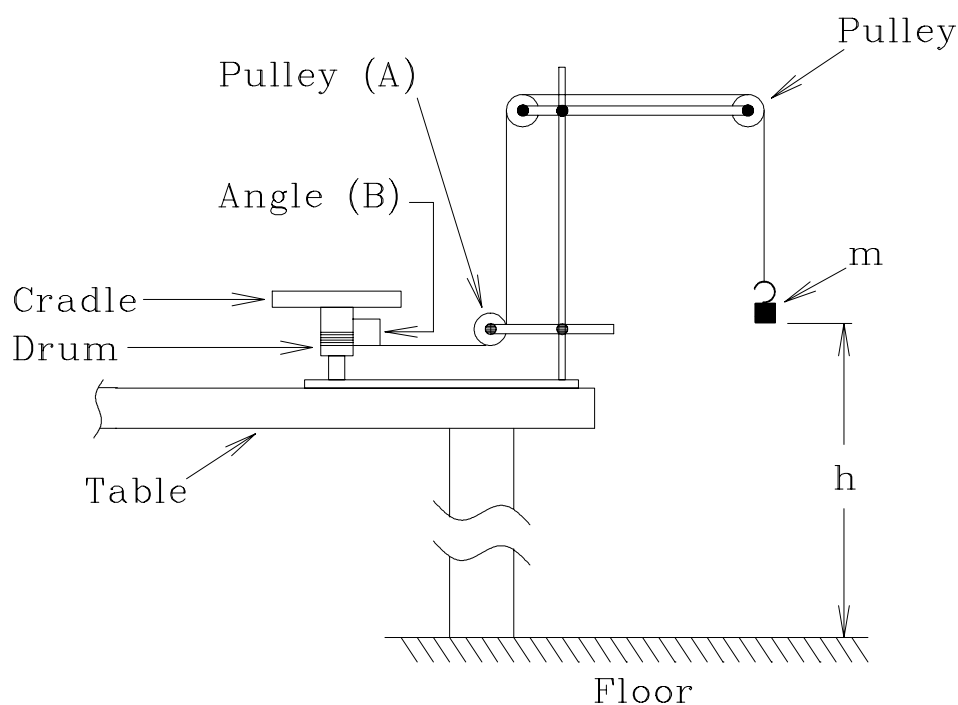


Experiment 4

Angular Motion

Before starting the experiment, you need to be familiar with the concept of angular position θ , angular velocity ω , angular acceleration α , torque τ , moment of inertia I . See Kleppner and Kolenkow, Chapter 6.



The equipment consists of a plastic disc bolted to a metal drum (this combination is called the cradle), which can rotate nearly friction free, around a vertical axis. A torque τ can be applied to the cradle with a string-and-pulley combination, at the end of which a mass m can be attached. We arrange the pulleys so that the string leaving the drum is perpendicular to the axis of rotation. Then the torque exerted on the cradle by the string is

$$\vec{\tau} = \vec{r} \times \vec{T}, \quad (4.1)$$

where \vec{r} is the radius of the pulley and \vec{T} is the tension in the string. If the mass m at the other end of the string is falling, starting from rest, over a distance h in a time t , its acceleration is

$$|a| = g - \frac{T}{m} = \frac{2h}{t^2}. \quad (4.2)$$

The linear acceleration a of m is related to the angular acceleration α of the cradle by

$$|a| = |r| |\alpha|. \quad (4.3)$$

α is, in turn, related to the moment of inertia of the cradle I_c by

$$\vec{\tau} = I_c \vec{\alpha}. \quad (4.4)$$

ⓘ **Exercise #1.**

Derive Equations (4.1)–(4.4), and combine these equations to show that

$$I_c = mr^2 \left[\left(\frac{gt^2}{2h} \right) - 1 \right]. \quad (4.5)$$

Using Equations (4.4) and (4.5) to rearrange Equation (4.3) gives

$$\alpha = \frac{a}{r} = \frac{\tau}{I} = \frac{Tr}{I}. \quad (4.6)$$

By combining Equation (4.2) with the result of (4.6), the rotational inertia can be expressed as

$$I = mr^2 \left(\frac{g}{a} - 1 \right). \quad (4.7)$$

All exercises will be completed with the string wound around the second smallest pulley ($r=0.02282$ m) on the cradle and the height h set to approximately 1 m.

ⓘ **Exercise #2.**

Using the PhysicaLab software, select the Dig2 photogate input channel and collect 20 points of data, with a timing of 0.5 seconds between points. To determine the acceleration, fit the data to $A+B*x+C*x**2/2$, and record the values for the acceleration of each run. Each weight will be run twice, and an average will be calculated. You will now evaluate the rotational inertia I_c of the

m (kg)	0.020	0.030	0.040	0.050
a_1 (m/s ²)				
a_2 (m/s ²)				
$\langle a \rangle$ (m/s ²)				

Table 4.1: Acceleration data for unloaded cradle assembly

unloaded cradle using Equation (4.7). Using the equation on its own would not lead to the correct I_c value. This is because the mass required to overcome the friction of the pulley system, m_0 , is not taken into account. There is a method that does not require m_0 to be known. This method starts with a rearrangement of Equation (4.7), so that m is expressed as a function of a :

$$I_c = m_T r^2 \left(\frac{g}{a} - 1 \right) \approx m_T r^2 \left(\frac{g}{a} \right) = (m - m_0) \left(\frac{gr^2}{a} \right) \rightarrow m = m_0 + \left(\frac{I_c}{gr^2} \right) a. \quad (4.8)$$

The equation is simplified by the fact that $a \ll g$, and by taking into account that the mass offset m_0 and the mass m used make up the total mass $m_T = m - m_0$ that causes the tension on the string. The resulting equation is that of a straight line with slope $I_c/(gr^2)$ and y-intercept equal to m_0 .

Using PHYSICA, make a plot of m vs. $\langle a \rangle$, fit to a straight line, and determine I_c .

ⓘ **Exercise #3. Rotational inertia of the cradle and disc.**

Place the disc on the cradle and repeat the measurements to obtain the moment of inertia I_t of the cradle-plus-disc. Subtract I_c from I_t to obtain I_{disc} , the moment of inertia of the disc alone. Compare the measured value of I_{disc} with the theoretical value which you can calculate from the disc's weight and dimensions.

m (kg)	0.020	0.030	0.040	0.050
a_1 (m/s ²)				
a_2 (m/s ²)				
$\langle a \rangle$ (m/s ²)				

Table 4.2: Acceleration data for cradle-and-disc assembly

ⓘ **Exercise #4. Rotational inertia of the cradle and ring.**

Replace the disc with the ring on the cradle and repeat the measurements to obtain the moment of inertia I_t of the cradle-plus-ring. Subtract I_c from I_t to obtain I_{ring} , the moment of inertia of the ring alone. Compare the measured value of I_{ring} with the theoretical value which you can calculate from the ring's weight and dimensions.

m (kg)	0.020	0.030	0.040	0.050
a_1 (m/s ²)				
a_2 (m/s ²)				
$\langle a \rangle$ (m/s ²)				

Table 4.3: Acceleration data for cradle-and-ring assembly

ⓘ **Exercise #5. Rotational inertia of the cradle and cube.**

Replace the ring with the cube on the cradle and repeat the measurements to obtain the moment of inertia I_t of the cradle-plus-cube. Subtract I_c from I_t to obtain I_{cube} , the moment of inertia of the cube alone. Compare the measured value of I_{cube} with the theoretical value which you can calculate from the cube's weight and dimensions.

m (kg)	0.020	0.030	0.040	0.050
a_1 (m/s ²)				
a_2 (m/s ²)				
$\langle a \rangle$ (m/s ²)				

Table 4.4: Acceleration data for cradle-and-cube assembly

ⓘ **Exercise #6. Rotational inertia of the cradle and sphere.**

Replace the cube with the sphere on the cradle and repeat the measurements to obtain the moment of inertia I_t of the cradle-plus-sphere. Subtract I_c from I_t to obtain I_{sphere} , the moment of inertia of

the sphere alone. Compare the measured value of I_{sphere} with the theoretical value which you can calculate from the sphere's weight and dimensions.

m (kg)	0.020	0.030	0.040	0.050
a_1 (m/s ²)				
a_2 (m/s ²)				
$\langle a \rangle$ (m/s ²)				

Table 4.5: Acceleration data for cradle-and-sphere assembly

ⓘ **Exercise #7. Rotational inertia of the cradle and cone.**

Replace the sphere with the cone on the cradle and repeat the measurements to obtain the moment of inertia I_t of the cradle-plus-cone. Subtract I_c from I_t to obtain I_{cone} , the moment of inertia of the cone alone. Compare the measured value of I_{cone} with the theoretical value which you can calculate from the cone's weight and dimensions.

m (kg)	0.020	0.030	0.040	0.050
a_1 (m/s ²)				
a_2 (m/s ²)				
$\langle a \rangle$ (m/s ²)				

Table 4.6: Acceleration data for cradle-and-cone assembly

ⓘ **Exercise #8. Rotational inertia of the cradle and pyramid.**

Replace the cone with the pyramid on the cradle and repeat the measurements to obtain the moment of inertia I_t of the cradle-plus-pyramid. Subtract I_c from I_t to obtain $I_{pyramid}$, the moment of inertia of the pyramid alone. Compare the measured value of $I_{pyramid}$ with the theoretical value which you can calculate from the pyramid's weight and dimensions.

m (kg)	0.020	0.030	0.040	0.050
a_1 (m/s ²)				
a_2 (m/s ²)				
$\langle a \rangle$ (m/s ²)				

Table 4.7: Acceleration data for cradle-and-pyramid assembly

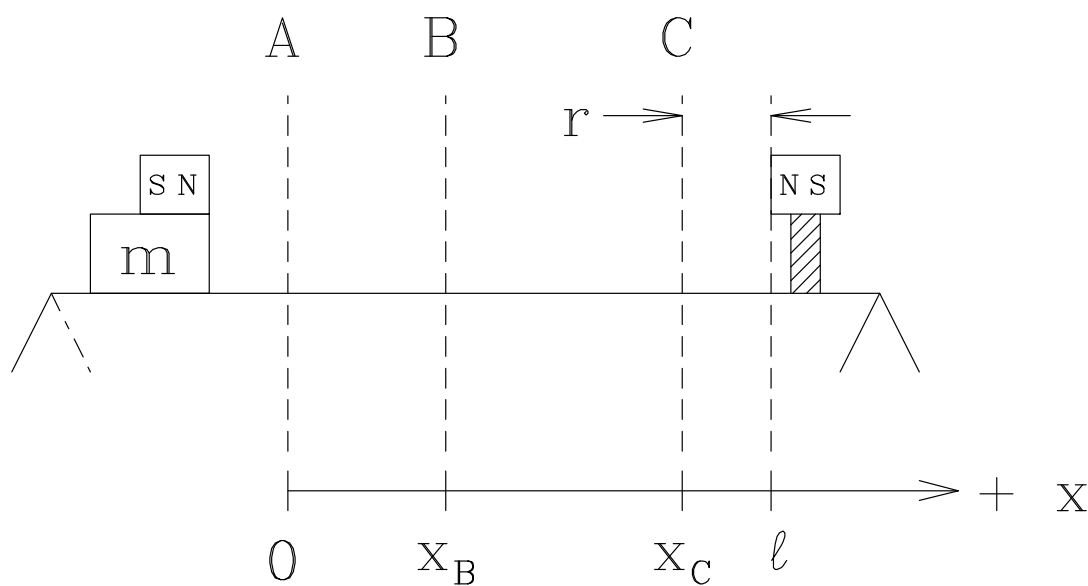
Experiment 5

Determination of a Potential Energy Curve on a Linear Air Track

5.1 Theory

A glider of mass m is moving with a velocity \vec{v}_0 towards a stationary magnet. The force between the two magnets is repulsive, and at point C the glider will reverse its direction of motion. The position of C depends on m, v_0 and the strength of the magnetic interaction. This strength is given by a potential function $U(r)$, with r as the separation of the two magnets. If the track was truly free of friction, then $U(r)$ is easy to determine: measure v_0 at a position, say at A, where $U(r)$ is negligibly small. If the turning point C is at position x_c , then conservation of mechanical energy gives (where $r = \ell - x_c$):

“Total Energy at A” (i.e. $x = 0$) equals “Total Energy at C” (i.e. $x = x_c$).



$$\frac{1}{2}mv_0^2 + 0 = 0 = U(r). \quad (5.1)$$

Regrettably, the track is not exactly friction-free, and the frictional loss in kinetic energy while the glider travels from A to C is not a negligible quantity. We can take the friction losses into account as