Assignment No. 7

Physics 2P20

Due April 9, 2018
(optional; solutions will be discussed in class on April 9)

1. The force on a charged particle in an electric field $\vec{E}$ and a magnetic field $\vec{B}$ is given by
$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$
in an inertial systen, where q is the charge and $\vec{v}$ is the velocity of the particle in the inertial system. Show that the differential equation of motion referred to a rotating coordinate system with angular velocity $\vec{\omega} = -(q/2M)\vec{B}$ is, for small ω,
$$m\ddot{\vec{r}}' = q\vec{E}$$
that is, the term involving $\vec{B}$ is eliminated. This result is known as *Larmor's theorem.*

2. A uniform ladder of length l leans against a smooth vertical wall, making an initial angle θ_0 with the floor. It starts to slip downward without friction. Show that the top of the ladder loses contact with the wall when it is at two-thirds of its initial height.

   Hint: Only a single variable is needed to describe the system. Note the motion of the center of mass.

3. A uniform block of mass m and dimensions a by $2a$ by $3a$ spins about a long diagonal with angular velocity $\vec{\omega}$. Using a coordinate system with origin at the center of the block,

   (a) find the kinetic energy; and

   (b) find the angle between the angular velocity vector and the angular momentum vector about the origin.

4. A rigid body consists of six particles, each of mass m, fixed to the ends of three light rods of length $2a$, $2b$, and $2c$, respectively; the rods being held mutually perpendicular to one another ath their midpoints.

   (a) Show that a set of coordimate axes defined by the rods are principal axes, and write down the inertia tensor for the system in these axes.

   (b) Use matrix algebra to find the angular momentum and the kinetic energy of the system when it is rotating with angular velocity $\vec{\omega}$ about an axis passing through the origin and point (a, b, c).