Theory

The ballistic pendulum is used to explore the transfer and conservation of energy and momentum in a collision of two objects. One of these objects is a small projectile of mass m that is projected at a certain speed v by a launcher. The second object is a pendulum that is initially stationary. As the projectile hits the pendulum, a re-distribution of energy and momentum takes place.

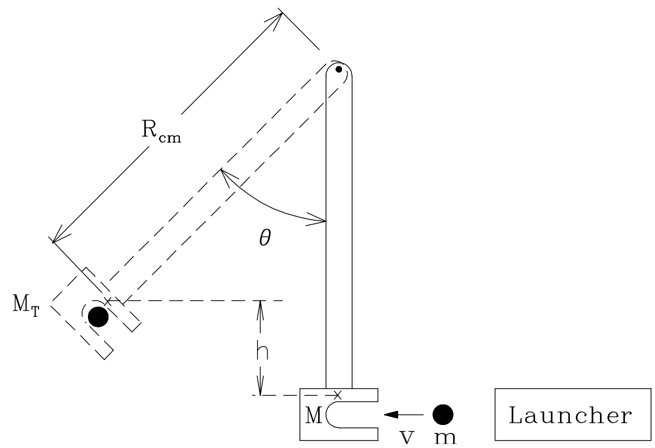

Figure 3.2: Ballistic Pendulum

In a certain class of collisions, the projectile is captured by the pendulum. For such *inelastic collisions*, we can consider the pendulum and projectile to be one single object after the collision, and this one combined object carries all of the kinetic energy K_{after} and momentum P_{after} after the collision. If the mass of the pendulum is M then the total mass of the combined object after the collision is $M_T = M + m$. If the pendulum is stationary when the projectile hits, the pendulum contributes nothing to the total kinetic energy and momentum of the system before the collision. Thus the principle of conservation of momentum in this case yields (where v_T is the speed of the combined object immediately after the collision):

$$\begin{aligned} P_{before} &= P_{after} \\ mv &= M_T v_T \\ mv &= (M+m)v_T \end{aligned} \tag{3.3}$$

Solving Equation 3.3 for v_T, we obtain

$$v_T = \frac{mv}{M_T} \tag{3.4}$$

Using the expression for the speed of the combined object immediately after the collision from the previous equation, we can show that the kinetic energy of the combined object immediately after the collision is *less* than the kinetic energy of the projectile just before the collision:

$$\begin{aligned} K_{after} &= \frac{1}{2}M_T v_T^2 \\ K_{after} &= \frac{1}{2}M_T\left(\frac{mv}{M_T}\right)^2 \\ K_{after} &= \frac{1}{2}M_T\left(\frac{m^2v^2}{M_T^2}\right) \\ K_{after} &= \frac{1}{2}\left(\frac{m^2v^2}{M_T}\right) \\ K_{after} &= \frac{m}{M_T}\left(\frac{1}{2}mv^2\right) \\ K_{after} &= \frac{m}{M_T}K_{before} \end{aligned}$$

Because the mass m of the projectile is less than the mass M_T of the combined object after the collision, it follows that

$$\frac{m}{M_T} < 1$$

and therefore

$$K_{after} < K_{before}$$

As the pendulum begins to swing after the collision, the kinetic energy of the combined object is gradually converted to gravitational potential energy as the pendulum rises. At the peak of its swing, the pendulum's mechanical energy is all in the form of gravitational potential energy, and its kinetic energy is zero. As the pendulum begins to move downwards after it reaches the peak of its motion, its gravitational potential energy is gradually converted to kinetic energy as the pendulum falls. At the lowest point of its motion, the pendulums's mechanical energy is all in the form of kinetic energy, and its graivational potential energy can be considered to be zero. Thus, we can write

$$E_{top} = M_T g h \tag{3.5}$$

and

$$E_{bottom} = \frac{1}{2} M_T v_T^2 \tag{3.6}$$

Assuming that mechanical energy is conserved during the initial swing of the pendulum, we can equate the expressions in Equations 3.5 and 3.6. While doing this, we can combine Equations 3.4 and 3.6 to eliminate v_T to obtain an expression that relates the initial speed v of the projectile to the final elevation h of the combined object, as follows.

$$\begin{aligned}
\frac{1}{2} M_T v_T^2 &= M_T g h \\
\frac{1}{2} M_T \left(\frac{mv}{M_T}\right)^2 &= M_T g h \\
\frac{1}{2} M_T \left(\frac{m^2 v^2}{M_T^2}\right) &= M_T g h \\
\frac{1}{2} \left(\frac{m^2 v^2}{M_T}\right) &= M_T g h \\
\left(\frac{m^2 v^2}{M_T}\right) &= 2 M_T g h \\
m^2 v^2 &= 2 M_T^2 g h \\
v^2 &= \left(\frac{M_T^2}{m^2}\right) 2gh \\
v &= \frac{M_T}{m} \sqrt{2gh} \qquad (3.7)
\end{aligned}$$

Equation 3.7 can be expressed in terms of the maximum angle of the pendulum's motion, as follows. The length R_{cm} describes the radius of the arc from the pivot point to the centre of mass of combined rod, block, and block contents. With the vertical orientation of R_{cm} as the base of a right-angled triangle, h can be expressed in terms of the maximum angle θ of the swing: $R_{\text{cm}} \cos\theta = (R_{\text{cm}} - h)$. Solving for h, we obtain

$$h = R_{\text{cm}} - R_{\text{cm}} \cos\theta = R_{\text{cm}} (1 - \cos\theta) \tag{3.8}$$

Inserting this expression for h into Equation 3.7, we obtain

$$v = \frac{M_T}{m}\sqrt{2gR_{cm}\left(1-\cos\theta\right)} \tag{3.9}$$

There is another energy conversion taking place, even before the collision. In the experiment, the launcher transfers some of its elastic potential energy into the projectile's initial kinetic energy. (Similarly, the force exerted by the spring on the projectile provides the projectile's initial momentum.) Applying the principle of conservation of energy to the transfer of energy from the spring to the projectile yields (where x is the maximum displacement of the spring from its equilibrium position and k is the stiffness constant of the spring)

$$\begin{aligned}
\text{elastic potential energy of spring} &= \text{initial kinetic energy of projectile} \\
\frac{1}{2}kx^2 &= \frac{1}{2}mv^2 \\
k &= \frac{mv^2}{x^2} \qquad (3.10)
\end{aligned}$$

Calculating v using Equation 3.7 and measuring the value of x allows us to use Equation 3.10 to determine the stiffness constant of the spring.

Note that there is an additional complication: The spring has a "pre-load" displacement. That is, when the spring is in its "relaxed" state, there is some compression in the spring. Think about how the previous analysis has to be modified to account for this pre-load compression.