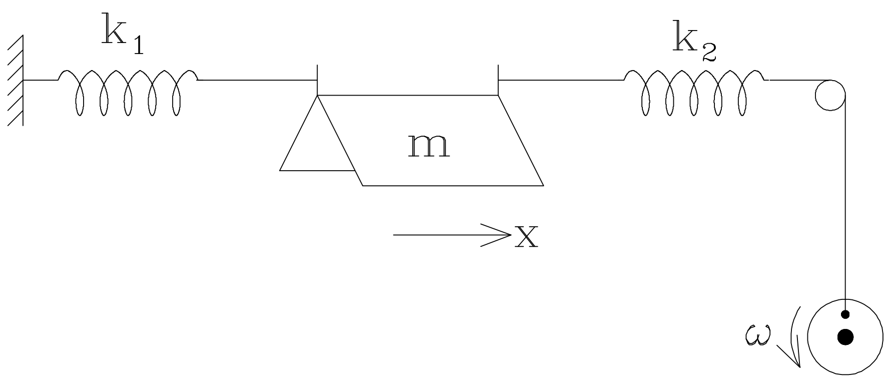
Experiment 3

Oscillators

The oscillating system consists of a car (mass m) on the Linear Air Track, connected to two stretched springs (spring constants k_1 and k_2); viscous damping can be provided by taping magnets to the sides of the car, and a periodic external force of variable frequency ω can be applied to m via a turntable. The position $x(t)$ of m can be either measured directly on the track, or a record of $x(t)$ can be graphed directly using PHYSICA. The period T (and therefore the frequency $\omega = 2\pi/T$) can be determined with a timer and a "flag" on the car.



(!) NOTE
Make sure to clean and level the air track prior to beginning the experiment for this and every subsequent lab that requires the use of the linear air track.

Please note that the sonar range finder has a minimum distance of 35 cm.

3.1 Undamped, Harmonic Oscillator

When the force F on the car is proportional to the displacement q from its equilibrium position, then the car will perform harmonic oscillations around $q = 0$. We then have for the force F,

$$F = -kq,$$

for the potential energy U,

$$U = \frac{1}{2}kq^2,$$

and for the differential equation of motion,

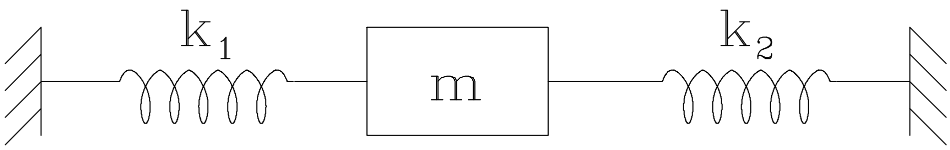
$$\frac{d^2q}{dt^2} + \omega_0^2 q = 0, \tag{3.1}$$

with $\omega_0^2 = k/m$. The motion is described by the solution to Equation (3.1),

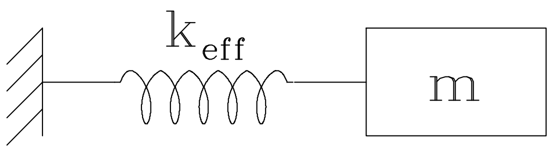
$$q = A\cos(\omega_0 t + \psi). \tag{3.2}$$

Amplitude A and phase angle ψ depend only on initial conditions, and are independent of ω_0.

(!) **Exercise #1.**
Remove the damping magnets from the sides and put them on top of the car. Carefully determine $\omega_0 \pm \sigma(\omega_0)$ (using the timer) of the oscillator as a function of amplitude, for the range $12 \geq A \geq 1$ cm. Determine m. Show theoretically, that the system



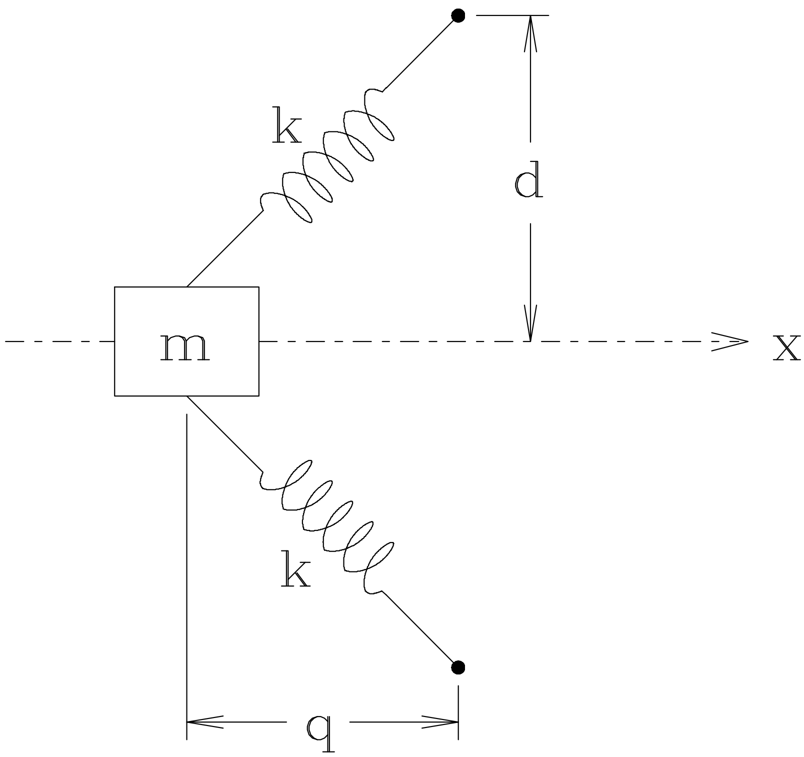
is equivalent to the system:



and derive an expression for k_{eff} in terms of k_1, k_2 and any other experimentally relevant parameters. Use the balance to determine m. From ω_0 and m, calculate k_{eff}.

3.2 Undamped, Anharmonic Oscillator

When the restoring force on the car is not exactly proportional to q, the car still oscillates around $q = 0$, but the oscillations will not be strictly harmonic, and amplitude and frequency are not independent. An example of an anharmonic oscillator is:



(!) **Exercise #2.**
Show that for the system above the restoring force (in the x-direction) on the car is

$$F = -kq \left[2 - 2 \left(\frac{\ell_0}{d\sqrt{1 + \frac{q^2}{d^2}}} \right) \right], \tag{3.3}$$

where ℓ_0 is the length of the relaxed spring, and $d > \ell_0$. Show that for small values of q/d this is approximately equal to:

$$F = -kq \left(2 - \frac{2\ell_0}{d} \right) - kq^3 \left(\frac{\ell_0}{d^3} \right). \tag{3.4}$$

Set up the oscillator as sketched above; make $d \cong 20$ cm. Carefully measure ω as a function of A adjusting d from 2 to 20 cm in 2 cm steps, and plot the results.

3.3 Damped, Harmonic Oscillator

Return to the set-up used in Section 3.1, but tape the damping magnet to the side of the car. This will provide a retarding, "frictional" force proportional to the velocity v of the car. The differential equation of motion is now:

$$m\frac{d^2q}{dt^2} = -kq - bv, \tag{3.5}$$

or

$$\frac{d^2q}{dt^2} + \gamma\frac{dq}{dt} + \omega_0^2 q = 0, \tag{3.6}$$

with $\gamma = b/m$ and $\omega_0^2 = k/m$.

For reasonably weak damping ($\gamma < 2\omega_0$) the motion of the car is still an oscillation, but with a frequency $\omega_1(\neq \omega_0)$ and a steadily decreasing amplitude:

$$q(t) = A \exp\left(\frac{-\gamma t}{2}\right) \cos(\omega_1 t + \psi), \tag{3.7}$$

with

$$\omega_1^2 = \omega_0^2 - \left(\frac{\gamma}{2}\right)^2. \tag{3.8}$$

(!) **Exercise #3.**
With the magnet taped to the top of the car, weigh the car and determine ω_0. Take this measurement three times and use standard deviation to determine $\sigma(\omega_0)$.

Place the magnet on the side of the car, set the car in a damped oscillation, and measure $\omega_1 \pm \sigma(\omega_1)$ carefully. Then calculate γ from Eqns. (3.7) and (3.8).

3.4 Forced, Damped, Harmonic Oscillator and Resonance

Set the turntable in motion. The record of $x(t)$ will now be a rather irregular motion (the transient), followed by a steady-state motion of the form $x = A\cos(\omega t + \psi)$. Determine the frequency ω. The steady-state amplitude and phase angle are given by

$$A = \frac{F_0}{m} \frac{1}{\sqrt{(\omega^2 - \omega_0^2)^2 + (\gamma\omega)^2}}. \tag{3.9}$$

$$\tan\psi = \frac{-\gamma\omega}{\omega_0^2 - \omega^2}. \tag{3.10}$$

(!) **Exercise #5.**
Determine A (from the scale attached to the Linear Air Track) for a large number of ω values, and plot a graph (the resonance curve) of A vs. ω. Fit to Equation (3.9) to determine F_0/m, ω, and γ.

Show theoretically that:

1. $A_0 = A(\omega \to 0) = F_0/k_{\text{eff}}$.
2. A is a maximum for $\omega = \omega_r$, with

$$\omega_r^2 = \omega_0^2 - \frac{1}{2}\gamma^2, \tag{3.11}$$

and

$$A_r = A(\omega = \omega_r) = \frac{F_0}{m} \cdot \frac{1}{\gamma\omega_r} \cong \frac{F_0}{m}\frac{1}{\gamma\omega_0} = \frac{A_0\omega_0}{\gamma}. \tag{3.12}$$

Determine A_0, A_r and ω_r from the resonance curve and determine F_0 and γ from them. Compare this γ value with the values determined in Section 3.2.

Two final comments:

1. Note qualitatively the behaviour of the phase angle ψ when ω increases from a value less than ω_r to a value larger than ω_r.

2. The behaviour of a damped oscillator is determined by ω_0 and γ, which are often combined in a "quality factor" $Q = \omega_0/\gamma$.

- Calculate Q for this oscillator. Q determines the "shape" of the resonance curve as follows: Let the two frequencies for which

$$A = \frac{1}{\sqrt{2}} A_r$$

be ω_+ and ω_-. Then the half width of the resonance curve, defined as $\omega_+ - \omega_- = \Delta\omega$, is approximately given by $\Delta\omega \cong \gamma$. Compare the theoretical half width of your resonance curve with the experimental value.

Reference

1. D. Kleppner and R.J. Kolenkow, *An Introduction to Mechanics.* McGraw Hill, 1973. Chapter 10.