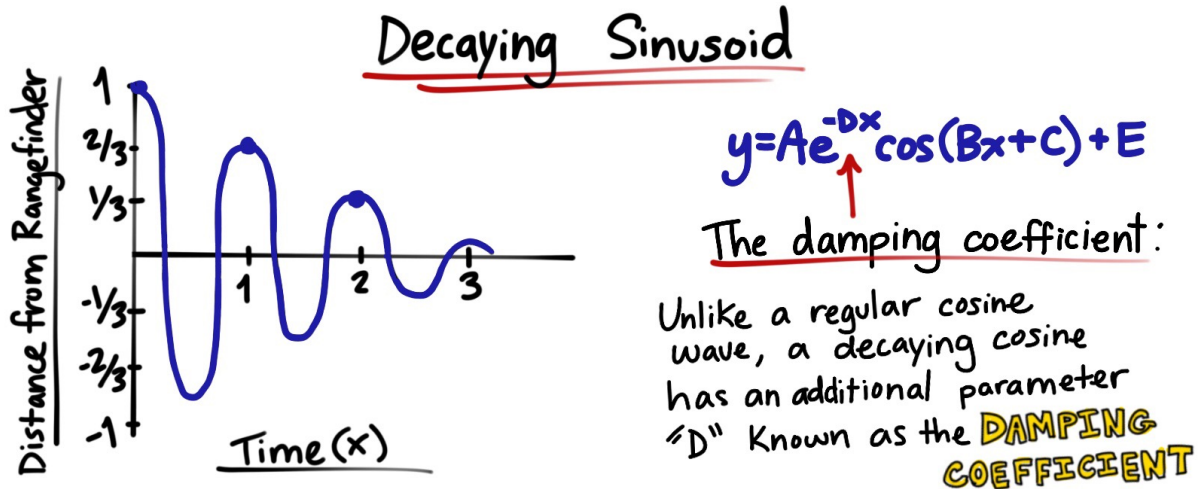


Tips for Fitting a Decaying Sinusoid

$y = A_0 e^{(-\gamma t)} \cos(\omega_d t + \phi)$ ← Formula in lab manual

$y = A \cos(Bx + C) e^{Dx} + E$ ← Formula in Physicalab

When fitting a decaying sinusoid we need to account for the damping coefficient: D.

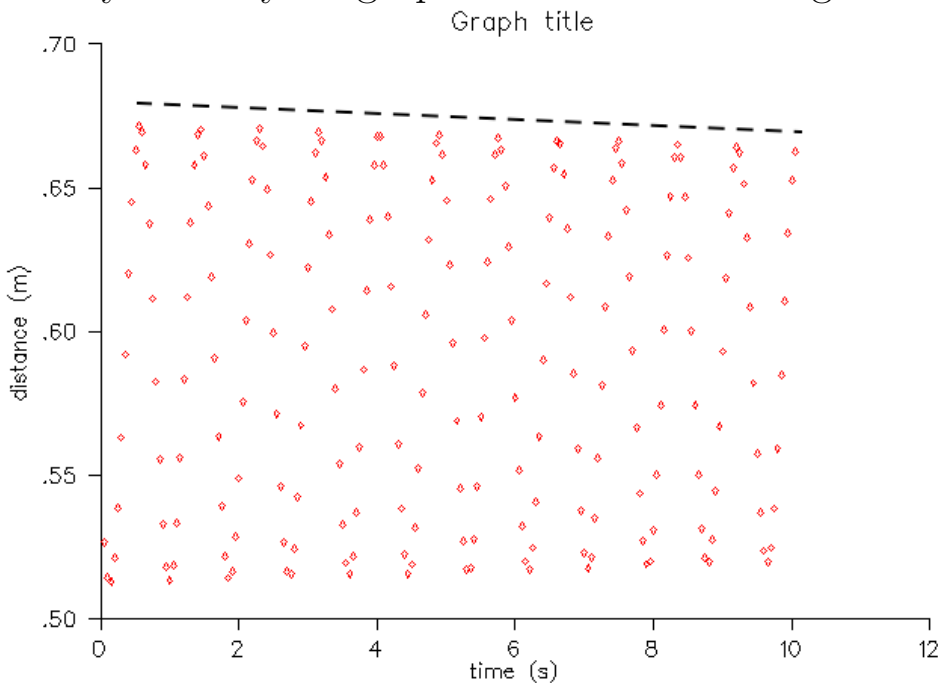


Estimating "D": If $D = 1/x$, Ae^{-Dx} becomes $Ae^{-\frac{1}{x}x} \rightarrow Ae^{-1}$ or $\frac{A}{e}$
 $\frac{A}{e} = \frac{A}{2.718...} \approx \frac{A}{3}$ So, when $D = 1/x$ the amplitude is approx. one third of its initial value.

In the case of this graph, the initial amplitude decreases by $2/3$ after 2 sec. Therefore $D = 1/2$

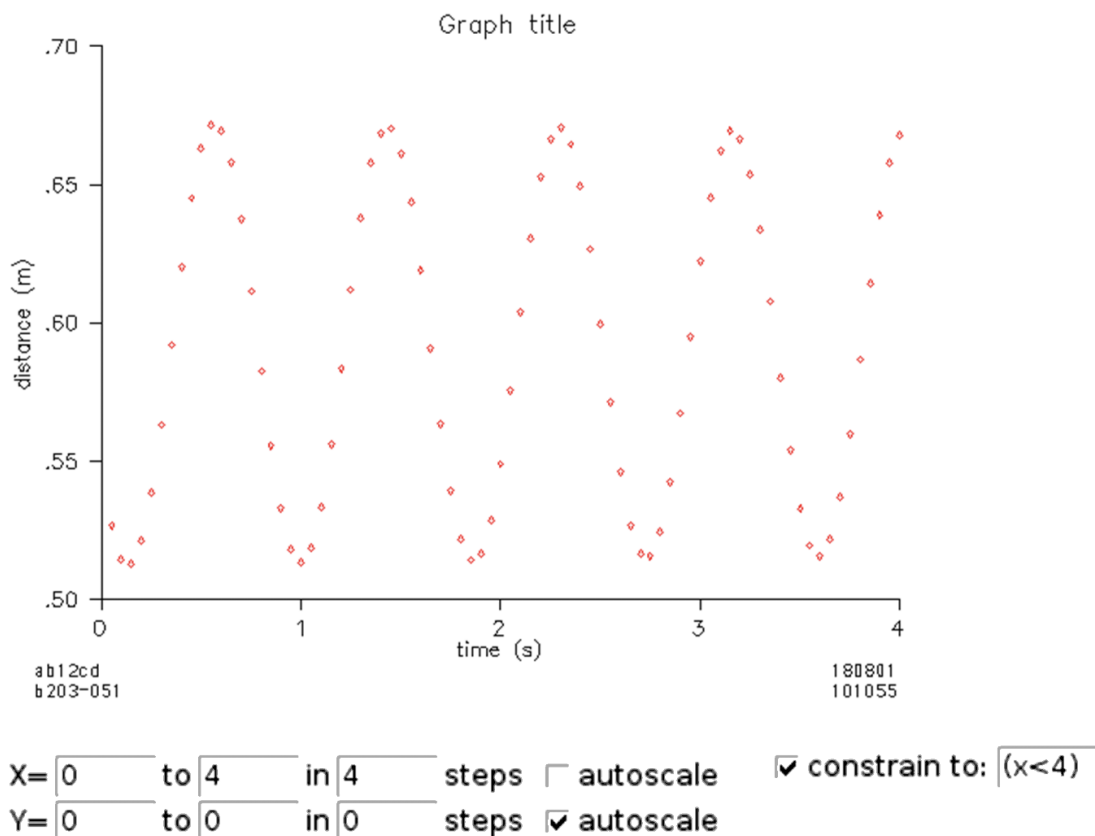
Solving the other parameters:

1. In Physicalab your graph will look something like this:

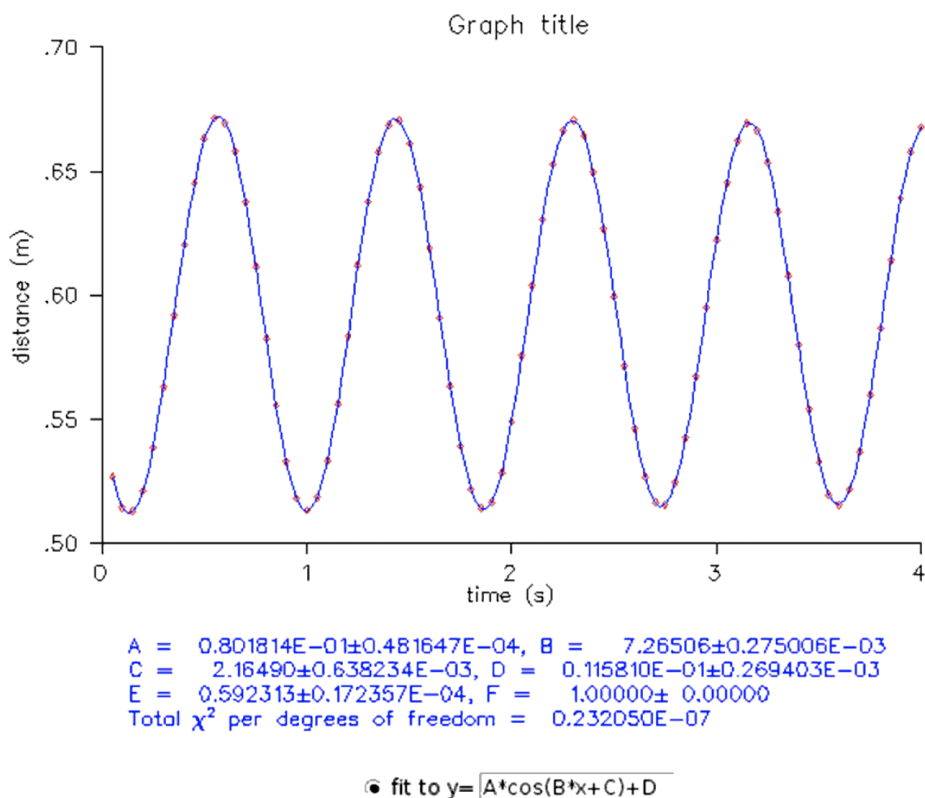


You can see the amplitude slowly decreases over time. However the first few cycles look very similar to an undamped sinusoidal function.

2. Change the limits on the x-axis to view the first few cycles. With a constraint, the fitting routine will use only the specified range of data, in this case the data from 0 to 4 seconds.

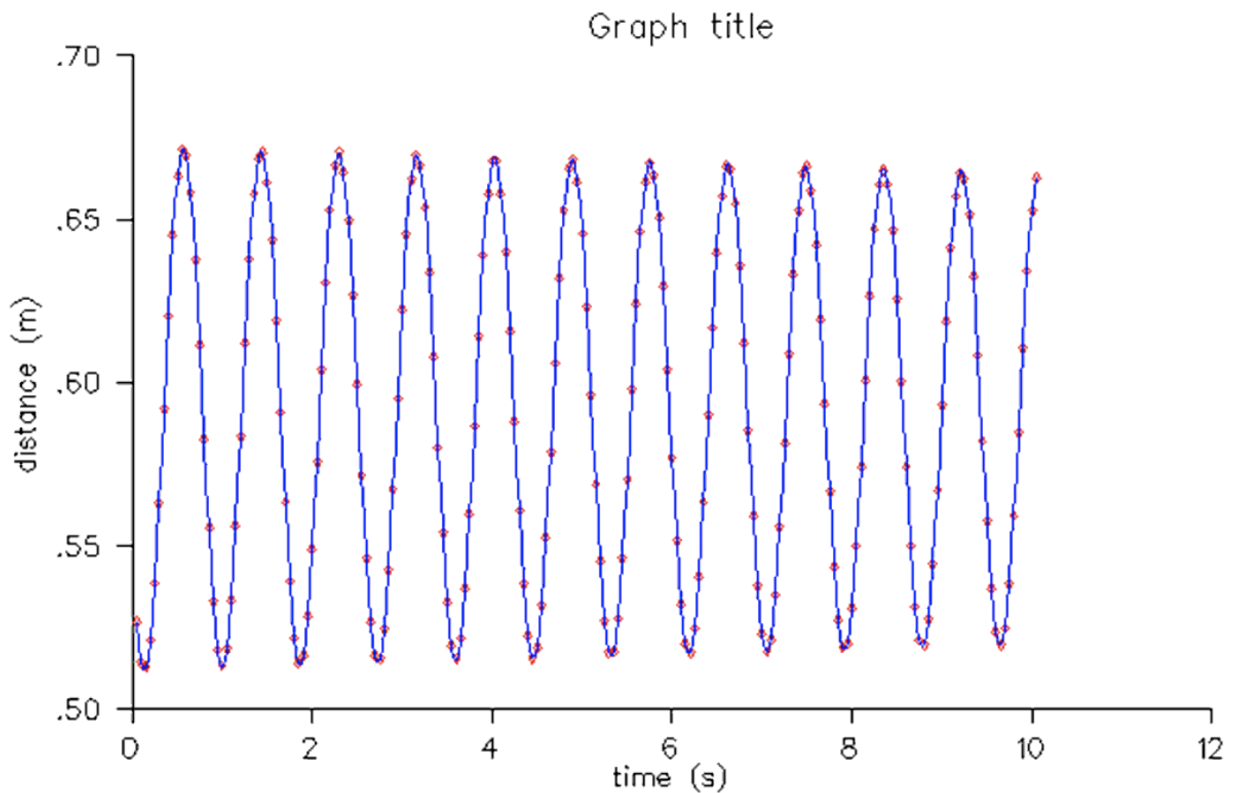


3. Try fitting a cosine wave $y = A \cos(Bx + C) + D$ to this data by estimating the amplitude A, initial phase angle C, the angular velocity B from the wave period $T=2\pi/B$ and the average distance of the wave from the x-axis D.



4. Select auto scale and remove any constraints. Select fit to $y = A \cos(Bx + C)e^{Dx} + E$. Note that now the average distance of the wave from the x-axis is given by E, not D.

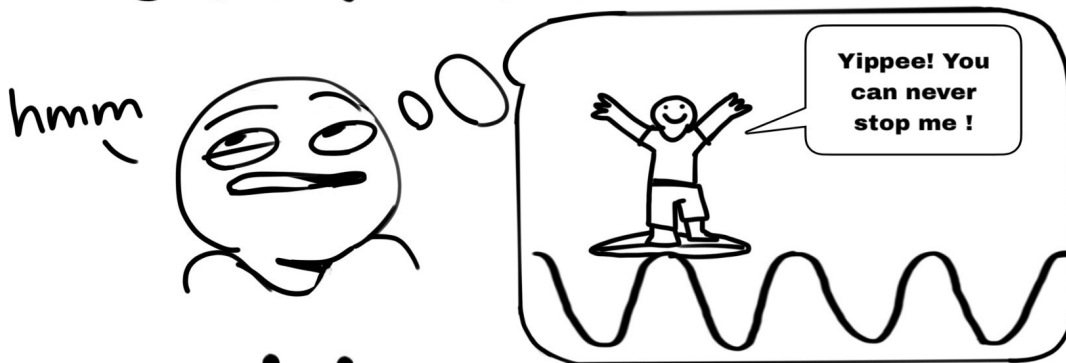
Enter your estimates for A, B, C and E from the previous fit, then estimate the value of the damping coefficient D as described above until you get a good fit of your decaying sinusoid data.



A = 0.800173E-01±0.572884E-04, B = 7.26530±0.129381E-03
 C = 2.16442±0.736910E-03, D = 0.103204E-01±0.126756E-03
 E = 0.592175±0.199570E-04, F = 1.00000±0.00000
 Total χ^2 per degrees of freedom = 0.798960E-07

fit to $y = A \cdot \cos(B \cdot x + C) \cdot \exp(-D \cdot x) + E$ constrain to: $(x < 4)$
 initial guesses: A = 0.8 B = 7.3 C = 2.16 D = 0.1 E = 1 F = 1
 X = 0 to 4 in 4 steps autoscale X grid X log X label time (s)

Idealized



Reality

