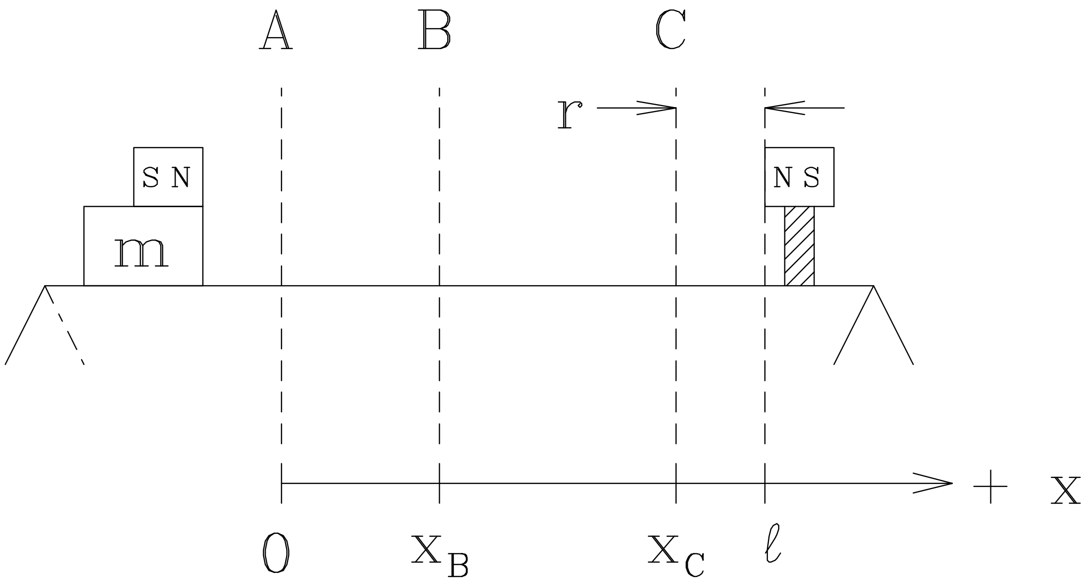
Experiment 5

Determination of a Potential Energy Curve on a Linear Air Track

5.1 Theory

A glider of mass m is moving with a velocity $\vec{v}_0$ towards a stationary magnet. The force between the two magnets is repulsive, and at point C the glider will reverse its direction of motion. The position of C depends on m, v_0 and the strength of the magnetic interaction. This strength is given by a potential function $U(r)$, with r as the separation of the two magnets. If the track was truly free of friction, then $U(r)$ is easy to determine: measure v_0 at a position, say at A, where $U(r)$ is negligibly small. If the turning point C is at position x_c, then conservation of mechanical energy gives (where $r = \ell - x_c$):

"Total Energy at A" (i.e. $x = 0$) equals "Total Energy at C" (i.e. $x = x_c$).



$$\frac{1}{2}mv_0^2 + 0 = 0 = U(r). \tag{5.1}$$

Regrettably, the track is not exactly friction-free, and the frictional loss in kinetic energy while the glider travels from A to C is not a negligible quantity. We can take the friction losses into account as follows: Apply the Work-Energy theorem to the motion from A to C:

$$W_m + W_f = \left(\frac{1}{2}mv^2\right) \text{ (at C)} - \left(\frac{1}{2}mv^2\right) \text{ (at A)}, \tag{5.2}$$

where W_m is the work done by the magnetic forces and W_f is the work done by the friction forces. Since the magnetic force is zero at A, we have

$$W_m = -U(x_r). \tag{5.3}$$

The right-hand-side of Equation (5.2) equals $-\frac{1}{2}mv_0^2$, where v_0 is the speed at $A(x=0)$.

5.1.1 Calculation of the Term W_f

The origin of the frictional force F_f is the friction in the air layer between the glider and the track. For small glider velocities v this force is proportional to v, producing an acceleration $a = -kv$, where k is a constant. For this type of acceleration, we know that velocity v and position x depend on time t as follows:

$$a = -kv \tag{5.4}$$

$$v = v_0 \exp(-kt) \tag{5.5}$$

$$x = \left(\frac{v_0}{k}\right)(1 - \exp(-kt)). \tag{5.6}$$

Therefore the relations between a, v and x are:

$$a = k^2 x - kv_0 \tag{5.7}$$

$$v = v_0 - kx. \tag{5.8}$$

Therefore, the work done by the frictional force while the glider moves from A to C is approximately[1] given by

$$W_f = \int_0^{x_r} \vec{F}_f \cdot d\vec{x} = m \int_0^{x_r} (k^2 x - kv_0)\, dx = m\left(\frac{1}{2}k^2 x_r^2 - kv_0 x_r\right). \tag{5.9}$$

Combining Equations (5.2), (5.3), and (5.9) gives $U(x_r)$:

$$U(x_r) = \frac{1}{2}mv_0^2 + \frac{1}{2}mk^2 x_r^2 - mkv_0 x_r. \tag{5.10}$$

We can determine k by measuring the velocity v_B of the glider at a point B (coordinate x_B) where the magnetic force is still negligible; from Equation (5.5) we have

$$k = \left(\frac{v_0 - v_B}{x_B}\right). \tag{5.11}$$

Therefore, all quantities in Equation (5.10) can be measured, and $U(x_r)$ can be determined.

5.2 Procedure

Carefully level the air track; position the fixed magnet and the two photogates. Set $x_B \simeq 0.3$ m and $\ell \simeq 0.6$ m. The photogates at A and B measure the velocities of the glider by recording the times it takes the glider to interrupt a light beam. Let these times be Δt_0 (at A) and Δt_B (at B); then $v_0 = \ell^*/\Delta t_0$ and $v_B = \ell^*/\Delta t_B$, where ℓ^* is the length of the glider. First determine carefully the quantities: m, x_B, ℓ and ℓ^*. Then give the glider a velocity in the range of $0.1 - 0.2$ ms^{-1}, and determine, for a given run, $\Delta t_0, \Delta t_B$ and x_c. Do a number of runs, so that you can calculate $U(x_r)$ for a range of x_r values. Arrange the data in tabular form.

[1] The result is not exact, since close to C, where the magnetic force becomes important, the acceleration $\vec{a}$ is not proportional to $-\vec{v}$ anymore.

5.3 Data Analysis

Do all calculations in the MKS system of units; record all calculated results in tabular form. For each run:

1. Calculate v_0 and v_B from $\ell^*, \Delta t_0$ and Δt_B.

2. Calculate k, using Equation (5.11), with v_0, v_B, and x_B.

3. From the values v_0, m, k and x_c for each run, calculate each term in Equation (5.10), and sum them to obtain $U(r)$. (You really obtain first $U(x_c)$), but you can convert the x_c values into r values using $r = \ell - x_c$.

4. So now you have $U(r)$ values for different r values. Can you now "guess" at the mathematical relation between $U(r)$ and r? For example: Is $U(r)$ linear in r, that is: $U(r) = ar + b$? You can check this by plotting $U(r)$ vs. r on linear graph paper, and see if your data points fall on a straight line. If they do, you can calculate a and b. If they do not, then plot $U(r)$ vs. r on full logarithmic graph paper. If the data points fall on a straight line now, then you know that $U(r) = Kr^n$, and from your graph you can determine K and n. If this does not work, plot your data on semi-logarithmic graph paper, to see if they obey $U(r) = A\exp(Br)$.

In your report, derive Equations (5.7) and (5.8).