of star
m = "mass" of photon
r_2 = Radius of star
r_1 = Distance from star to Earth
f_1 = Source frequency
f_2 = Observed frequency
λ_1 = Source wavelength
λ_2 = Observed wavelength
G = Gravitational Constant
c = Speed of light
h = Planck's Constant

DERIVATION OF THE GRAVITATIONAL REDSHIFT EQUATION

$$E = hf_1 - \frac{GMhf_1}{r_2c^2}$$

$$E = hf_1\left[1 - \frac{GM}{r_2c^2}\right]$$

$$E = hf_2$$

$$hf_2 = hf_1[1 - \frac{GM}{r_2c^2}]$$

$$f_2 = f_1[1 - \frac{GM}{r_2c^2}]$$

$$\frac{c}{\lambda_2} = \frac{c}{\lambda_1}[1 - \frac{GM}{r_2c^2}]$$

$$\lambda_1 = \lambda_2[1 - \frac{GM}{r_2c^2}]$$

$$\boxed{\frac{\lambda_2 - \lambda_1}{\lambda_1} = \frac{1}{[\frac{r_2c^2}{GM} - 1]}}$$

THEORY OF GENERAL RELATIVITY

- Overview of General Relativity: curvature of space-time and the bending of light
- Corrections to the derived classical equation must be made
- The Schwarzschild metric for gravitational redshift:

$$\frac{\lambda_2 - \lambda_1}{\lambda_1} = \frac{1}{\sqrt{(1 - \frac{2GM}{r_2 c^2})}} - 1$$

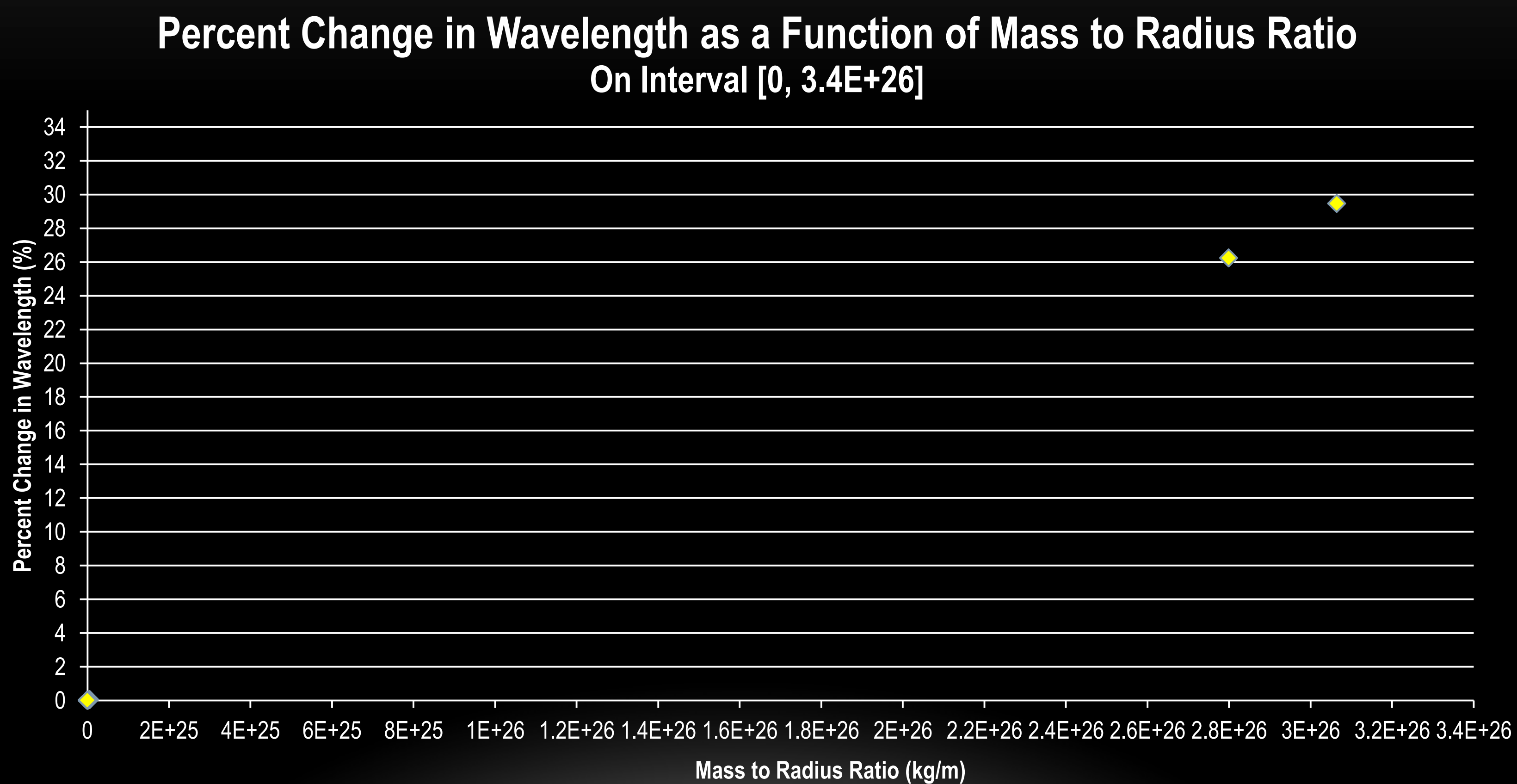
DATA AND ANALYSIS

Star Name	Type	Mass (kg)	Radius (m)	Relative Velocity (m/s)
Rigel A	Blue-White Supergiant	4.18E+31	54874950000	17800
Epsilon Eridani	Brown Dwarf	1.52E+30	509106000	-40400
61 Cygni A	K-Type Dwarf Star	1.39E+30	462507000	-64300
Wolf 359	N/A	1.79E+29	111280000	19000
PSR J0348+0432	Neutron Star	4.00E+30	13041.78932	-12000
Crab Pulsar	Neutron Star	2.78E+30	9945.65	N/A
Proxima Centauri	Red Dwarf	2.45E+29	98065000	-21700
DX Cancri	Red Dwarf	1.79E+29	76505000	9000
Ross 128	Red Dwarf	2.98E+29	146055000	-31000
Barnard's Star	Red Dwarf M4Ve	2.86E+29	136483032	-110600
UY Scuti	Red Hypergiant	1.59E+31	1.18791E+12	N/A
Altair	Type-A Main Sequence	3.55E+30	1279720000	-26100
Procyon B	White Dwarf	1.19711E+30	8592860.28	-3200
Sirius B	White Dwarf	1.9448E+30	5849272.8	-7600
IK Pegasi B	White Dwarf	2.29E+30	4178052	-11400
Van Maanen 2	White Dwarf	1.35E+30	7659762	-38000
40 Eridani B	White Dwarf	9.94E+29	9748788	-43000
Procyon A	White Main Sequence	2.82E+30	1424384000	-3200
The Sun	Yellow Dwarf G2V	1.98855E+30	696342000	N/A
Alpha Centauri A	Yellow Dwarf G2V	2.19E+30	854411634	-21600
Zeta Ophiuchi	Yellow Supergiant	3.98E+31	5911750000	-15000

DATA AND ANALYSIS

Star Name	Type	Mass to Radius Ratio (kg/m)	Percent Change in Wavelength due to Gravitational Redshift (%)	Percent Change in Wavelength due to Doppler Effect (%)	Equivalent Doppler Velocity (m/s)
Rigel A	Blue-White Supergiant	7.61732E+20	5.65598E-05	-0.005937088	-169.5620265
Epsilon Eridani	Brown Dwarf	2.98563E+21	0.000221688	0.013477806	-664.6022797
61 Cygni A	K-Type Dwarf Star	3.00958E+21	0.000223466	0.021452773	-669.9336297
Wolf 359	N/A	1.60766E+21	0.000119371	-0.006337316	-357.8653726
PSR J0348+0432	Neutron Star	3.06475E+26	29.46035162	0.004002929	-68221591.51
Crab Pulsar	Neutron Star	2.79911E+26	26.23690087	N/A	-62308445.05
Proxima Centauri	Red Dwarf	2.49426E+21	0.000185203	0.007238865	-555.2248227
DX Cancri	Red Dwarf	2.33972E+21	0.000173728	-0.003001987	-520.8224133
Ross 128	Red Dwarf	2.04221E+21	0.000151638	0.010341556	-454.5973287
Barnard's Star	Red Dwarf M4Ve	2.09807E+21	0.000155785	0.036905804	-467.0321733
UY Scuti	Red Hypergiant	1.33848E+19	9.93843E-07	N/A	-2.979467014
Altair	Type-A Main Sequence	2.77404E+21	0.000205978	0.008706781	-617.5041061
Procyon B	White Dwarf	1.39314E+23	0.010345368	0.001067416	-31011.42411
Sirius B	White Dwarf	3.32486E+23	0.024693717	0.002535151	-74011.62635
IK Pegasi B	White Dwarf	5.47344E+23	0.0406577	0.003802775	-121839.1804
Van Maanen 2	White Dwarf	1.76535E+23	0.013109702	0.012677042	-39296.74751
40 Eridani B	White Dwarf	1.0199E+23	0.007573463	0.014345314	-22702.95287
Procyon A	White Main Sequence	1.9798E+21	0.000147004	0.001067416	-440.7055053
The Sun	Yellow Dwarf G2V	2.85571E+21	0.000212041	N/A	-635.6826803
Alpha Centauri A	Yellow Dwarf G2V	2.56013E+21	0.000190094	0.007205504	-569.8866735
Zeta Ophiuchi	Yellow Supergiant	6.72728E+21	0.000499514	0.005003712	-1497.497083

DATA AND ANALYSIS



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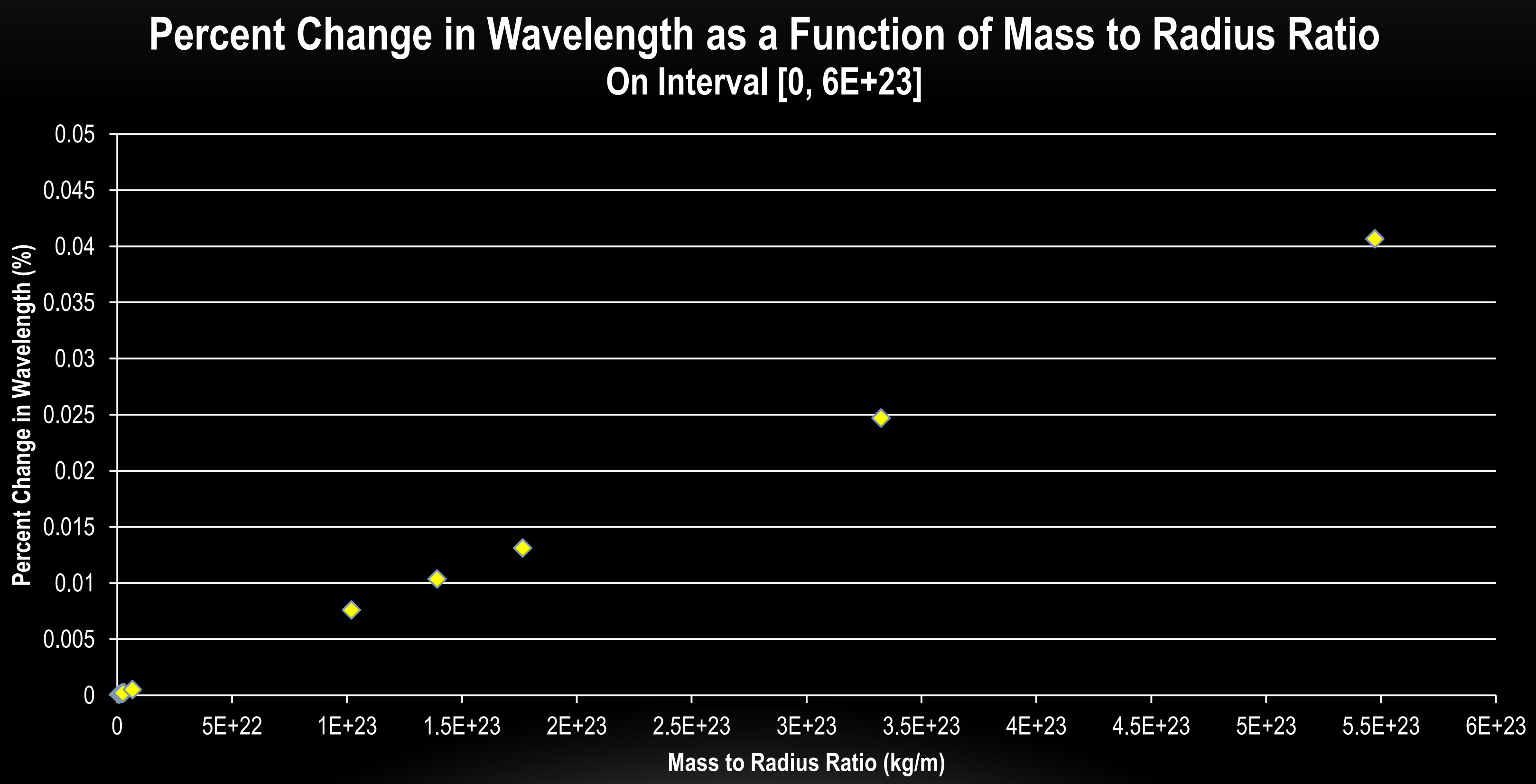

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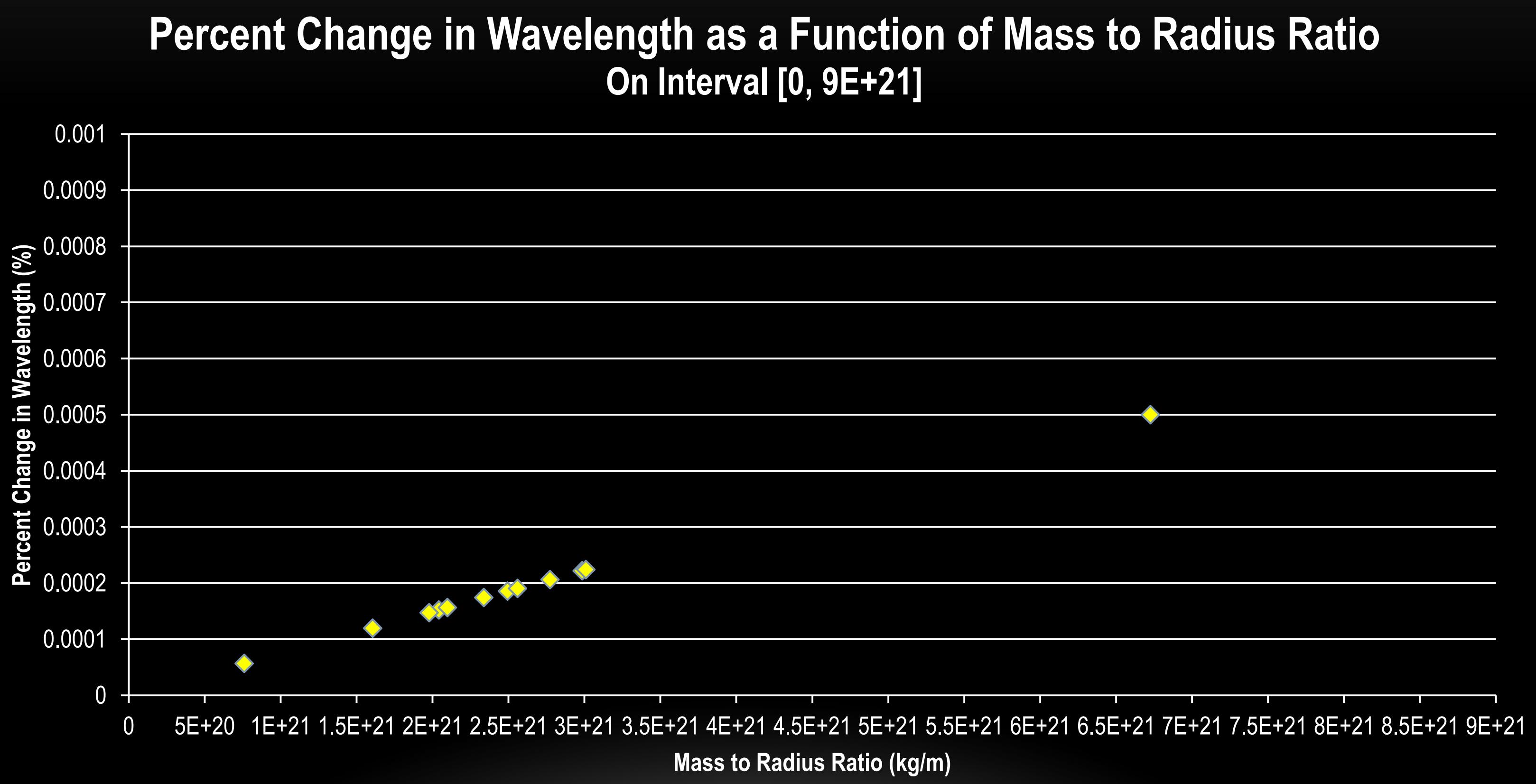

DATA AND ANALYSIS

Star Name	Type	Percent of Shift from Gravitational Redshift (%)	Percent of Shift from Doppler Effect (%)	Equivalent Doppler Velocity to Relative Velocity Ratio (%)
Rigel A	Blue-White Supergiant	0.943662922	99.05633708	-0.952595654
Epsilon Eridani	Brown Dwarf	1.618220023	98.38177998	1.645055148
61 Cygni A	K-Type Dwarf Star	1.030927487	98.96907251	1.041887449
Wolf 359	N/A	1.848799164	98.15120084	-1.883501961
PSR J0348+0432	Neutron Star	99.98641433	0.013585668	568513.2626
Crab Pulsar	Neutron Star	N/A	N/A	N/A
Proxima Centauri	Red Dwarf	2.49463505	97.50536495	2.558639736
DX Cancri	Red Dwarf	5.470515321	94.52948468	-5.786915703
Ross 128	Red Dwarf	1.445104113	98.55489589	1.466442996
Barnard's Star	Red Dwarf M4Ve	0.420341945	99.57965805	0.422271404
UY Scuti	Red Hypergiant	N/A	N/A	N/A
Altair	Type-A Main Sequence	2.311042336	97.68895766	2.365916115
Procyon B	White Dwarf	90.64718589	9.352814109	969.1070035
Sirius B	White Dwarf	90.68947225	9.310527752	973.8371888
IK Pegasi B	White Dwarf	91.44684062	8.553159381	1068.76474
Van Maanen 2	White Dwarf	50.8389193	49.1610807	103.4124935
40 Eridani B	White Dwarf	34.55239959	65.44760041	52.79756481
Procyon A	White Main Sequence	12.10484999	87.89515001	13.77204704
The Sun	Yellow Dwarf G2V	N/A	N/A	N/A
Alpha Centauri A	Yellow Dwarf G2V	2.570368232	97.42963177	2.638364229
Zeta Ophiuchi	Yellow Supergiant	9.076745099	90.9232549	9.983313885

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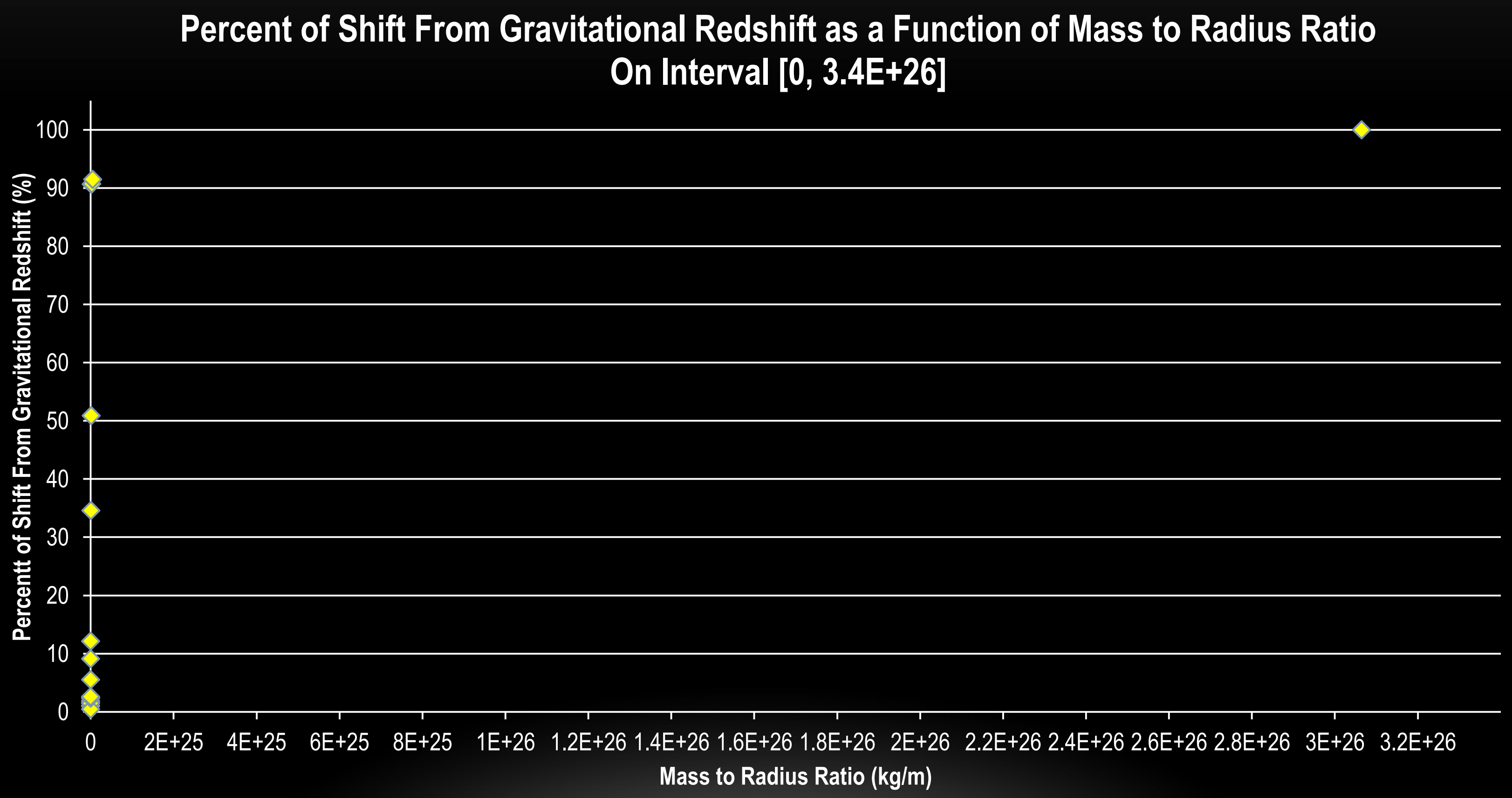

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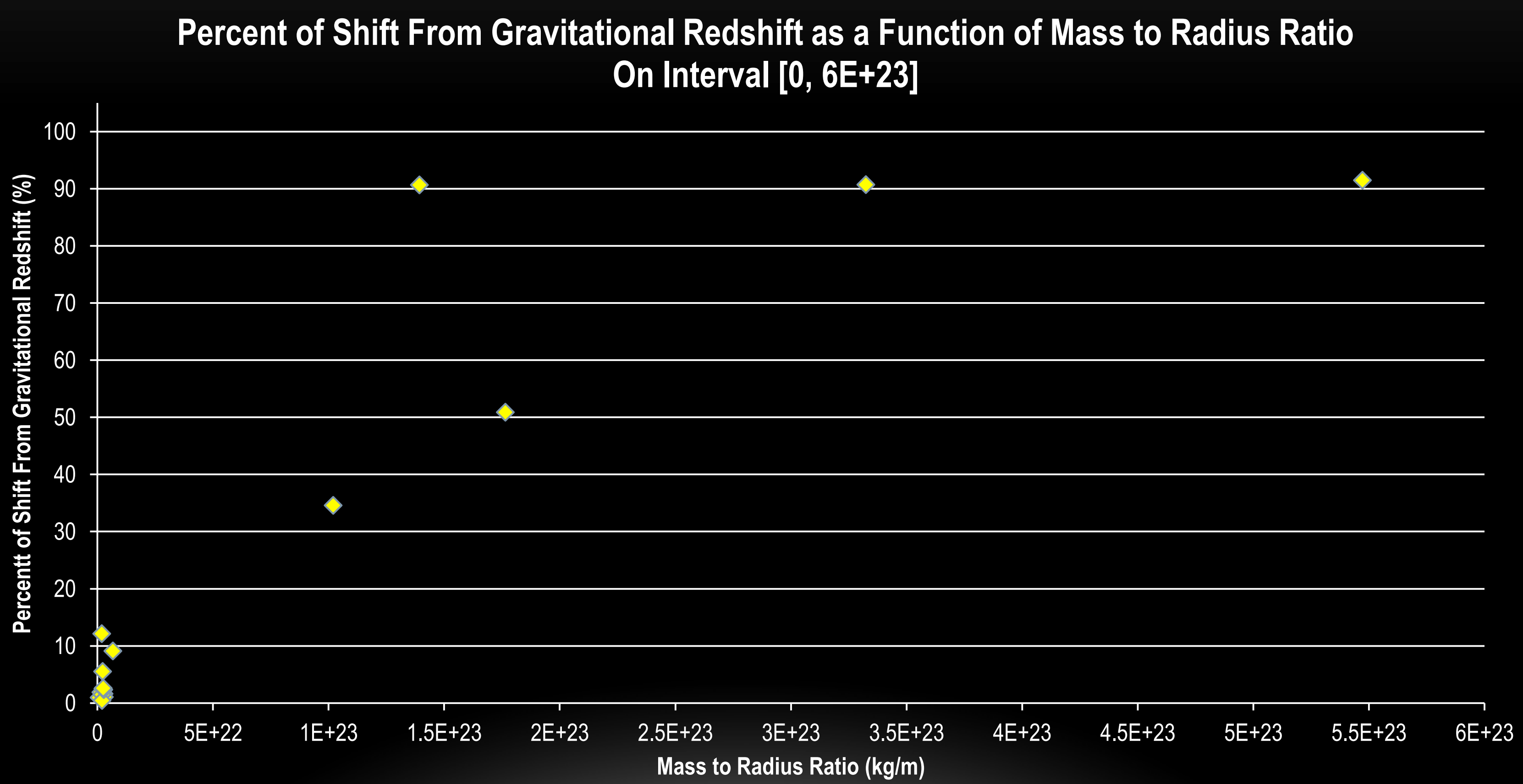

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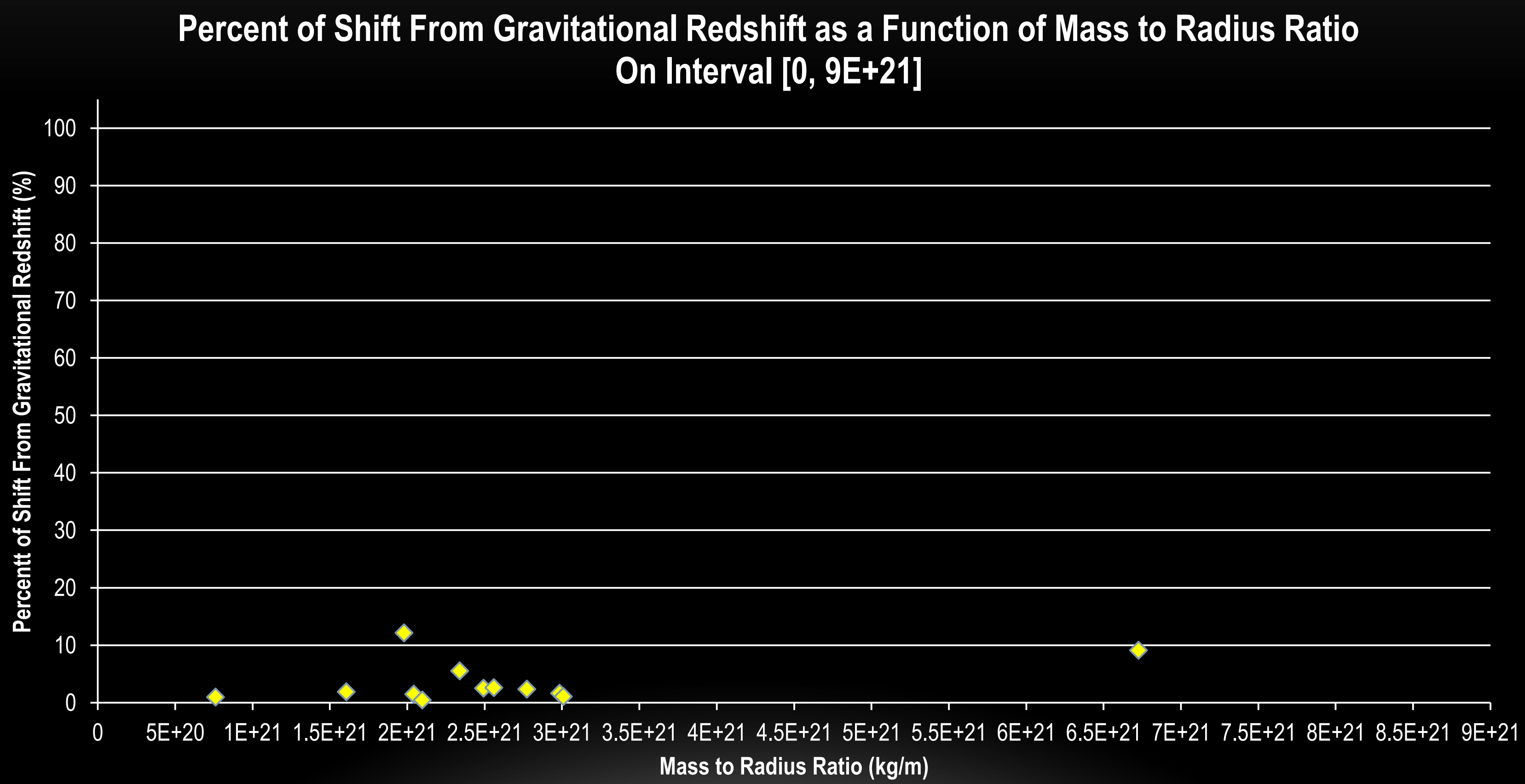

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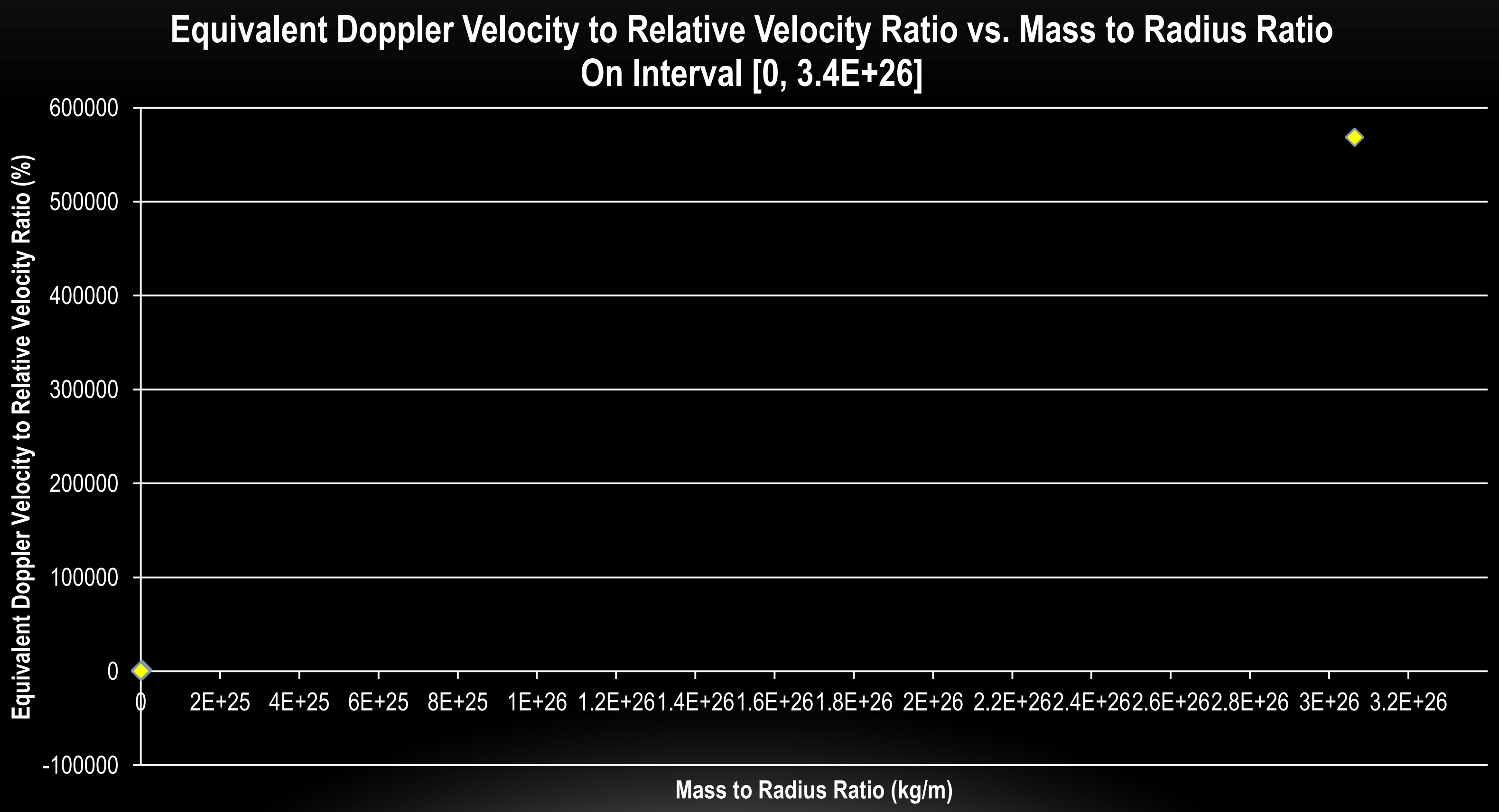

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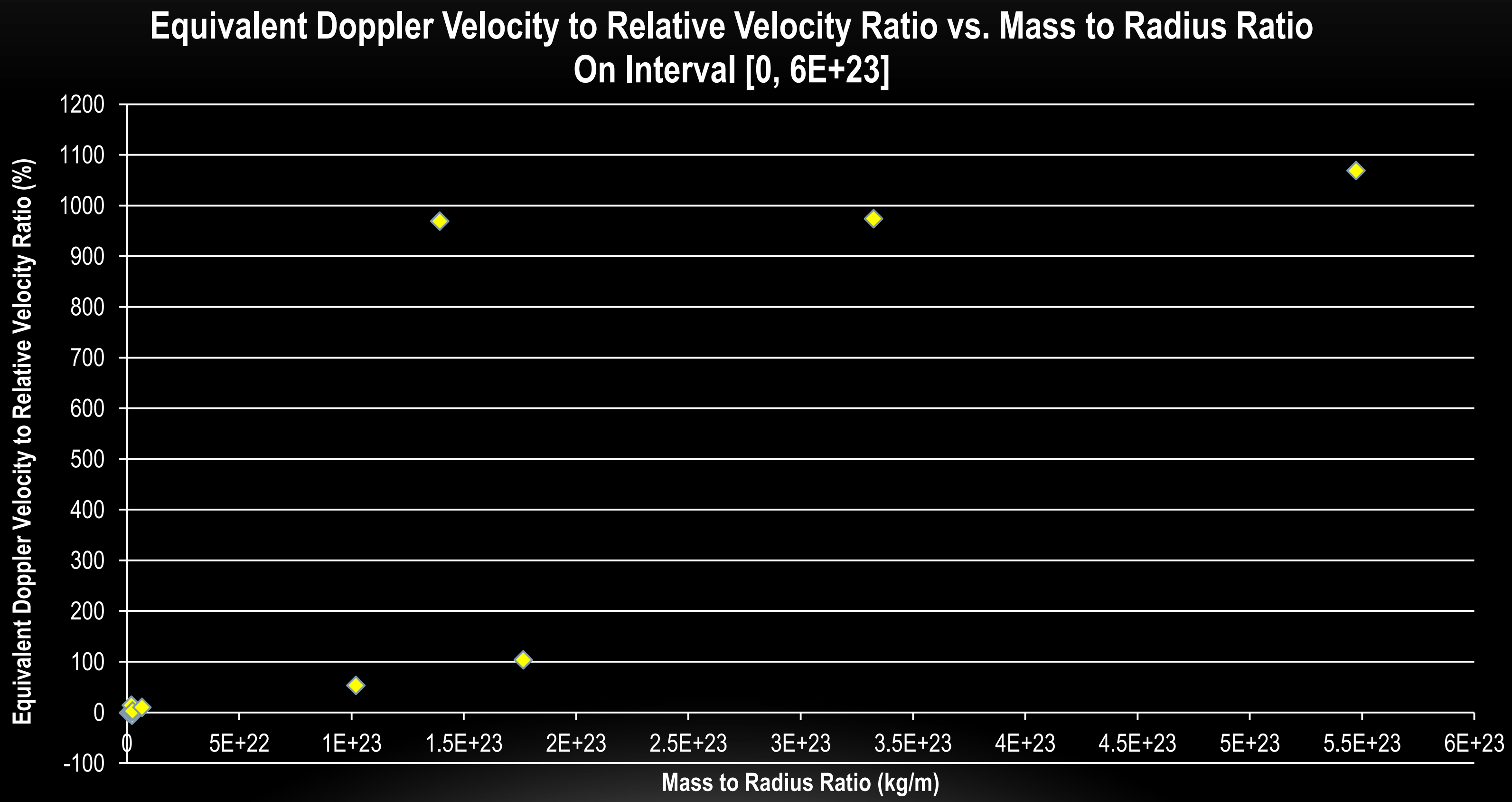

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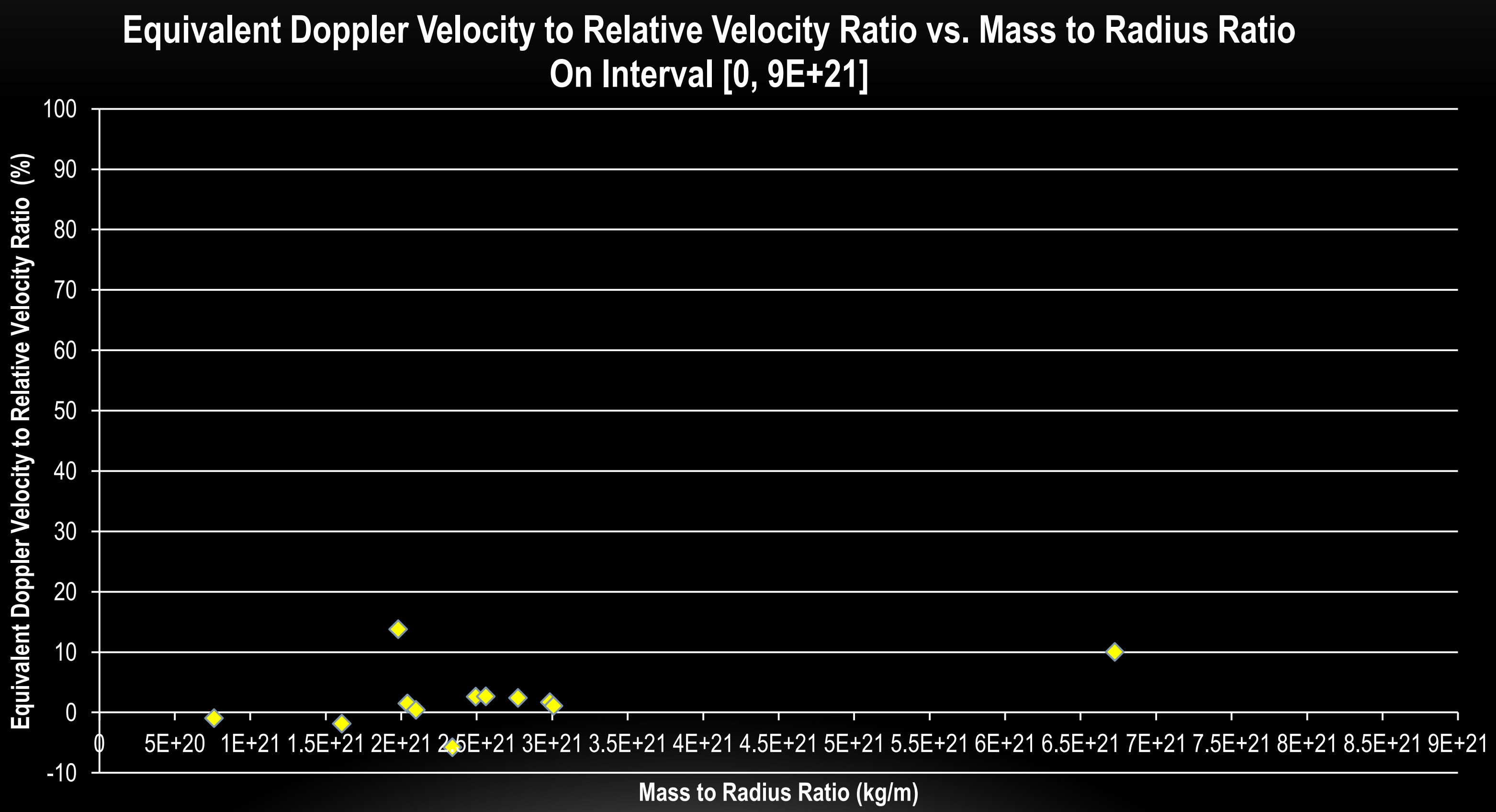

CONCLUSIONS

- Is gravitational redshift negligible or appreciable?
- The contribution of gravitational redshift cannot be generalized as it depends on the characteristics of a star
- Stars with a large mass-to-radius ratio will exhibit a large gravitational redshift
- The effects of gravitational redshift for a star will be dampened as the relative velocity of the star increases

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