

Experiment 3

The ballistic pendulum

Procedure

The launcher component of the ballistic pendulum consists of a sliding metal rod surrounded by a precision spring. When the lever that is connected to the sliding rod is pulled to the left, the spring is compressed and the rod extends out of the left side of the launcher. This extension can be measured to determine the distance that the spring was compressed.

There are four slots on the side of the launcher. The lever is moved from its relaxed position at the rightmost slot of the launcher and is then lowered into one of the other three slots to lock the spring at one of three compression settings that we will name, from right to left: *short*, *medium*, or *long* range.

A cylindrical barrel on the right side of the launcher holds the projectile, a steel ball. As the lever is *slowly* raised in the slot, the launcher suddenly discharges with the spring extending and pushing the rod to strike the ball. The ball exits the launcher and impacts the pendulum.

The pendulum consists of a rod and bob of combined mass M attached to a pivot point. When an impact takes place, the pendulum catches the impacting mass m , changing the total mass of the pendulum to $M_T = M + m$, and swings about the pivot point to a maximum angle of deviation θ , relative to the initial vertical position of $\theta = 0^\circ$.

The pendulum drags with it a pointer that stops at the limit of the swing and identifies the value of θ on a degree scale concentric with the pivot. The pendulum then free falls back to the vertical position to stop against the barrel. A small amount of friction between the pointer and the scale prevents the pointer from falling back along with the pendulum. The effect of this friction and the mass of the pointer on the system is negligible.

The pointer, initially at rest, is accelerated along with the the pendulum arm on impact. Could it keep moving past the limit of the pendulum arm, after the arm has stopped, and thus give inaccurate angle readings?

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Note: When the launcher is in the discharged position, the spring is not fully extended but is subjected to a compression preload x_0 and is thus storing some potential energy. You will determine this preload at the end of the experiment.

Caution: Do not place ball in the launcher until the lever is properly lowered and secure in a slot, otherwise an unintended spring discharge could occur, ejecting the ball and possibly causing an injury.

Data gathering and analysis

To determine the physical characteristics of the ballistic pendulum apparatus:

- remove the pendulum arm from the ballistic pendulum assembly by unscrewing the pivot screw.
Replace the screw for safekeeping;
- measure with a digital scale ($\sigma = \pm 0.01$ g) the mass of the ball m and ball/pendulum assembly M_T ;

$$m = \dots \pm \dots \text{ kg} \quad M_T = \dots \pm \dots \text{ kg}$$

- determine the centre of mass point of the pendulum/ball combination by balancing it on the edge of the steel ruler and noting the position of the balance point on the scale located on the pendulum arm, then measure with the ruler the distance from this point to the centre of the pendulum pivot (see schematic Figure 4.2).
- The centre-of-mass distance R_{cm} , from the balance point to the centre of the pivot hole on the arm of the pendulum of mass M_T , is

$$R_{cm} = \dots \pm \dots \text{ m.}$$

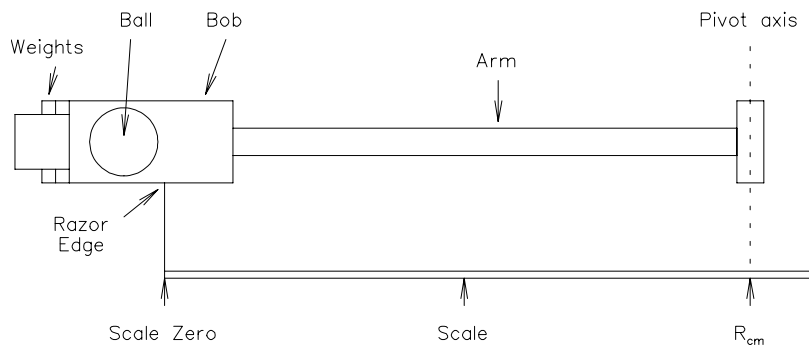


Figure 3.1: Generic diagram for determining a pendulum centre-of-mass

- With the launcher discharged, measure the distance in mm that the rod extends from the left end of the launcher. You will subtract this rod offset from all of the following measurements. The offset is

$$\text{offset} = \dots \pm \dots \text{ m.}$$

- Compress and lock the spring at the *short* range setting. Measure the distance that the rod now extends from the end of the launcher. Subtract from this length the rod offset and record the result below and as x in Table 3.2:

$$x_{short} = \dots \pm \dots \text{ m.}$$

- Repeat the above step for the *medium* and *long* range settings:

$$x_{medium} = \dots \pm \dots \text{m} \quad x_{long} = \dots \pm \dots \text{m}.$$

- Finally, replace the pendulum arm, making sure that the pivot screw is tight.

Before you begin to gather ballistic data remember to avoid sitting directly in front of the discharge path of the launcher barrel and

CAUTION: Always wear safety glasses while using the launcher.

- Move the lever from the discharged position to the first slot and push it down to lock the spring at the *short* range setting. Further compression selects the *medium* range and finally, the *long* range setting.
- Load the launcher by raising the pendulum and placing the ball at the end of the barrel, then lower the pendulum to freely hang in the vertical position.
- Hold the pendulum in the vertical position and move the angle indicator to the 0° mark. If the indicator does not reach zero, you will need to subtract this difference from all your angle readings.
- To fire the launcher, gently raise the lever from the retaining slot to release the spring.
- ⓘ **Note:** if the ball falls out of the pendulum when launched, the assembly is not level or the pendulum bob position needs adjustment. Make small adjustments to the base leveling screws to centre the pendulum with the launcher barrel. If the problem persists, see a TA for assistance.
- ⓘ **Note:** be sure that the angle indicator does not move when the pendulum falls back and strikes the launcher barrel, otherwise you will get an incorrect angle reading. If this happens, gently stop the falling pendulum with your finger before it strikes the launcher.
- Perform five *short* range launches, recording the angle θ_i reached in trial $i = 1 \dots 5$ in the appropriate spaces of Table 3.1.
- Perform five launches using the *medium* and then the *long* range settings.

Because the five θ_i values at a given range setting *are expected to be the same*, you can perform a statistical analysis to get an average value $\langle \theta \rangle$ and the standard deviation $\sigma\theta$ for each of the three settings.

- Enter the five θ values as a column in Physicalab, then from the **Edit** menu select **Insert X Index** to add a column of index values. Check **bellcurve** and click **Draw**. The results for $\langle \theta \rangle$ and $\sigma\theta$ appear at the bottom of the graph as $\langle \theta \rangle \pm \sigma\theta$.
- Convert the degree values $\langle \theta \rangle$ and $\sigma\theta$ to radians values $\theta(\text{rads})$ and $\sigma\theta(\text{rads})$. Recall that $360^\circ = 2\pi$ radians.

You now need to calculate v and δv for the three range settings. The equation that relates the projectile velocity to the pendulum angle is derived in the Theory lab document and reproduced here:

$$v = \frac{M_T}{m} \sqrt{2gR_{cm} (1 - \cos \theta)} \quad (3.1)$$

range	θ_1°	θ_2°	θ_3°	θ_4°	θ_5°	$\langle\theta\rangle^\circ$	$\sigma\theta^\circ$	$\theta(\text{rads})$	$\sigma\theta(\text{rads})$
short									
medium									
long									

Table 3.1: Experimental angle values at three force settings

Note that in Equation 3.1 only the $\sqrt{(1 - \cos\theta)}$ factor changes with θ . Chances of mistakes in the error propagation process will be minimized if the constant quantities are represented by C and δC and are evaluated only once. Use the radian values of θ and $\sigma\theta$ from Table 3.1 in the following calculations.

$$C = \frac{M_T}{m} \sqrt{2gR_{cm}} \qquad \frac{\delta C}{C} = \sqrt{\left(\frac{\delta M_T}{M_T}\right)^2 + \left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta R_{cm}}{2R_{cm}}\right)^2}$$

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$$v_s = C\sqrt{(1 - \cos\theta)} \qquad \delta v_s = |v_s| \sqrt{\left(\frac{\delta C}{C}\right)^2 + \left(\frac{\sin\theta \delta\theta}{2\cos\theta}\right)^2}$$

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$$v_s = \dots \pm \dots \text{ m/s}$$

? What should be the dimensions of C ? And those of $\delta C/C$?

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- Calculate the maximum kinetic energy of the steel ball at the moment that it lost contact with the launcher rod and enter the value in Table 3.2. Show a complete calculation for the *short* range setting:

$$K_s = \frac{1}{2}mv_s^2 \qquad \delta K_s = |K_s| \sqrt{\left(\frac{\delta m}{m}\right)^2 + \left(2\frac{\delta v_s}{v_s}\right)^2}$$

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$$K_s = \dots \pm \dots \text{ J}$$

If no energy is lost (or gained) during the interaction, the kinetic energy K is equal to the potential energy $V = kx^2/2$ stored by the spring before the ball was discharged, and so

$$K = \left(\frac{k}{2}\right) x^2 \tag{3.2}$$

Comparing Equation 3.2 with the equation of a straight line $Y = AX + B$ and matching terms we see that $Y = K$, $X = x^2$ and the slope is $A = k/2$. The Y-intercept B (which is the value of Y when $X = 0$) is $B = 0$ since there is no corresponding term in Equation 3.2.

Plotting K as a function of x^2 and fitting these points to a straight line yields from the slope of the fitted line a value for the stiffness constant k of the launcher spring.

<i>range</i>	<i>v</i> (m/s)	<i>x</i> (m)	x^2 (m ²)	<i>K</i> (J)
short	±	±	±	±
medium	±	±	±	±
long	±	±	±	±

Table 3.2: Parameters for the calculation of the kinetic energy K and the stiffness constant k

- Shift focus to the Physicalab software and enter in the data window the three data pairs and corresponding errors as four space-delimited numbers: $x^2 K \delta K \delta x^2$.
- Select **scatter plot**. Click **Draw** to generate a graph of your data. Your graphed points should well approximate a straight line.

Draw an imaginary line through the three points; the points should lie close to the line. If they are not, then the computer fit of a straight line through these points will not yield a good result. Look for a mistake in your calculations before proceeding. If the problem persists, consult the TA.

- Select **fit to: y=** and enter **A*x+B** in the fitting equation box. Click **Draw** to perform a linear fit of the data. Label the axes and include a descriptive title. Click **Send to:** to email yourself a copy of the graph for later inclusion in your lab report.
- Record the values of the fit parameters **A**, **B** and their associated errors, then calculate k and δk :

$$\mathbf{A} = \dots \pm \dots \qquad \mathbf{B} = \dots \pm \dots$$

$$k = \dots = \dots = \dots$$

$$\delta k = \dots = \dots = \dots$$

$$k = \dots \pm \dots$$

? The stiffness constant k is typically expressed in units of N/m. Using dimensional analysis, verify that your dimensions for k obtained from the graph agree with those of N/m.

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- From the slope and Y-intercept, calculate the X-intercept (i.e. the value of X when $Y = 0$) by setting Y to zero and solving for X :

$$X = \dots = \dots = \dots$$

$$\delta X = \dots = \dots = \dots$$

$$X = \dots \pm \dots m^2.$$

- From this result estimate the spring preload distance x_0 :

$$x_0 = \dots = \dots = \dots$$

$$\delta x_0 = \dots = \dots = \dots$$

$$x_0 = \dots \pm \dots m.$$

! Important! Be sure to have this printout signed and dated by a TA before you leave at the end of the lab session. All your work needs to be kept for review by the instructor, if so requested.

Lab report

Go to the “Lab Documents” web page to access the online lab report template for this experiment. Complete the template as instructed and submit it to Turnitin before the lab report submission deadline, late in the evening six days following your scheduled lab session. Do not wait until the last minute. Turnitin will not accept overdue submissions. Unsubmitted lab reports are assigned a grade of zero.