(TA INITIALS)

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Experiment 2

Harmonic motion

When an object of mass m is attached to a hanging spring, the object experiences a gravitational force $F_g = mg$ and a force due to the additional stretch of the spring. The latter force is described by Hooke's law,

$$F_s = -ky\ , \tag{2.1}$$

where y is the magnitude of the additional stretch of the spring. nd k is called the stiffness constant of the spring (also known as the spring constant, or the spring's force constant). By drawing a free-body diagram for the hanging object, and doing a bit of algebraic manipulation, you can arrive at the following conclusions:

$$F_g = -F_s \quad \Rightarrow \quad mg = ky \quad \Rightarrow \quad m = \frac{k}{g}\ y. \tag{2.2}$$

If the mass is displaced from its new equilibrium position and released, it will begin to oscillate according to

$$y = A_0 \cos(\omega_0 t + \phi), \qquad \omega_0 = \sqrt{\frac{k}{m}} = 2\pi f_0 = \frac{2\pi}{T_0} \tag{2.3}$$

where A_0 and ϕ are the initial amplitude and phase angle of the oscillation, T_0 is the period in seconds, and ω_0 is the angular speed in radians/second. If a damping force F_d is present, the oscillation decays exponentially at a rate determined by the damping coefficient γ:

$$y = A_0 e^{(-\gamma t)} \cos(\omega_d t + \phi), \qquad \omega_d = \sqrt{\omega_0^2 - \gamma^2} \tag{2.4}$$

Procedure and analysis

The experimental setup consists of a vertical stand from which hangs a spring. A platform attached to the bottom of the spring accepts various mass loads. The spring-mass system is free to move in the vertical direction.

When the platform is static, its height above the table can be measured with a ruler.

When the system is in motion, Physicalab is used to record the time-dependent change in distance from a rangefinder to the bottom of the platform. The rangefinder measures distance by emitting an ultrasonic pulse and timing the delay for the echo to return.

Static determination of the stiffness constant k

In the following exercise you will determine the stiffness constant of a spring. **Note:** Use *positive k values* in all the following steps.

Equation 2.2 gives the relationship between the mass m and displacement y for a series of points (y, m). This is the equation of a straight line with slope $k/g = \Delta m/\Delta y$, so that $k = |\Delta m/\Delta y| * g$.

By varying m, measuring y_0, and fitting the resulting data to the equation of a straight line, the stiffness constant k for the spring can be determined. Note that the absolute distance y_0 does not matter, but the change in distance Δy with the change in mass Δm is critical.

- Suspend the spring from the holder and load it with the 50 g platform. Use a ruler to carefully measure the distance y from the top of the table to the bottom of the platform to a precision of 1 mm. Record your result in Table 2.1.

- Add masses to lower the platform until it is close to but is not touching the top of the table. Record this distance, then select several intermediate masses and fill in Table 2.1.

[?] Why do you want to determine a mass that brings the platform near to the table?

. . . .

- Enter the data pairs (y, m) in the Physicalab data window. Select **scatter plot** and click [**Draw**] to generate a graph of your data.

m (kg)							
y (m)							

Table 2.1: Data for determining the spring force constant k

[?] Did you choose to include the mass of the platform (50 g) as part of the stretching mass m in Table 2.1? How would the graph and the resulting value for the stiffness constant k be affected by your choice?

. . . .

- Select **fit to: y=** and enter **A+B*x** in the fitting equation box. Click [**Draw**] to perform a linear fit on your data, then evaluate k and the associated error $\delta(k)$. Click [**Send to:**] to email yourself a copy of the graph for later inclusion in your lab report.

B = ... ± . . . . $A =$ . . . . ± . . . .

k = ... = ... = ...

δk = ... = ... = ...

$k =$ . . . . ± . . . .

Damped harmonic oscillator

You will now explore the behaviour in time of an oscillating mass with the aid of a computer-controlled rangefinder. This device sends out an ultrasonic pulse that reflects from an object in the path of the conical beam and returns to the rangefinder as an echo. The rangefinder measures the elapsed time to determine the distance to the object, using the speed of sound at sea level as reference.

By accumulating a series of coordinate points (ω, m) and fitting your data to Equation 2.4, you can use several methods to determine the spring constant for the oscillating mass system.

- With each of the masses used previously with this spring, raise the platform a small distance, then release it to start the mass oscillating vertically. Wait until the spring/mass system no longer exhibits any erratic oscillations.

- Login to the Physicalab software. Check the **Dig1** box and choose to collect **200** points at **0.05** s/point. Click **Get data** to acquire a data set.

- Select **scatter plot**, then Click **Draw**. Your points should display a smooth slowly decaying sine wave, without sharp peaks, stray points, or flat spots. If any of these are noted, adjust the position of the rangefinder and acquire a new data set.

- Select **fit to: y=** and enter **A*cos(B*x+C)*exp(-D*x)+E** in the fitting equation box. Click **Draw**. If you get an error message the initial guesses for the fitting parameters may be too distant from the required values for the fitting program to properly converge.

  The fitting equation is equivalent to Equation 2.4. Look at your graph and enter some reasonable initial values for the amplitude **A** of the wave and the average (equilibrium) distance **E** of the wave from the detector. **C** corresponds to the initial phase angle of the sine wave when $\mathbf{x} = 0$.

  The angular speed **B** (in radians/s) is given by $\mathbf{B} = 2\pi/T$. Estimate the time x $= T$ between two adjacent minima of the sine wave then estimate and enter an initial guess for **B**.

  The damping coefficient **D** determines the exponential decay rate of the wave amplitude to the equilibrium distance **E**. When $\mathbf{D} = 1/\mathbf{x}$, the envelope will have decreased from $\mathbf{A}_0$ to $\mathbf{A}_0 e^{-1} = \mathbf{A}_0/e = \mathbf{A}_0/2.718 \approx \mathbf{A}_0/3$.

  Make an initial guess for **D** by estimating the time $t = \mathbf{x}$ required for the envelope to decrease by a factor of 2/3.

- Check that the fitted waveform overlaps your data points well, then label the axes and include as part of the title the value of mass m used. Email yourself a copy of all graphs as part of your lab data set.

- Record the results of the fit in Table 2.2, then complete the table as before.

m (kg)							
y_0 (m)							
ω_d (rad/s)							
γ (s^{-1})							

Table 2.2: Experimental results for damped harmonic oscillator

- The stiffness constant k can be determined as before from the slope of a line fitted through the (y_0, m) data in Table 2.2. Generate and save the graph, then record the results below:

B = ... ± .... $A =$ .... ± ....

k = ... = ... = ...

δk = ... = ... = ...

$k(y_0) =$ .... ± ....

The stiffness constant k can also be determined from the period of oscillation of the mass. Rearranging the terms in Equation 2.3 yields $m = k/\omega_d^2$.

- Enter the coordinate pairs from Table 2.2 in the form (ω_d, m) and view the scatter plot of your data.
- Select **fit to: y=** and enter **A+B/x**2** in the fitting equation box. This is more convenient than squaring all the ω_0 values and fitting to **A+B/x**.
- Click **Draw** to perform a quadratic fit on your data. Print the graph, then enter the value for the stiffness constant below:

$k(\omega_d) =$ .... ± ....

The damped harmonic oscillator equation (Equation 2.4) predicts that the frequency of oscillation depends on the damping coefficient γ so that your experiment actually yields values for ω_d rather than ω_0. Calculate ω_0 from ω_d using an average value for γ.

? Which ω_d value should be used? Why?

....

$\omega_0 =$ .... = ... = ...

? Is the difference between ω_d and ω_0 significant? Explain.

....

Calibration of rangefinder

It was assumed that the rangefinder is properly calibrated and measuring the correct distance. It is always a good practice to check the calibration of instrument scales against a known good reference, in this case the ruler scale. Calibration errors are systematic errors and can be corrected. Once the amount of mis-calibration is determined, the mis-calibrated data can be adjusted to represent correct values.

- Check the calibration of your rangefinder against the ruler. Compare the slope values from the (y_0, m) data. If the rangefinder is calibrated, the slope results should agree within experimental error.

`ruler :`	`slope` =	....	±	y`-int =`	± ....
`rangefinder :`	`slope` =	....	±	y`-int =`	± ....

? Do your two results for the slopes **B** agree? Explain.

....

? Should the values of the y-intercepts **A** be the same? Explain.

....

! Important! Be sure to have this printout signed and dated by a TA before you leave at the end of the lab session. All your work needs to be kept for review by the instructor, if so requested.

Lab report

Go to the "Lab Documents" web page to access the online lab report template for this experiment. Complete the template as instructed and submit it to Turnitin before the lab report submission deadline, late in the evening six days following your scheduled lab session. Do not wait until the last minute. Turnitin will not accept overdue submissions. Unsubmitted lab reports are assigned a grade of zero.