Experiment 7

Kater's Pendulum

The period T of a physical pendulum is given by

$$T = 2\pi\sqrt{\frac{I}{Mgd}}, \tag{7.1}$$

where

I: Moment of Inertia with respect to the axis of oscillation (AO).
M: Total mass of the pendulum.
g: Acceleration due to gravity.
d: Distance between the centre of mass (CM) of the pendulum and the AO.

When the amplitude of oscillation is not small, so that the approximation $\sin\theta \cong \theta$ ceases to be valid, then T is given by

$$T = 2\pi\sqrt{\frac{I}{Mgd}}\left[1 + \frac{1}{2^2}\cdot\sin^2\left(\frac{\theta_m}{2}\right) + \frac{1}{2^2}\cdot\frac{3^2}{4^2}\cdot\sin^4\left(\frac{\theta_m}{2}\right) + \cdots\right], \tag{7.2}$$

where θ_m is the maximum angular displacement.

Kater's pendulum consists of a bar with two fixed knife edges, AO_1 and AO_2; two different masses M_1 and M_2 can be clamped to the bar. The positions of M_1 and M_2 along the bar are adjusted until the period of oscillations T_1 around AO_1 is equal to the period of oscillation T_2 around AO_2. In this condition we then have

$$T_1 = T_2 = T_0 = \sqrt{\frac{4\pi^2\ell_0}{g}}, \tag{7.3}$$

where ℓ_0 is the distance between the knife edges.

7.1 Procedure

Set up Kater's pendulum, making sure it swings freely. Fasten (and fix) the small mass M_1 to the bar, outside AO_1 and AO_2, and measure its distance ℓ to AO_1. Swing the pendulum, with small θ_m, with AO_1 as the pivot point, and measure T_1 a few times with the photogate. Be sure that the pendulum swings "straight" through the gate.

Now reverse the pendulum; swing it around AO_2 and measure T_2. Unless you are very lucky, T_1 and T_2 will be appreciably different. Change the position of M_2 (that is, change ℓ), and measure T_1 and T_2 again. Repeat this procedure a few times and plot curves of T_1 vs. ℓ and T_2 vs. ℓ on one graph. The intersection of these two curves indicates the value of ℓ for which T_1 is approximately equal to T_2. Guided by this graph, change ℓ by small amounts in the right direction until you experimentally find $T_1 = T_2 = T_0$ within experimental error.

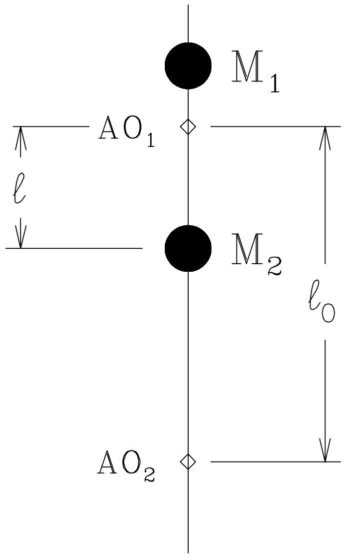

Figure 7.1: Dimensions of Kater's Pendulum

For our pendulum, $\ell_0 = 99.48 \pm 0.01$ cm; with your T_0 value you can now calculate g from Equation (7.3). There are in fact *two* positions of M_2 for which $T_1 = T_2 = T_0$; find both positions, and compare the two values of g, and their error.

The inherent accuracy of this experiment is quite high. Give a complete error analysis, including a discussion of whether your measured T values should be corrected for large oscillation amplitudes θ_m. Use Equation (7.2) to prove this.

References

1. D. Kleppner and R.J. Kolenkow, *An Introduction to Mechanics.* McGraw Hill, 1973. Chapter 6.

2. N. Feather, *An Introduction to the Physics of Mass, Length and Time.* pp. 187–194.