(TA INITIALS)

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Experiment 1

The pendulum

A simple pendulum consists of a compact object of mass m suspended from a fixed point by a string of length L, as shown in Fig. 1.1. The gravitational force exerted on the object of mass m is $F = mg$, where g is the acceleration due to gravity.

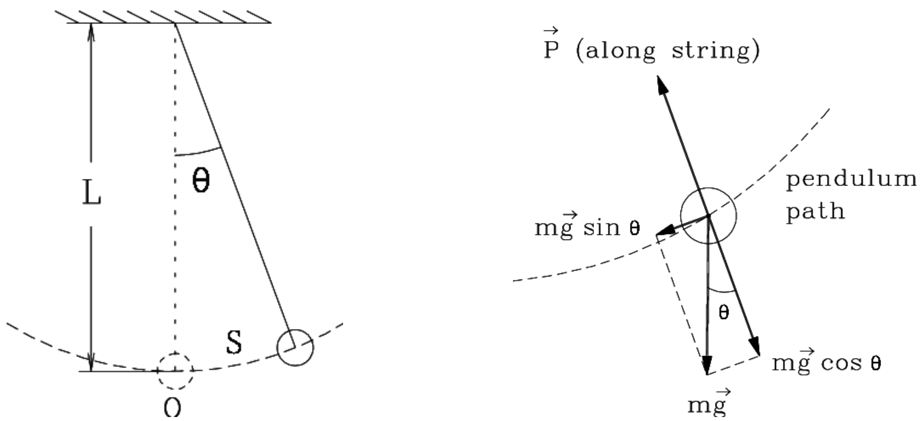

Figure 1.1: Two-dimensional (plane) trajectory of the simple pendulum

If the suspended object (often called a pendulum bob) is displaced slightly from its equilibrium vertical position by an angle θ, it will swing back and forth. The motion of the pendulum can be predicted using Newton's laws of motion and Newton's law of gravity. Assuming that there is no air resistance or other kinds of resistance, that the string has zero mass, and θ is a limited to few degrees, (the smaller the better) this analysis yields that the period T of the motion of this ideal pendulum satisfies to a good approximation

$$T = 2\pi\sqrt{\frac{L}{g}}. \tag{1.1}$$

? How would the period T of the pendulum change if the length L were doubled and everything else remained the same?

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? How would T change if the mass m were doubled and everything else remained the same?

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? How would T change if the gravitational acceleration g were less; for example, at the surface of the Moon instead of the Earth?

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Procedure

In this experiment, you will explore the relationship between the period T and length L of a swinging pendulum. The pendulum apparatus consists of:

- a vertical post and fixed arm from which the pendulum bob, an aluminum ball of diameter d, is suspended by a light nylon string of negligible mass ($m \approx 0$);
- a sliding arm used to adjust the pendulum swing length. If the arm is properly calibrated to the scale on the pendulum post, so that the scale reads '0' when the arm touches the top of the ball, then the scale can be used to directly measure, or set, this length;
- a scale that, when calibrated to the sliding arm, displays the length of the string s from **the bottom of the sliding arm**, the pivot point of the pendulum, to the **top of the ball**, to a precision of one millimetre (mm).

This length s is not exactly equal to the pendulum length L; the pendulum length is measured from the top of the string (at the fixed point) to the centre of the ball. Therefore, L is the sum of the length of the string s and half the ball's diameter d given in Table 1.1:

$$L = s + \frac{1}{2}d. \tag{1.2}$$

To calibrate the pendulum:

Click the "pendulum calibration" links (Parts 1 and 2) in "Lab Documents" to view a graphical description of the following steps:

1. Loosen the clamping nut to release the string and lower the ball to the table.
2. Align the bottom of the sliding arm, labelled **Index**, with the zero mark on the scale.
3. Adjust the string length so that the top of the ball just contacts the bottom of the arm, ensuring that the string is not stretched.
4. Gently tighten the string under the clamping nut. Do not just wrap the string around the nut; it will slip and result in length measurements that are incorrect.

To check the calibration:

1. Raise the arm away from the ball and carefully reposition the arm until it once again just contacts the top of the ball.
2. The index at the bottom of the sliding arm should be at the zero mark on the scale. If it is not, repeat the calibration adjustments until the bottom of the arm is in line with the zero mark of the scale *and* the arm lightly contacts the top of the pendulum ball.

Data gathering and analysis using Physicalab

(!) At the start of every lab session, click on the desktop icon to open a new Physicalab application, then enter your *Brock* email address. Without a valid email address, you will not be able to send yourself the graphs made during the experiment for inclusion in your lab report.

(!) At the end of the lab session, be sure to close the Physicalab application, otherwise your email address will be accessible to the next person using the work station.

As shown in the short video “Introduction to using rangefinder and Physicalab,” you are going to use a computer-controlled range-finder and the Physicalab software to monitor the change in distance over time of the pendulum bob from the device.

- Mount the **larger ball** m_1 and **calibrate the pendulum**.
- Adjust the sliding arm so that the string length s is approximately 0.3 m.

  Record the actual length to a precision of 1 mm (0.001 m).

Acquire distance/time data of the pendulum motion

1. Set the pendulum swinging in a straight line, keeping θ small (less than approximately 15°). Wait several seconds to allow for any stray oscillations present in the bob to dissipate before beginning to collect data.

[?] Does the angle of swing need to be precisely 15°? How might other choices of angle affect the results?

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2. Shift focus to the Physicalab software. Check the **Dig1** box and choose to collect **50** points at **0.1** s/point. Click [**Get data**] to acquire a set of data points $(\mathbf{x}, \mathbf{y})$ of distance $\mathbf{y}$ as a function of time $\mathbf{x}$.

3. Click [**Draw**] to graph your data. Your points should look like a smooth sine wave, without spikes, stray points or flat spots. If any of these are noted, adjust the position of the range-finder and acquire a new data set. Flat spots at the bottom of the graph occur when the ball is too close to the range-finder.

[?] If the pendulum was not swinging in-line with but at a significant angle β to the range-finder, how would the appearance of your graph change? Try to explain what is going on mathematically. Would this likely affect your results?

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Fit the pendulum distance/time data to a sine wave

1. Select **fit to: y=** and enter **A*sin(B*x+C)+D** in the fitting equation box.

   As reviewed in the Appendix, **x** represents the independent variable (here in units of time), **A** is the amplitude of the sine wave, **C** is the initial phase angle (in radians) of the wave when $\mathbf{x} = 0$, and **D** is the average distance of the pendulum from the detector; i.e. **D** is the distance from the pendulum to the detector when the pendulum is vertical or motionless.

   The fit parameter **B** (in radians/s) is the rate of change in angle with time, which is also called the angular frequency, so that **B*x** is an angle in radians. After one period of oscillation, where

T represents the period in seconds, **x** increases by T and the angle **B***x increases by 2π radians. Therefore $\mathbf{B}T = 2\pi$, and this allows us to relate the fitting parameter **B** to the period T of the pendulum's oscillation:

$$\mathbf{B} = \frac{2\pi}{T} \qquad \text{which is equivalent to} \qquad T = \frac{2\pi}{\mathbf{B}}$$

2. Click **Draw**. If you get a **Fit timed out** message on the bottom left of the screen, the initial guesses for the fitting parameters may be too distant from the required values for the fitting program to properly converge.

   Look at your graph and enter some reasonable *approximate* values for the fitting parameters. You can get an initial guess for **B** by estimating the time **x** between two adjacent minima, or one period, of the sine wave.

? Why are you fitting an equation to your data when you can estimate the period T of the pendulum directly from the graph?

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3. Label the axes and give your graph a descriptive title that includes the length s of the string. Click **Send to:** to email yourself a copy of the graph for later inclusion in your lab report.
4. Record in Table 1.1 the trial length s, the fitting parameter B, the values of T and T^2, and a value for g using Equation 1.1. Do not round values at this time.

? Is your value for g reasonable? Explain.

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Run, i	mass	m (kg)	d (m)	s (m)	L (m)	B (rad/s)	T (s)	T^2 (s^2)	g_i (m/s^2)
1	m_1	0.0225	0.02540						
2	m_1	0.0225	0.02540						
3	m_1	0.0225	0.02540						
4	m_1	0.0225	0.02540						
5	m_1	0.0225	0.02540						
1	m_2	0.0095	0.01904						

Table 1.1: Table of experimental results

- Repeat the above steps for m_1 with $s = 0.45$ m, 0.60 m, 0.75 m, and 0.90 m.

? Do you have to use these specific lengths? Could another set of values be used just as well? In terms of measurement errors, how might your choice of length affect the results?

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- Mount the second ball m_2, **recalibrate the pendulum** and verify the calibration. Set the string length to approximately $s = 0.5$ m, then repeat steps 3–9 for m_2 to complete Table 1.1.

Determining δg from a single value of g

Because you made only one trial using the small ball of mass m_2, error propagation rules need to be used to determine error estimates for L and g. The *measurement errors* in s and d, represented by δs and δd, are determined from the scales of the measuring instruments. The micrometer used to measure the ball diameter d has a resolution, or scale increment, of 0.00001 m.

$$\delta s = \pm \ldots\ldots\ldots\ldots\ldots \qquad\qquad \delta d = \pm \ldots\ldots\ldots\ldots\ldots$$

- Equation 1.2 is used to calculate L and derive δL. Show in three steps below the relevant equation, then the variables replaced by the appropriate unrounded values, and finally show the numerical result. Do not include units at this step.

$$L = s + \tfrac{1}{2}d = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\delta L = \sqrt{(\delta s)^2 + \left(\tfrac{1}{2}\delta d\right)^2} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Present the final result for $L \pm \delta L$, properly rounded and with the correct units

$$L = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$$

- Equation 1.1 expresses the relationship between g, L, and T. You now have δL but not δT, the error in T. Because the value of T is derived from the fit parameter **B** and $\delta\mathbf{B}$ is given by the fit, you could derive δT from $\delta\mathbf{B}$. However, a more direct approach is to use the relationship $\mathbf{B} = 2\pi/T$ to rewrite Equation 1.1 in terms of **B** instead of T and solve for g to get:

$$g = \mathbf{B}^2 L = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\delta g = g\sqrt{\left(\tfrac{2\delta\mathbf{B}}{\mathbf{B}}\right)^2 + \left(\tfrac{\delta L}{L}\right)^2} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$g = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$$

Determining δg from a set of g values

You performed five trials ($i = 1, \ldots, N = 5$) using the large ball of mass m_1 to obtain five results for g that are *expected to have the same value* if Equation 1.1 is valid. In this case, you can invoke the theory of statistics to evaluate a sample average $\langle g \rangle$ of the five trials as well as the standard deviation of the sample $\sigma(g)$.

- In Physicalab, enter in a column your five g values, then from the **Edit** menu select **Insert X Index column 1** to add a column of index values. Check **bellcurve** to view your data as a distribution. Also check **Bargraph** to display the data in bins.
- Copy $\langle g \rangle$ and $\sigma(g)$ below and remember to email yourself a copy of the graph.

$$g = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots, \qquad N = 5 \texttt{ samples}$$

- Now, click **File, Upload your g data** to add your five g values to the pendulum database. As the data from all the different groups of students doing the experiment is accumulated, a nice statistical distribution of the value of g should evolve and the standard deviation $\sigma(g)$ should systematically decrease.

- Click **File, Get class g data** to download the list of N values of g so far collected, then click **Draw** to display a distribution of the data. Record below the values for the group mean $\langle g \rangle$ and the group standard deviation $\sigma(g)$.

$g = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots,$ $N = \ldots$ `samples`

? How do $\langle g \rangle$ and $\sigma(g)$ for the N currently accumulated group values compare with your result? Is this what you expect?

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Determining g and δg from the slope of a graph

A graphical method can also be used to determine g and δg. Here, a line of best fit that represents the relationship between the x and y coordinates is drawn through your data. The software chooses the line of best fit so that it minimizes some quantity related to the distances of the data points from the line of best fit. (As a challenge, you might like to think about the details; specifically which quantity is minimized?) Rewriting Equation 1.1 in terms of L as a function of T^2 as follows

$$L = \left(\frac{g}{4\pi^2}\right) T^2 \tag{1.3}$$

yields a linear relationship $y = mx + b$ between $y = L$ and $x = T^2$, with slope $m = g/(4\pi^2)$ and y-intercept $b = 0$.

- Enter the five coordinates (T^2, L) in the Physicalab data window. Select **scatter plot**. Click **Draw** to generate a graph of your data. Select **fit to: y=** and enter **A*x+B** in the fitting equation box. Click **Draw**. The computer will evaluate a line of best fit through the data points and output the fit results. The χ^2 (chi square) value is a measure of the goodness of the fit; the value of χ^2 is smaller for data that more closely lies along a straight line.

- Send yourself the graph, then record the slope A and error δA below:

$A = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$

- Calculate values for g and δg. Refer to the Appendix to determine the error equation for δg.

$g = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$

$\delta g = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots$

$g = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$

- Redraw the preceding graph, this time including the data point from the single trial using mass m_2 and repeat the linear fit. This graph can provide you with insight on the dependence of g on the mass of the pendulum bob. Record the slope for comparison with the previous result.

$$A = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$$

(!) Important! Be sure to have this printout signed and dated by a TA before you leave at the end of the lab session. All your work needs to be kept for review by the instructor, if so requested.

Lab report

Go to the "Lab Documents" web page to access the online lab report template for this experiment. Complete the template as instructed and submit it to Turnitin before the lab report submission deadline, late in the evening six days following your scheduled lab session. Do not wait until the last minute. Turnitin will not accept overdue submissions. Unsubmitted lab reports are assigned a grade of zero.