The simple harmonic oscillator

A simple harmonic oscillator (SHO) is a model system that is used to describe numerous real physical systems. The reason for this is profound: the same fundamental equations that describe the motion of a mass-on-a-spring also describe, to a very good approximation, the inter-atomic forces that hold all matter together. At least in a first approximation, we can fairly accurately pretend that *every* solid object we touch is held together by springs connecting pairs of atoms.

With no external forces applied to a solid material, the "inter-atomic springs" are at their equilibrium lengths, neither stretched nor compressed. The application of an external stretching force to the material will cause these springs to extend, thereby increasing the bulk length of the material. When the applied external force is removed, the springs return to their equilibrium lengths, restoring the material to its original dimensions. Such restoring forces may be overcome by a large enough applied external force that will cause the object to deform permanently or to break. The maximum force applicable without permanent distortion is called the *elastic limit* of the material.

Hooke's law states that the stretch or compression x of a material is directly proportional to the applied force F. The proportionality constant, called the *stiffness constant* k, has units of newtons per metre (N/m), and is also frequently called the spring constant or the spring's force constant. This proportionality between force and deformation has been found to be an excellent approximation for any solid object, as long as the elastic limit of the material is not exceeded.

For an object attached to a spring, Hooke's law is:

$$F_s = -kx \ ,$$

where F_s is the force exerted by the spring on the object and x is the displacement of the object from its equilibrium position. The negative sign expresses the fact that the force exerted *by the spring on the object* is in the direction *opposite* to the object's displacement.

For this reason, the force exerted by a spring on an attached object is often described as a *restoring force*, because it tends to restore equilibrium. That is, when the object is not at its equilibrium position, the force that the spring exerts on the object is directed towards the equilibrium position.

If the spring force and gravity are the only forces acting on the object, the system is called a simple harmonic oscillator, and the object undergoes simple harmonic motion — sinusoidal oscillations about the equilibrium point, with a constant amplitude, and a constant frequency that does not depend on the amplitude.

Simple harmonic motion is equivalent to an object moving around the circumference of a circle at constant speed, in the sense that the same formulas describe each kind of motion.

In the absence of friction, the oscillations will continue forever. Friction robs the oscillator of its mechanical energy, transferring it to thermal energy, and so the oscillations decay, and eventually stop altogether. This process is called damping, and so in the presence of friction, this kind of motion is called damped harmonic oscillation.

If the mass is displaced from its new equilibrium position and released, it will begin to oscillate according to

$$y = A_0 \cos(\omega_0 t + \phi), \qquad \omega_0 = \sqrt{\frac{k}{m}} = 2\pi f_0 = \frac{2\pi}{T_0}$$

where A_0 and ϕ are the initial amplitude and phase angle of the oscillation, T_0 is the period in seconds, and ω_0 is the angular speed in radians/second.

Recall that pendulum motion was also analyzed using this same equation. In essence, the pendulum was assumed to be undergoing simple harmonic motion during the short time interval that was sampled.

If a damping force F_d is present, the oscillation decays exponentially at a rate determined by the damping coefficient γ:

$$y = A_0 e^{(-\gamma t)} \cos(\omega_d t + \phi), \qquad \omega_d = \sqrt{\omega_0^2 - \gamma^2}, \qquad \gamma = \frac{R}{2m}$$

Note an interesting detail; the damped frequency of oscillations, ω_d, is smaller than ω_0 because of the subtraction of γ^2 under the square root. This reduction is not all that noticeable, even though the decrease in the amplitude due to the $e^{-\gamma t}$ term may be readily observed.

Note also that the hanging spring is stretched by its own weight and may exhibit twisting as well as lateral oscillations when stretching, factors that are neglected in our analysis.

One other simplifying assumption: this experiment assumes an ideal massless spring connected to a point mass m. Even with all these approximations, the damped harmonic oscillator lends itself very nicely to an experimental investigation.

Note further that the pendulum motion would, if given enough time, have decreased in amplitude so that for a long data sample of the pendulum motion over time, the damped harmonic oscillator equation would have to be used to make a proper fit of the data set.