

## Experiment 4

# Collisions and conservation laws

### Procedure

The equipment consists of an air table that discharges streams of air from a series of closely spaced holes on its flat surface. Two plastic pucks float on this cushion of air with little friction. The pucks are launched and made to collide. An overhead camera records a movie of the collision.

A program called Tracker is then used to make a frame-by-frame analysis of the movie. For each frame, the software determines the position of the two pucks and records their coordinates in a table.

These dots map the trajectory of the two pucks in time, as shown in Figure 4.1. From this data, vectors of the velocities before and after the collision are determined for each puck.

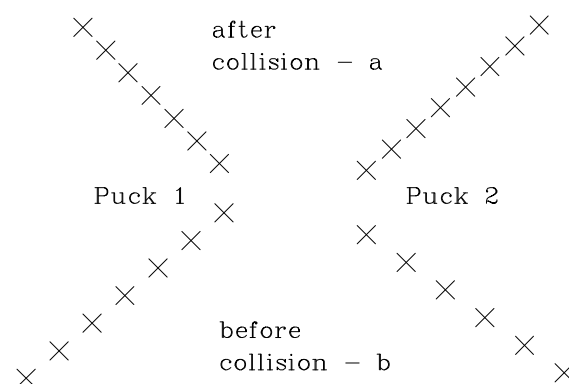


Figure 4.1: Trails left by moving pucks

### To record a video of the collision:

**Note:** There may not be sufficient air tables for every student to use simultaneously. In this case, the equipment will have to be shared. Once a collision is recorded, the video analysis can be done on any of the other workstations in the lab room, making the air table available for the next user.

**Every student** is required to record and analyse their own collision video.

1. At an airtable workstation, login to Physicalab and select “USB camera” from the “Hardware” menu.
2. Click “Start video stream”; a window should open to display the video from the camera situated above the air table. Verify that the table is framed evenly within the displayed picture. You can adjust the brightness of the image with the scrollbox above the video frame.
3. Place the pucks on the air table and turn on the air until the pucks begin to float freely, then, if necessary, level the frame by adjusting the height of the two legs on the side until the pucks float in place.

4. Place the launchers in adjacent corners of the frame. Load the pucks and press the trigger to release them toward the centre of the air table, where they will collide. Do several practice runs.

While *any* collision is in principle usable, *for ease of analysis* your collision should be fairly symmetric as in Figure 4.1 and take about 2–3 seconds to complete. Small initial velocities will cause the pucks to slow down due to friction and yield unsatisfactory results.

5. To record the collision, have an assistant press the “Capture video” button and then release the pucks. The collision process will be recorded at 30 frames per second for about 5 seconds.
6. Review your collision video by clicking on the “Review video” button. Repeat the above procedure until you get a nice collision to analyse. The white dots on the pucks should be fairly sharp as these are the objects that the software will track.
7. When done, logout and let the next student login to Physicalab to record their own video.

### To track the puck trajectories:

To access your video, login to Physicalab at any workstation in the lab and select “USB camera” from the “Hardware” menu, then click “Analyse video” to start the Tracker software. Your video should appear there for analysis. If two video windows appear, click **View** and uncheck **Bottom View**. Stretch the Tracker window to maximize the size of the video image.

The actual centre of the puck is elevated from the air table surface, causing its position to be distorted in the video image as the puck moves around the field of view of the camera. To avoid this problem, you will use the two white dots on the flat surface of the puck to define a virtual centre-of-mass for each puck that will be tracked accurately. Here is a summary of the required steps:

1. Calibrate the video frame to the air table grid by clicking **Track, New, Calibration Tools, Calibration Points**. An  $xy$ -coordinate system will appear. The grid is divided into 5-cm squares.

Shift-click on intersecting grid lines about 15 cm (3 squares or divisions) below and 15 cm left of the grid centre and **enter -0.15** for the  $x$  and  $y$  coordinates of **point 1**, then move 30 cm (6 divisions) up and 30 cm to the right of point 1, shift-click and **enter 0.15** for the  $x$  and  $y$  coordinates of **point 2**. The  $xy$ -axes should centre between these two reference points. You can review or change the coordinates of these points by clicking on them.

The grid in the movie is now calibrated to the air table grid.

2. Use the scrollbar under the viewer window to advance the video frame-by-frame to the start of the collision sequence after the pucks have left the launchers and both white dots are visible. Right-click on the scroll pointer and select this frame as the start frame for the collision analysis.
3. Use the scrollbar to advance the movie to the the end of the collision before the pucks reach the edge of the air table. Right-click the pointer and select this frame as the end frame for the collision analysis.

You can review this section of the video at various playback rates or step through frames. Adjust the start and end frames, if necessary, using the pointer as before.

4. Click the arrow to the left of the scrollbar to reset the video to the first frame of the selected collision sequence.

5. Referring to the puck on the left as Puck 1, click **Track, New, Point Mass**. A **Track Control** window opens with a tracking object called **mass A**. Hold down ‘ctrl+shift’ and click on the centre of one of the white dots to set it as the tracking object. You can zoom in with the mouse wheel to get a close-up view of the region of interest.
  6. When the **Autotracker** window appears, click **Search**. The software will then proceed to track the object in every frame and generate a table of  $xy$ -coordinates as a function of time for that mass. When done, close the window.
  7. Reset the video, then identify the other white dot on puck 1 as **mass B** and click **Search** to track it.
  8. Repeat these steps to define and track the two white dots on puck 2 as **mass C** and **mass D**.
  9. Reset the video, then click **Track, New, Center Of Mass** and check **mass A** and **mass B** to define for puck 1 a centre-of-mass point **cm A** using masses A and B. There will now be three traces visible. Click **View, Track Control** then double-click **mass A** and uncheck the **Visible** option to hide the track for mass A. Repeat for mass B, leaving only the centre-of-mass **cm A** trace visible.
  10. Reset the video, then click **Track, New, Center Of Mass** and check **mass C** and **mass D** to define for puck 2 a centre-of-mass point **cm B**. Hide the mass traces.
- Run the video. Were the pucks rotating, thus having angular momentum? Were the dots that will be used to define the before and after vectors evenly spaced and in a straight line?

### Analysis of puck trajectories:

You will now use the centre-of-mass coordinate data to generate before and after collision vectors for the two pucks. Note that four velocity vectors ( $\vec{v}_{1b}$ ,  $\vec{v}_{2b}$ ,  $\vec{v}_{1a}$ , and  $\vec{v}_{2a}$ ) are required.

The Tracker software is able to analyse your data in various convenient ways. For example, after a set of  $x$  and  $y$  coordinates have been recorded, Tracker can calculate and display for each time interval between two sequential dots, the components of the velocity of the puck in the  $x$  and  $y$  directions,  $v_x$  and  $v_y$ , as well as the speed  $v$  and direction angle  $\theta_v$ .

Recall that a vector  $\vec{v}$  confined to an  $xy$ -plane can be resolved into two orthogonal vectors, an  $x$ -component  $\vec{v}_x$  with magnitude  $v_x$  and a  $y$ -component  $\vec{v}_y$  with magnitude  $v_y$ .

The vector  $\vec{v}$  has a magnitude  $v = \sqrt{(v_x)^2 + (v_y)^2}$  and points in the  $xy$  plane at an angle of  $\theta = \arctan(v_y/v_x)$  from the origin.

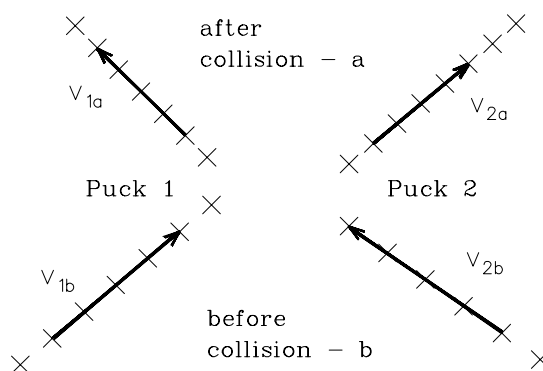


Figure 4.2: Collision vector labels

1. Starting with Puck 1, select **cm A** in the Tracker **Plot** and **Table** windows. The track plot should consist of two collinear series of dots. The series of dots from time  $t = 0$  to the moment just before the track changes direction due to the collision are used to define the velocity vector before the collision for Puck 1, labelled  $v_{1b}$  in Figure 4.2. The series of dots that are made after the collision define the velocity vector after the collision for Puck 1, labelled  $v_{1a}$  in Figure 4.2.

- Click on **Table** and in the window that appears uncheck  $x$  and  $y$  to hide the puck displacements, then check  $v_x, v_y$  to display columns of the puck component velocities and  $v, \theta_v$  to display the speed and direction angle.

**?** Review the speed and angle data. Are the speeds fairly constant or do they display a systematic decrease in time, suggesting that friction is significant?

- Right-click on one of the table column labeled  $t$  and select **Analyze**. The **Data Tool** window appears. Note that angles here are given in radians, although Tracker may be set to display angles in degrees or radians.

**Note:** If the velocity data in the Data Tool does not match the data in the Tracker table, right-click any tabs under **File**, select **Close all**, then **No** to save and finally, re-open the **Data Tool** window.

- In the **Data Tool** click **Analyze** and verify that **Statistics** is checked, then highlight a range of velocities  $v_{1bx}$  and  $v_{1by}$  for the puck *before* the collision. In the **mean** row are displayed the average values of  $v_{1bx}, v_{1by}, v_{1b}$ , and  $\theta_{1b}$  for the selected data.

The **sd** row contains the standard deviations (errors),  $\delta v_{1bx}, \delta v_{1by}, \delta v_{1b}$ , and  $\delta \theta_{1b}$ . Enter these results in Table 4.1.

- Highlight in the data table the range of velocities *after* the collision to get the mean and error values  $v_{1ax}, v_{1ay}, v_{1a}, \theta_{1a}, \delta v_{1ax}, \delta v_{1ay}, \delta v_{1a}$ , and  $\delta \theta_{1a}$ . Enter these results in Table 4.1.

- Now repeat the preceding steps for Puck 2. Select **cm B** in the Tracker **Plot** and **Table** windows, then display vectors  $v_{2b}$  and  $v_{2a}$ .

- Measure the masses  $m_1$  and  $m_2$  of pucks 1 and 2 using a digital scale and record these values to an accuracy of 0.01 g in Table 4.1. Note the scale measurement error. Be sure to match each mass with the corresponding puck trace, otherwise your  $Q$  value will not make sense.

Puck 1	Puck 2
$m_1 \pm \delta m =$	$m_2 \pm \delta m =$
$v_{1bx} \pm \delta v_{1bx} =$	$v_{2bx} \pm \delta v_{2bx} =$
$v_{1by} \pm \delta v_{1by} =$	$v_{2by} \pm \delta v_{2by} =$
$v_{1b} \pm \delta v_{1b} =$	$v_{2b} \pm \delta v_{2b} =$
$\theta_{1b} \pm \delta \theta_{1b} =$	$\theta_{2b} \pm \delta \theta_{2b} =$
$v_{1ax} \pm \delta v_{1ax} =$	$v_{2ax} \pm \delta v_{2ax} =$
$v_{1ay} \pm \delta v_{1ay} =$	$v_{2ay} \pm \delta v_{2ay} =$
$v_{1a} \pm \delta v_{1a} =$	$v_{2a} \pm \delta v_{2a} =$
$\theta_{1a} \pm \delta \theta_{1a} =$	$\theta_{2a} \pm \delta \theta_{2a} =$

Table 4.1: Results of collision analysis

The equation, derived in the Theory section of the lab and used to calculate the experimental mass ratio of the pucks is given by:

$$\frac{m_1}{m_2} = \frac{|\vec{v}_{2a} - \vec{v}_{2b}|}{|\vec{v}_{1a} - \vec{v}_{1b}|}.$$

The vectors  $(\vec{v}_{2a} - \vec{v}_{2b})$  and  $(\vec{v}_{1b} - \vec{v}_{1a})$  needed to solve this equation depend on the **magnitude and direction** of the individual vectors, hence a **vector subtraction** must be performed.

Since the magnitudes of the  $x$  and  $y$  components of these vectors are available in Table 4.1, the component vectors can be subtracted separately and then combined to give the magnitude and direction angle of the resultant difference vector.

- Evaluate the magnitudes and directions of these difference vectors, as follows:

$$v_{1x} = (v_{1bx} - v_{1ax}) = \dots = \dots$$

$$v_{1y} = (v_{1by} - v_{1ay}) = \dots = \dots$$

$$(v_{1b} - v_{1a}) = \sqrt{v_{1x}^2 + v_{1y}^2} = \dots = \dots$$

$$\theta_1 = \arctan\left(\frac{v_{1y}}{v_{1x}}\right) = \dots = \dots$$

$$v_{2x} = (v_{2ax} - v_{2bx}) = \dots = \dots$$

$$v_{2y} = (v_{2ay} - v_{2by}) = \dots = \dots$$

$$(v_{2a} - v_{2b}) = \sqrt{v_{2x}^2 + v_{2y}^2} = \dots = \dots$$

$$\theta_2 = \arctan\left(\frac{v_{2y}}{v_{2x}}\right) = \dots = \dots$$

- Since the change in momentum of one puck is equal and opposite to that of the other puck, then the two resultant vectors should then be nearly equal in magnitude since  $m_1 \approx m_2$  but now *parallel*.

? Compare the lengths and orientations (angle) of vectors  $(\vec{v}_{2a} - \vec{v}_{2b})$  and  $(\vec{v}_{1b} - \vec{v}_{1a})$ . Does the result make sense in light of the preceding explanation?

### Analysis:

- Determine a theoretical value  $(m_1/m_2)$  and error  $\delta(m_1/m_2)$  for the mass ratio from the given values of  $m_1$  and  $m_2$ .

$$\frac{m_1}{m_2} = \dots = \dots = \dots$$

$$\delta\left(\frac{m_1}{m_2}\right) = \dots = \dots = \dots$$

$$\text{(Theoretical)} \frac{m_1}{m_2} = \dots \pm \dots$$

- Approximate the errors in  $(v_{2a} - v_{2b})$  and  $(v_{1b} - v_{1a})$  using  $\delta v_{1b}$ ,  $\delta v_{1a}$ ,  $\delta v_{2b}$ , and  $\delta v_{2a}$  from Table 4.1 and the appropriate error equation.

$$\delta(v_{1b} - v_{1a}) = \dots = \dots = \dots$$

$$\delta(v_{2a} - v_{2b}) = \dots = \dots = \dots$$

$$\frac{m_1}{m_2} = \dots = \dots = \dots$$

$$\delta\left(\frac{m_1}{m_2}\right) = \dots = \dots = \dots$$

$$\text{(Experimental)} \frac{m_1}{m_2} = \dots \pm \dots$$

- The quality factor for the collision is derived in the Theory section of this lab and is given by

$$Q = \frac{m_1 v_{1a}^2 + m_2 v_{2a}^2}{m_1 v_{1b}^2 + m_2 v_{2b}^2}$$

Use it and the **theoretical**  $m_1$  and  $m_2$  values to determine  $Q$  and  $\delta Q$ .

When solving equations such as for  $Q$  and  $\delta Q$ , a good technique is to separate the equation into several terms and solve for each term separately. This avoids repetition and will expose possible calculation errors.

For example, there are four identical  $mv^2$  terms, each of which appears several times in the  $Q$  and  $\delta Q$  equations. Evaluate each term only once, then compare the four results. You would expect them to be similar since the  $m$  and  $v$  values are also similar.

Likewise, the equations for each error term are identical in form and should also yield similar results. You can then confidently complete the calculation of  $Q$  and  $\delta Q$ .

$$A = m_1 v_{1a}^2 = \dots = \dots$$

$$B = m_2 v_{2a}^2 = \dots = \dots$$

$$C = m_1 v_{1b}^2 = \dots = \dots$$

$$D = m_2 v_{2b}^2 = \dots = \dots$$

$$Q = \frac{A + B}{C + D} = \dots = \dots$$

$$\delta A = A \sqrt{\left(\frac{\delta m_1}{m_1}\right)^2 + \left(2 \frac{\delta v_{1a}}{v_{1a}}\right)^2} = \dots = \dots$$

$$\delta B = B \sqrt{\left(\frac{\delta m_2}{m_2}\right)^2 + \left(2 \frac{\delta v_{2a}}{v_{2a}}\right)^2} = \dots = \dots$$

$$\delta C = C \sqrt{\left(\frac{\delta m_1}{m_1}\right)^2 + \left(2 \frac{\delta v_{1b}}{v_{1b}}\right)^2} = \dots = \dots$$

$$\delta D = D \sqrt{\left(\frac{\delta m_2}{m_2}\right)^2 + \left(2 \frac{\delta v_{2b}}{v_{2b}}\right)^2} = \dots = \dots$$

$$\delta Q = Q \sqrt{\frac{(\delta A)^2 + (\delta B)^2}{(A + B)^2} + \frac{(\delta C)^2 + (\delta D)^2}{(C + D)^2}} = \dots = \dots$$

$$Q = \dots \pm \dots$$

ⓘ Important! Be sure to have this printout signed and dated by a TA before you leave at the end of the lab session. All your work needs to be kept for review by the instructor, if so requested.

## Lab report

Go to the “Lab Documents” web page to access the online lab report template for this experiment. Complete the template as instructed and submit it to Turnitin before the lab report submission deadline, late in the evening six days following your scheduled lab session. Do not wait until the last minute. Turnitin will not accept overdue submissions. Unsubmitted lab reports are assigned a grade of zero.