PhysicaLab

PhysicaLab is a computer-based graphing software used for data fitting and analysis. This program will be accessible through every computer in the lab room.

At the start of every lab session, click on the desktop icon to open a new Physicalab application, then enter one or two Brock email addresses for the member(s) of the group. Without valid emails, you will not be able to send yourself the graphs made during the experiment for inclusion in your lab report. At the end of the lab session, be sure to close the Physicalab application, otherwise your email address will be accessible to the next group using the workstation.

There is also a replica copy of PhysicaLab online on the Brock Physics Website called Physica Online. To access this program, go to the Brock Physics website, click courses on the left hand side and then click Physics Online or go to: http://landau.physics.brocku.ca/physica/

How To Use PhysicaLab

Inserting Data

In the bottom left portion of the screen is a text box. This is where you input your data points that will be plotted on the graph. In this text box, each horizontal line of text represents one data point. Each line can contain up to four numbers, each separated by a space, that make up one point. The first two numbers are your x and y values respectively. The next two numbers are the dy and dx values. These represent the possible error range for each data point. dy produces a vertical error while dx produces a horizontal error. If there is no error in your data, you can input 0's or leave those sections blank.

Here is an example of how the text box data should look like:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>dy</th>
<th>dy</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>36</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1: Example Data Set

Once you have data inserted into the text box, click Draw to generate a graph of your data.

Important! Whenever a change is made somewhere in the PhysicaLab program, remember to click Draw to generate an updated version of your graph.
Fitting the Data

To the right of the text box is where all of the fitting parameters are. Here you can choose which style of graph you want, what equation you want to fit your data to and what labels you want to use for your titles.

Throughout the various experiments that will be performed in this course, many sets of data will be collected. To best view the data before fitting it to an equation, the scatter plot feature is the best option. To fit the data points to a certain equation, you must first select the fit to $y =$ button and then type in the necessary equation. When typing in the equations, (*) represents multiplication, ($\wedge$) represents to the power, and $A$, $B$, $C$ and $D$ represent constants. So to properly type $y = x^2$ into PhysicaLab, you must type $A*x^2 + B*x + C$ into the fit to text box. The meaning of the four constants will change depending on the equation used in the fit. Example, for $y = A*x + B$, the $A$ is the slope, $B$ is the y-intercept and $C$ and $D$ are irrelevant.

⚠️ Important! Sometimes fitting your data to a certain curve will produce an error and the graph will not display. This is due to the initial guesses of the four constants. Adjusting these numbers to their approximate values will fix this problem.

To include labels on your graph, locate the X-Axis, Y-Axis and Title Label portion of PhysicaLab at the very bottom. Here you can check off which labels you wish to include.

⚠️ Important! Don’t forget to include your units in the X and Y labels!

Reading the Graph

Once data is successfully plotted and fit to an equation, blue text underneath the graph will appear which will give you different information about certain aspects. The four constants, $A$, $B$, $C$ and $D$, will each be displayed equal to some value.

NOTE: If a constant isn’t used in your fit equation, the value of that constant will be equal to one. These values are the reason you plot your data. Depending on the experiment the main two numbers you will most likely be looking for are your slope value and your y intercept.

Problem

The small angle approximation states that for small angles, $\sin(\theta)$ is approximately equal to $\theta$. To prove this complete the following table calculations:

<table>
<thead>
<tr>
<th>$\theta$ (in radians)</th>
<th>$\sin(\theta)$ (in radians)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\theta$ (in radians)</th>
<th>$\sin(\theta)$ (in radians)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td></td>
</tr>
</tbody>
</table>

Plot the above data into PhysicaLab Online as scatter points with an additional point at $(1, 1)$ and print off the graph. Remember to label your axes and give an appropriate title. Once the graph is printed, draw a straight line, using a ruler, from $(0, 0)$ to $(1, 1)$. This line represents if $\sin(\theta)$ was exactly equal to $\theta$. From the information on the graph, describe in a few sentences how accurate the small angle approximation is and what angles you consider to be classified as small angles.
Error Propagation

As mentioned in the measurement uncertainty section, no measurement is exact due to all of the possible errors. Error Propagation is the process of calculating these errors for an answer. The error propagation is considered the uncertainty range of an answer. This range is defined as all of the possible values the answer could be. To help clarify what this means, let's look at an example:

Example

A Brock Physics student does an experiment to find the force of gravity on a 5kg ball. His final answer, after performing error propagation on it, states that:

\[ F_g = 49N \pm 1N \]

This example answer simply states that the force of gravity is equal to 49N, plus or minus 1N. Therefore, the answer can be anywhere in the range from 48N to 50N. (49 - 1 = 48 and 49 + 1 = 50) This range of possible values for the answer is the error propagation and is simply the value that comes after the ±.

How To Calculate Error Propagation

Depending on the type of operations (addition, subtraction, multiplication, etc...) that are computed in the equation you are applying error propagation to, there are different rules that need to be followed. For an advanced list of all of these rules, see Appendix B at the end of the lab manual. The process for the basic operations can be found below.

In the following calculations, the notation of \( \Delta x \), where x is just a variable, defines the error of x. The error of x can vary depending on what x is. In most cases though, if the error value isn’t given to you directly, the error value is equal to half of the smallest resolution of the measuring tools. An example would be a ruler that can evaluate up to millimeter measurements. The error of anything measured by that ruler would equal 0.5mm or 0.0005m. Another example would be a scale that can weigh an object to the nearest tenth of a gram. The error of anything weighed by that scale would be equal to 0.05g or 0.00005kg.

Addition and Subtraction

To compute the error propagation for addition or subtraction, use the following rule:

\[ y = Ax \pm By \pm z \]

where A and B are coefficients and x, y, and z are variables becomes:

\[ \Delta y = \sqrt{(A\Delta x)^2 + (B\Delta y)^2 + (\Delta z)^2} \]

When computing addition and subtraction, if the constant values don’t have any error associated with them, for example gravity which equals exactly 9.8 m/s\(^2\), they are still needed in the error propagation equation.
Multiplication and Division

When computing multiplication and division, the constants factor out of relative errors and are not needed in the error propagation equation.

To compute the error propagation for multiplication and division, use the following rule:

\[ y = \frac{Ax \times By}{Cz \times w} \]

where A, B and C are constants and w, x, y, and z are variables becomes:

\[ \frac{\Delta y}{y} = \sqrt{\left( \frac{\Delta x}{x} \right)^2 + \left( \frac{\Delta y}{y} \right)^2 + \left( \frac{\Delta z}{z} \right)^2 + \left( \frac{\Delta w}{w} \right)^2} \]

Lab0, a second version

Help Desk, etc. goes here:

Data graphing and fitting using Physicalab

At the start of every lab session, click on the desktop icon to open a new Physicalab application, then enter your Brock email address. Without a valid email address, you will not be able to send yourself the graphs made during the experiment for inclusion in your lab report.

At the end of the lab session, be sure to close the Physicalab application, otherwise your email address will be accessible to the next person using the work station.

Graphing and fitting to a linear data set

A linear data set consists of a series of coordinate points \((x, y)\) where the relationship between the \(x\) and \(y\) coordinates is always such that a change \(\delta y\) in \(y\) is directly proportional to a change \(\delta x\) in the variable \(x\).

This relationship can be represented by a function \(y = m \cdot x + b\). This is the equation of a straight line with constant slope \(m = \delta y / \delta x\). The \(y\)-intercept \(b\) is the value of \(y\) when \(x = 0\).

- In Physicalab, click [File], then [Load linear data] to place some points in the data window.
- Click [Draw] to generate a graph of the data. You will note that the points appear to follow a linear behaviour; they appear to lie on an invisible straight line. This observation gives you the hint that if you were going to try and fit some function to this data set, then the equation for a straight line might be a good choice.
- Below the graph, select [fit to: \(y = \) ] and enter \(A \cdot x + B\) in the fitting equation box. Here, \(A\) represents the value of the slope and \(B\) is the value of the \(y\)-intercept.

The fitting algorithm calculates the best possible values for these fitting parameters, and uses them to determine a line of best fit that is drawn over your data points. The values of the fitting parameters appear below the graph.

Note that the points do not lie exactly on the line. This mismatch between the points and the line fitted to them is calculated by the fitting program and displayed as a number that represents the uncertainty \(\delta\) in the value of a given fit parameter.

Then the uncertainty, or error, in \(A\) is \(\delta A\) and the error in \(B\) is \(\delta B\). These results are typically displayed in the form \(A \pm \delta A\), \(B \pm \delta B\). The smaller the mismatch, the better the fit of the equation to the data set; if all the points were to lie exactly on the line, then these uncertainties would be zero.

Graphing and fitting to a sine data set

A data set that is periodic (where the \(y\) values seem to repeat after a given interval in \(x\)) can generally be fitted to a sine function, or sine wave, of the form \(y = \sin(x)\).

- Click [File], then [Load sinusoidal data]. This is some sample data obtained from the Pendulum experiment that you are going to perform later in the course.
- Click [Draw] to generate a graph of the data. Always check that the data points follow the basic shape of the function that you are going to fit to the data. In this case, you would like to see a nice smooth sine wave, without spikes, stray points or flat spots.
• Select fit to: \( y= \) and enter \( A*\sin(Bx+C)+D \) in the fitting equation box.

As reviewed in the Appendix, \( x \) represents the independent variable (here in units of time), \( A \) is the amplitude of the sine wave, \( C \) is the initial phase angle (in radians) of the wave when \( x = 0 \), and \( D \) is the average distance of the pendulum from the detector; i.e. \( D \) is the distance from the pendulum to the detector when the pendulum is vertical or motionless.

The fit parameter \( B \) (in radians/s) is the rate of change in angle with time, so that \( Bx \) is an angle in radians. If this angle is advanced by \( 2\pi \) radians, then the time changes by an amount \( x = T \), where \( T \) represents the period of the sine wave in seconds. Then \( B=2\pi/T \).

• Click Draw. If you get an error message the initial guesses for the fitting parameters may be too distant from the required values for the fitting program to properly converge.

Look at your graph and enter some reasonable approximate values for the fitting parameters. You can get an initial guess for \( B \) by estimating the time \( x \) between two adjacent minima, or one period, of the sine wave.

Introduction to uncertainty (error) analysis

Beware! In general conversation, the term error usually refers to some sort of mistake.

In the maths and sciences the term error refers specifically to the uncertainty \( \delta X \) in the magnitude of a given result \( X \). The result \( X \) without an associated error \( \delta X \) is meaningless, as there is no way to establish how reliable the value of \( X \) actually is.

A proper result is expressed as a pair of numbers \( X \pm \delta X \).

A sample data set

Table 3 contains a sample data set obtained from the Pendulum experiment. In the experiment, the length \( s \) of a string supporting a ball of diameter \( d \) that acts as the pendulum bob is adjusted and the length \( s \) is measured and recorded. The pendulum length \( L \) is the string length plus the radius, or \( 1/2 \) the diameter of the ball: \( L = s + 0.5d \).

Then the pendulum ball is set swinging and the motion is recorded and fitted to a sine function. The fit parameter \( B \) is obtained from the fit and the acceleration due to gravity \( g \) is obtained from \( g = B^2L \).

There were five trials done using a ball of mass \( m_1 \) and a single trial using a ball of mass \( m_2 \). The balls have a different diameter \( d \). We will use this data to determine by different methods values for \( g \) and their associated errors \( \delta g \).

<table>
<thead>
<tr>
<th>Run, i</th>
<th>mass</th>
<th>( m ) (kg)</th>
<th>( d ) (m)</th>
<th>( s ) (m)</th>
<th>( L ) (m)</th>
<th>( B ) (rad/s)</th>
<th>( g _i ) (m/s(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( m_1 )</td>
<td>0.0225</td>
<td>0.02540</td>
<td>0.300</td>
<td>0.3127</td>
<td>5.59641±0.00267</td>
<td>9.79370</td>
</tr>
<tr>
<td>2</td>
<td>( m_1 )</td>
<td>0.0225</td>
<td>0.02540</td>
<td>0.450</td>
<td>0.4627</td>
<td>4.60396±0.00224</td>
<td>9.80760</td>
</tr>
<tr>
<td>3</td>
<td>( m_1 )</td>
<td>0.0225</td>
<td>0.02540</td>
<td>0.600</td>
<td>0.6127</td>
<td>4.00688±0.00163</td>
<td>9.83695</td>
</tr>
<tr>
<td>4</td>
<td>( m_1 )</td>
<td>0.0225</td>
<td>0.02540</td>
<td>0.750</td>
<td>0.7627</td>
<td>3.58703±0.00182</td>
<td>9.81350</td>
</tr>
<tr>
<td>5</td>
<td>( m_1 )</td>
<td>0.0225</td>
<td>0.02540</td>
<td>0.900</td>
<td>0.9127</td>
<td>3.27880±0.00229</td>
<td>9.81201</td>
</tr>
<tr>
<td>1</td>
<td>( m_2 )</td>
<td>0.0095</td>
<td>0.01904</td>
<td>0.500</td>
<td>0.5095</td>
<td>4.38192±0.00240</td>
<td>9.78344</td>
</tr>
</tbody>
</table>

Table 3: Table of experimental results
Determining $g$ from a single measurement

Because there was only one trial using the ball of mass $m_2$, the only option is to use error propagation rules to determine error estimates for $L$ and $g$.

First, we need to determine the magnitude of the uncertainties in the measured values of the string length $s$ and the diameter of the ball $d$.

The measurement errors in $s$ and $d$, represented by $\delta s$ and $\delta d$, are determined from the scales of the measuring instruments. This error is expressed as ± one-half of the smallest increment, or resolution, of the scale used to make the measurement.

The scale used to set the string length $s$ had a resolution of 0.001 m, while the micrometer used to measure the ball diameter $d$ had a scale increment, of 0.00001 m. The errors are:

$$\delta s = \pm \ldots$$  \hspace{1cm} $$\delta d = \pm \ldots$$

A proper way to show a calculation is in three steps: first display the relevant equation, then replace the variables by their unrounded values and finally show the numerical result. Do not include units at this step.

The error equation for $L$ is obtained from the error rules in the Appendix. Then for $m_1$,

$$L = s + 0.5d = \ldots = \ldots$$

$$\delta L = \delta s + 0.5\delta d = \ldots = \ldots$$

Once these two results are obtained, the final result for $L \pm \delta L$, properly rounded and with the correct units is shown as follows:

$$L = \ldots \pm \ldots$$

To get a final result for $g$ using the ball of mass $m_2$, the error equation is needed. This time it is a good idea to perform a change of variables to determine $\delta g$. Note that the error equation is a product of two terms, $B^2$ and $L$. Let $A = B^2$ then $g = AL$. Using the power rule and the product rule respectively:

$$F = X^2 \rightarrow \frac{\delta F}{F} = 2\frac{\delta X}{X}, \hspace{1cm} F = XY \rightarrow \frac{\delta F}{F} = \sqrt{\left(\frac{\delta X}{X} + \frac{\delta Y}{Y}\right)}$$

(2)

Replacing the variables and substituting the $\frac{\delta A}{A}$ term in the $g$ error equation:

$$A = B^2 \rightarrow \frac{\delta A}{A} = 2\frac{\delta B}{B}, \hspace{1cm} g = AL \rightarrow \frac{\delta g}{g} = \sqrt{\left(\frac{\delta A}{A} + \frac{\delta L}{L}\right)} = \sqrt{\left(2\frac{\delta B}{B} + \frac{\delta L}{L}\right)}$$

(3)

$$g = B^2L = \ldots = \ldots$$

$$\delta g = g\sqrt{\left(2\frac{\delta B}{B}\right)^2 + \left(\frac{\delta L}{L}\right)^2} = \ldots = \ldots$$

$$g = \ldots \pm \ldots$$
Determining $g$ from a series of measurements

There are five trials ($i = 1 \ldots N = 5$) using the large ball of mass $m_1$. These five results for $g$ are expected to have the same value. In this case, you can invoke the theory of statistics to evaluate a sample average $\langle g \rangle$, or mean value, of the five trials as well as the standard deviation of the sample $\sigma(g)$ that gives a measure of the scattering of the trials around $\langle g \rangle$.

The sample average of $N$ values $g_i$ is given by the sum of the samples divided by the number of samples:

$$\langle g \rangle = \frac{1}{N} \sum_{i=1}^{N} g_i = \frac{(g_1 + g_2 + \ldots + g_N)}{N} \quad (4)$$

To get a feel for what is involved, let’s perform a manual standard deviation calculation.

- To begin, we use Equation 4 to calculate $\langle g \rangle$.
- Then for each $g_i$ we calculate the difference $\delta g_i$ and $(\delta g_i)^2$.
- Finally, we determine $\sigma(g)$ from the variance in the sample of $g$ values:

<table>
<thead>
<tr>
<th>$i$</th>
<th>$g_i$</th>
<th>$\delta g_i = g_i - \langle g \rangle$</th>
<th>$(\delta g_i)^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$$\langle g \rangle = \text{variance} = \sqrt{\text{variance} = \sigma(g) = \frac{1}{N-1} \sum_{i=1}^{N} (\delta g_i)^2}$$

Table 4: Calculation template for $g$ and $\sigma(g)$. Variance $= \sigma(g)^2 = \frac{1}{N-1} \sum_{i=1}^{N} (\delta g_i)^2$

$$g = \ldots \ldots \pm \ldots \ldots$$

- You can also enter your five $g$ values in Physicalab, then from the Edit menu select Insert X Index to add a column of index values. Check bellcurve to view your data as a distribution.

Compare the mean and standard deviation values from the graph with your results from above. They should be the same.

To view a more representative distribution with a larger number of samples, click File and Load distribution data. See how the graph changes as you vary the number of bins that the data is partitioned into. Checking the bargraph box will display your data as a frequency bar graph.