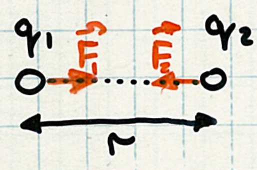
Basic physical concepts

Charge



$$F = K\frac{q_1 q_2}{r^2} \qquad K = 9.0\times10^{9}\ Nm^2C$$

Coulomb's Law

1 C = a very large charge $\qquad \bar{e} = -1.6\times10^{-19}C$

a historical convention

Current

= movement of charge , $I = \dfrac{dq}{dt}$

1A = 1C/1s , by definition

DC = direct current; entirely concerned with BULK movement of electrons

Note : electric current opposite to movement of $\bar{e}$'s

Metals : a "sea" of free electrons moving in a crystal lattice, accelerated by voltage, impeded by collisions

Cu : $8.5\times10^{22}\ \bar{e}/cm^3$ ⇒ electrons in a metal need to move very little to create a large flow of charge, a large current

Ex : - thermal motion of $\bar{e}$ @ room temp.: $\sim 10^5\ \frac{m}{s}$

- drift velocity due to I = 1A : $\sim 10^{-5}\ \frac{m}{s}$

Ex Calculate how many $\bar{e}$'s flow per second past a fixed point in a copper wire carrying 1A of current.

If the current moves from left to right, which way do the electrons move?

Ex How fast does the $\bar{e}$ cloud in the wire ($1\,mm^2$ cross-section) have to move to produce the current of 1A?

$\boxed{\text{Voltage}}$ = energy (work) involved in moving charge

$$\underset{\text{Voltage}}{V} \equiv \frac{W_{\leftarrow \text{work}}}{q} \quad , \quad \underset{\text{Volt}}{1V} = 1J/1C$$

Also, $\frac{d}{dt}\underset{\uparrow \text{work}}{W}$ = power $= V\frac{dq}{dt} = VI$, for V = const

or $\underset{\uparrow \text{Watt}}{1\,W} = 1V \times 1A$

Capitalize A, V, W → mA, μV, MW

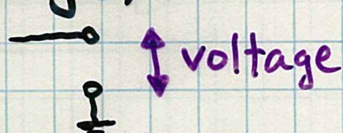
Def-n of voltage requires a reference point (i.e. moving charge <u>from where</u>?) ⇒ concept of

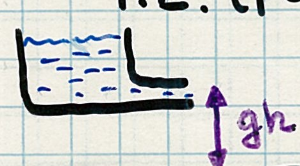
$\boxed{\text{Ground}}$ 

Other names for voltage:
- "potential" (w.r.t. ground)
- "potential difference" (between two points)
- "electromotive force", emf

<u>Analogy with water flow</u>

Charge, C	Mass, kg
Current, $Cs^{-1} = A$	Mass flow, $kg\,s^{-1}$
Voltage, $V = JC^{-1}$	P.E. (per unit mass), $J\,kg^{-1}$



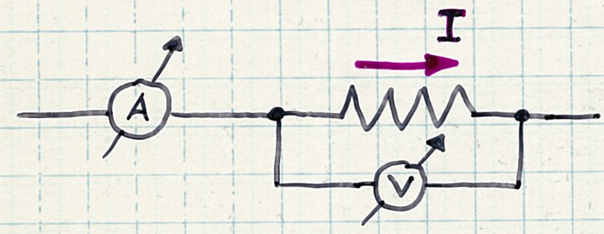


RESISTANCE

Drift velocity of charges in a conductor is limited by "frictional" effects:

$$I = \frac{V}{R}$$ Ohm's Law.

$$\Rightarrow \quad R = \frac{V}{I}$$



$$1\,\Omega = \frac{1\,V}{1\,A}$$

Resistance is a property of the conducting material:

$$R = \rho\, L/A$$

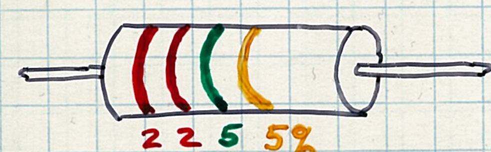
L = length
A = cross-sectional area
ρ = resistivity

Ex. Ag : $\rho = 1.5 \times 10^{-8}\ \Omega m$ — C : $\rho = 350 \times 10^{-8}\ \Omega m$

Magnitude of ρ determines whether the material is a conductor, an insulator, or a superconductor.

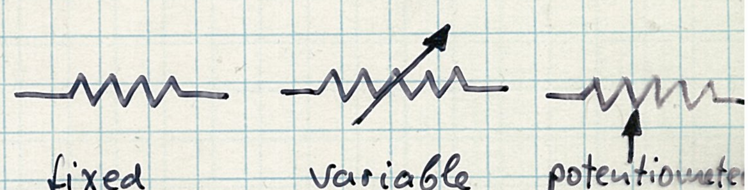
DH 1.7

<u>Colour coding</u>:

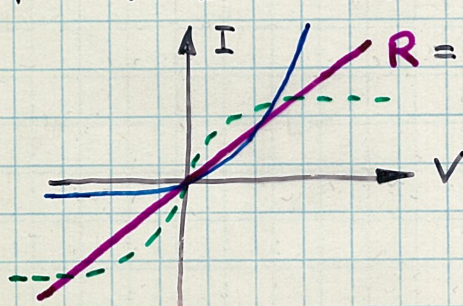


$\rightarrow$ 2200000
$= 2.2\ M\Omega \pm 5\%$

<u>Schematic representation</u>:



<u>Note</u>: R is never totally constant!



$R = \frac{V}{I} = \text{const}$

$\leftarrow$! Other V-I characteristics are possible

In general, $R = R(V, I, t, \ldots)$

POWER

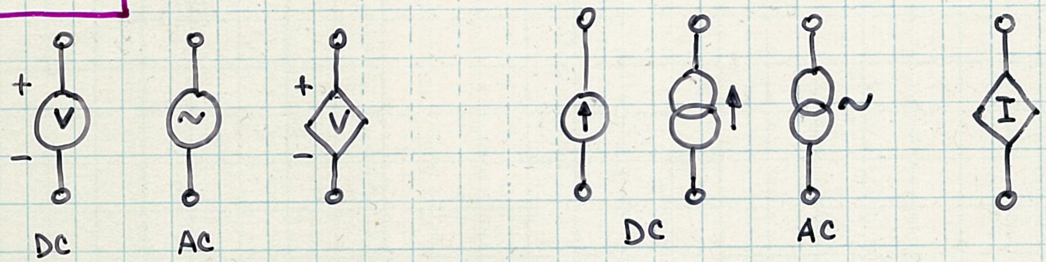
$$P = IV = I^2R = \frac{V^2}{R}$$

$$1W = 1A \cdot 1V$$

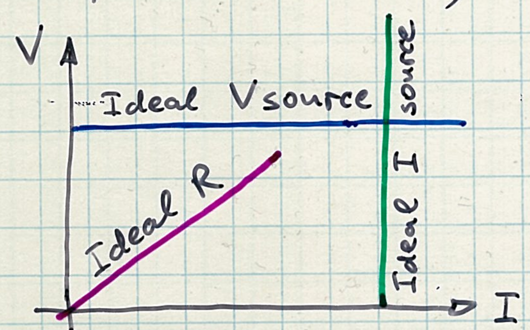
Dissipation: heat, light, sound

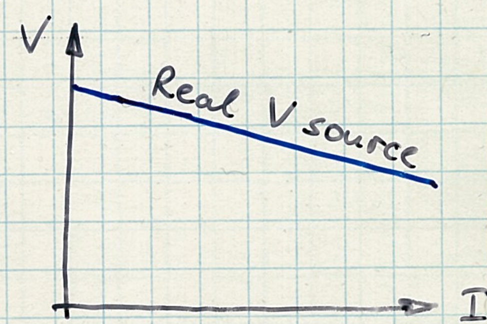
Rate: size, surrounding medium

SOURCES

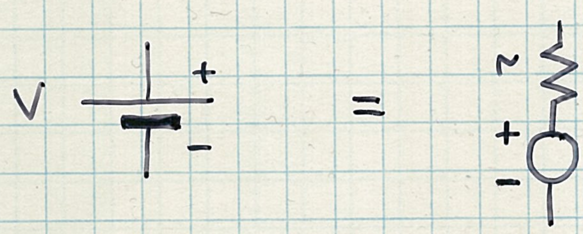


○ = independent (ideal) ◇ = dependent (e.g. battery)



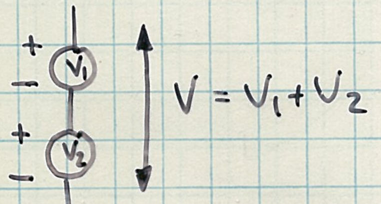


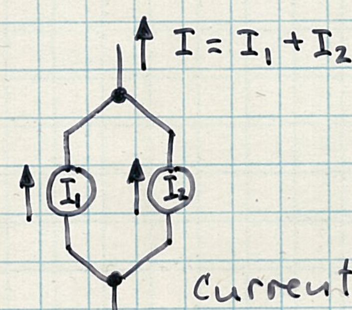
Representation of a dependent element via <u>lumped</u> parameter:



r = internal resistance

<u>Adding sources</u>:





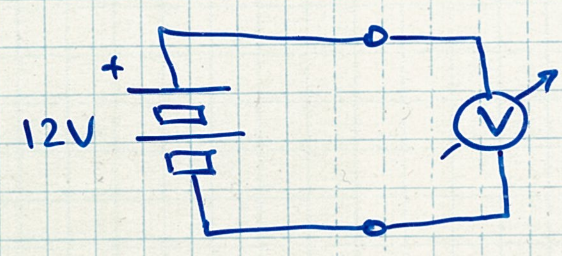
Voltage sources add in series

Ideal voltage sources cannot be added in parallel, unless $V_1 = V_2$

Current sources add in ||

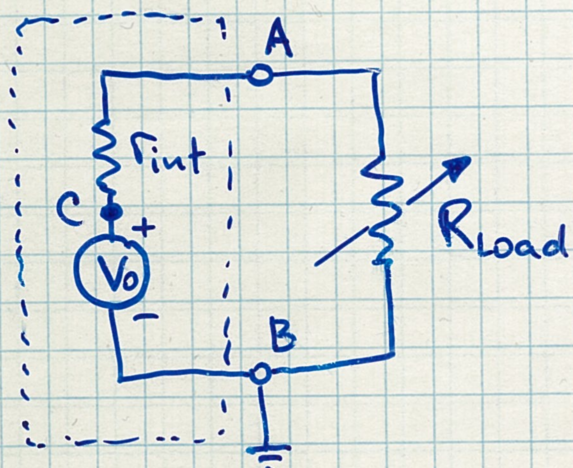
Ideal current sources cannot be added in series, unless $I_1 = I_2$

Ex What happens if an ideal 0-15V dc voltmeter is connected across a 12V car battery?



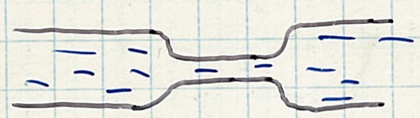
What happens if the voltmeter is replaced with an ammeter?

Ex. Check if the following lumped circuit is adequate to model a real voltage source



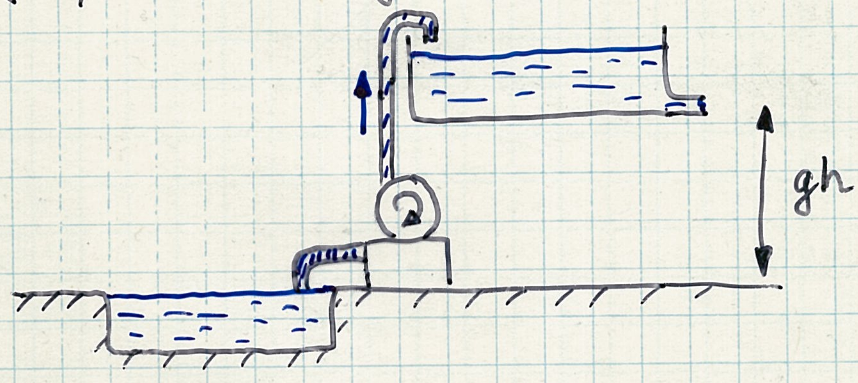
Water-flow analogy

Resistance :



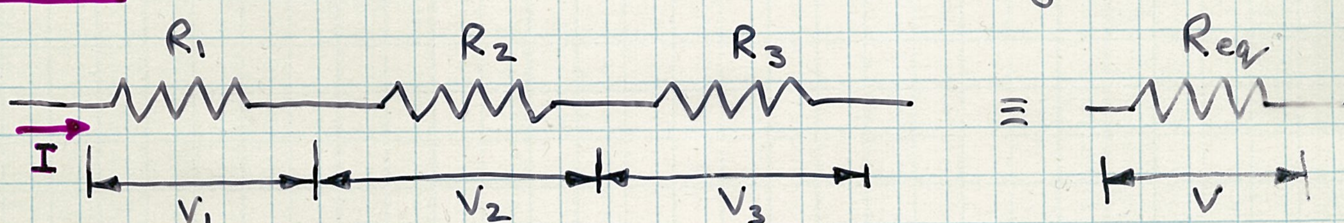
Ideal wires, $R = \phi$ $\longleftrightarrow$ large diameter pipes

Sources :



Circuit reduction

Series : the current is the same in every component



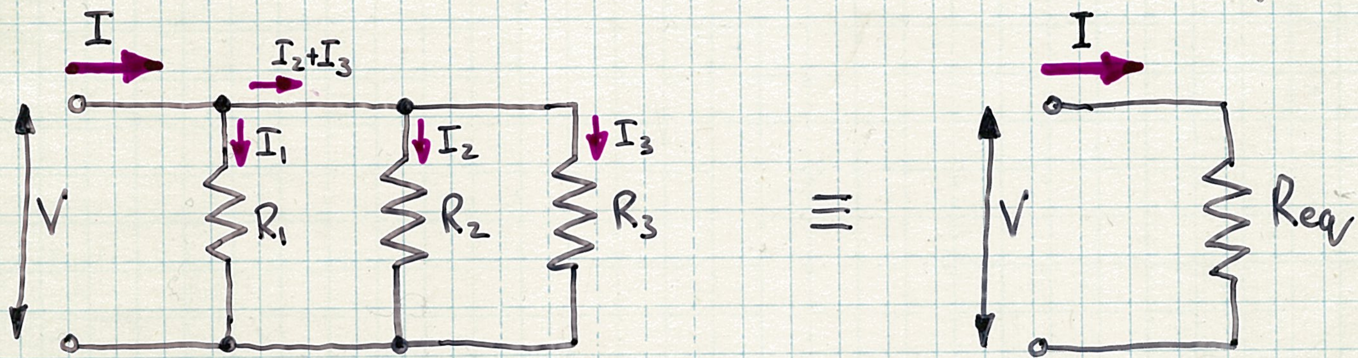
$$V = V_1 + V_2 + V_3 =$$
$$= IR_1 + IR_2 + IR_3 =$$
$$= I(R_1 + R_2 + R_3)$$
$$= I R_{eq}, \quad \text{where} \quad R_{eq} = R_1 + R_2 + R_3$$

$$\boxed{R_{eq} = \sum_i R_i}$$ for a series circuit

Voltage divider DH 1.9

$$V_1 = IR_1 = \frac{V}{R_{eq}} R_1 = \frac{R_1}{R_1 + R_2 + R_3} V$$

Parallel: the voltage is the same across every component



$$I = I_1 + I_2 + I_3 =$$
$$= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} =$$
$$= V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$
$$= \frac{V}{R_{eq}}$$

$$\boxed{\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}}$$

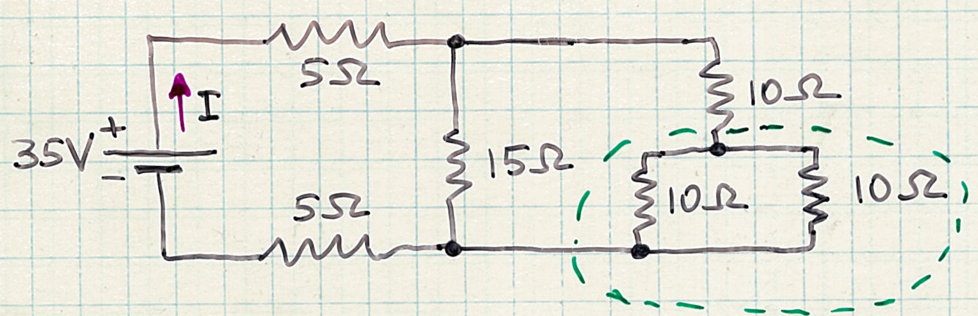
for a parallel circuit

Current divider

$$I_1 = \frac{V}{R_1} = \frac{I\,R_{eq}}{R_1} = I\,\frac{1/R_1}{1/R_{eq}} = I\,\frac{1/R_1}{1/R_1 + 1/R_2 + 1/R_3}$$

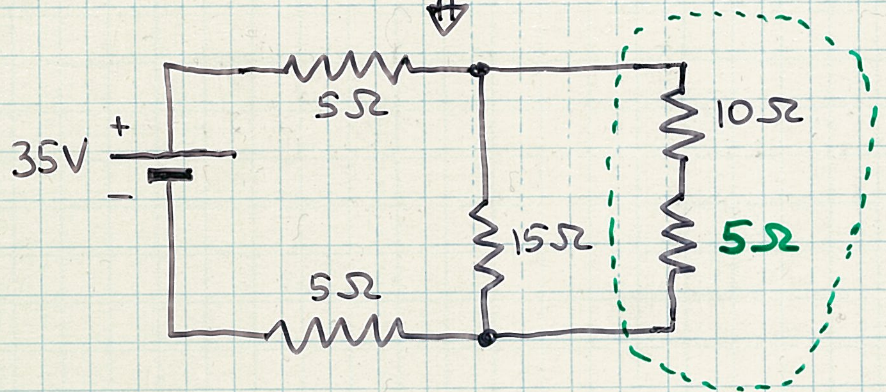
- current divides according to the inverse ratio of resistances.
- $1/R \equiv G$ conductance $[G] =$ mho $=$ siemens $= \Omega^{-1}$
- Complex circuits can be reduced by successive application of the above two rules.

Ex. Complex circuit reduction

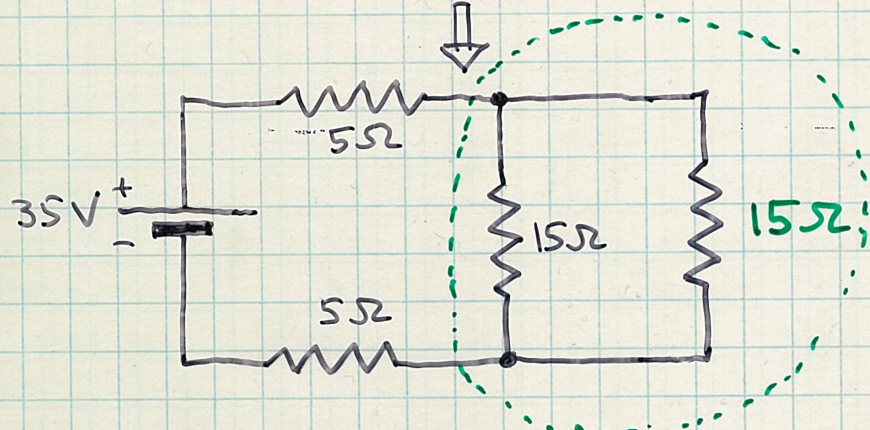


$$\frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10}$$

$$R_{eq} = 5\,\Omega$$

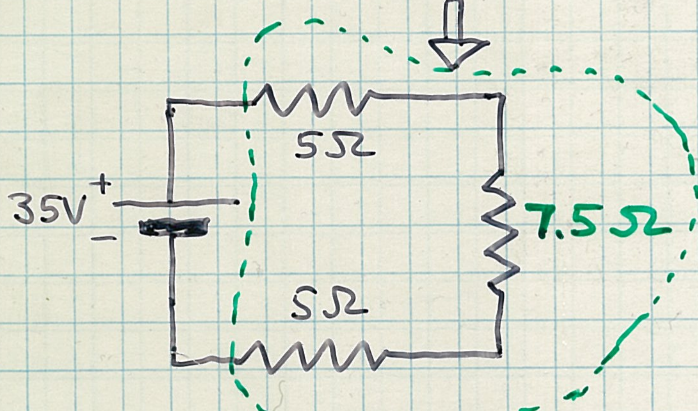


$$R_{eq} = 10 + 5 = 15\,\Omega$$

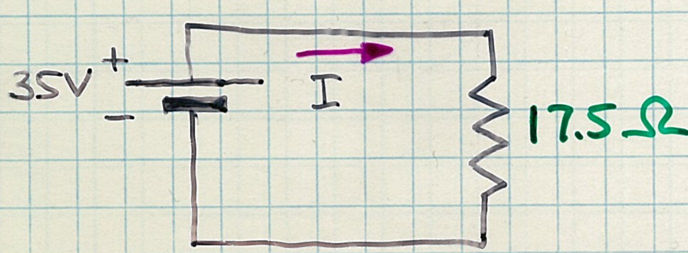


$$\frac{1}{R_{eq}} = \frac{1}{15} + \frac{1}{15} = \frac{2}{15}$$

$$R_{eq} = 7.5\,\Omega$$



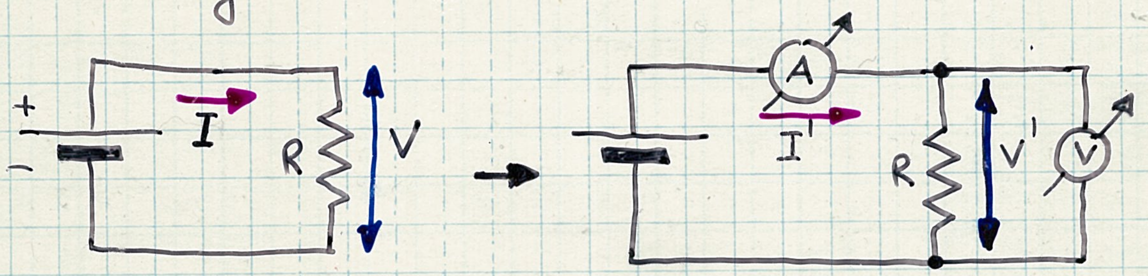
$$R_{eq} = 5 + 7.5 + 5 = \mathbf{17.5}\,\Omega$$



$$I = \frac{V}{R_{eq}} = \frac{35\,V}{17.5\,\Omega} = 2A$$

Meters

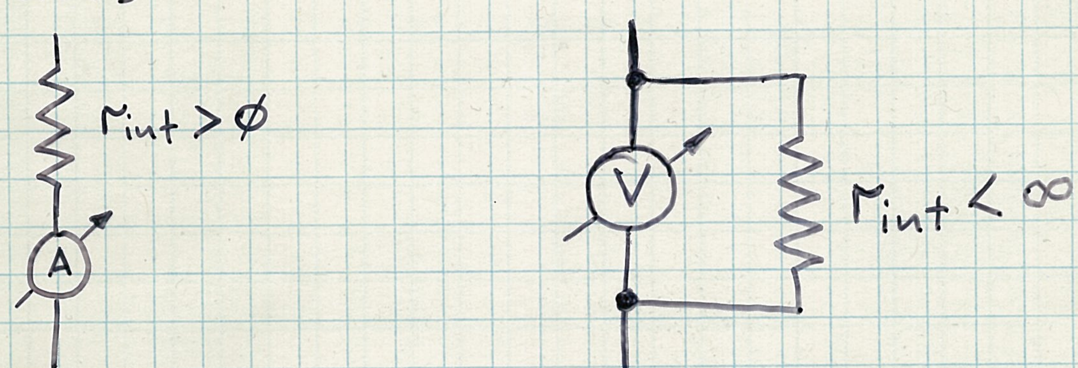
- Ideal meters do not disturb the circuits they are measuring



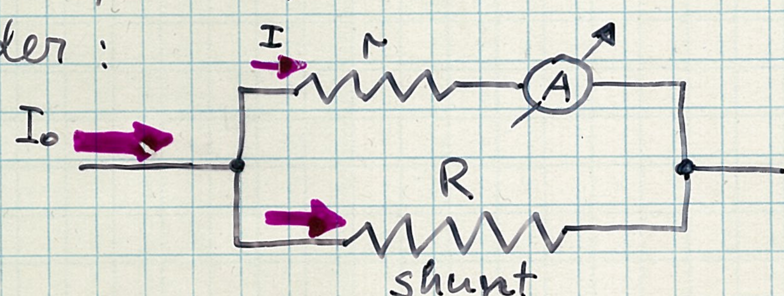
$$V = V'$$
$$I = I'$$

⇒ ideal ammeter has no resistance, $r_{int} = \emptyset$ } consume
ideal voltmeter has ∞ resistance, $r_{int} = \infty$ } NO power

- real meters



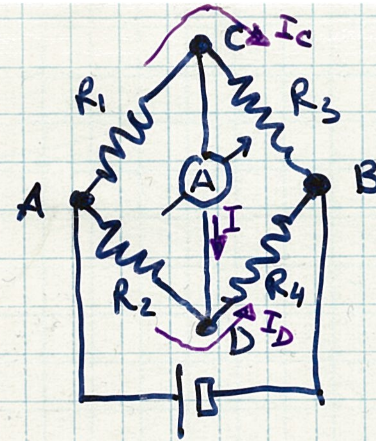
- measuring large currents: for large I, even a small resistance r_{int} will dissipate a lot of power, I^2R. ⇒ need to use a current divider:



$$I = I_0 \frac{R}{R+r}$$

$$R << r$$

Wheatstone bridge



Ex Null measurement

Adjust R_1, R_2, R_4 until $I = \phi$

Then: $V_c = V_D$ or $V_c - V_B = V_D - V_B$

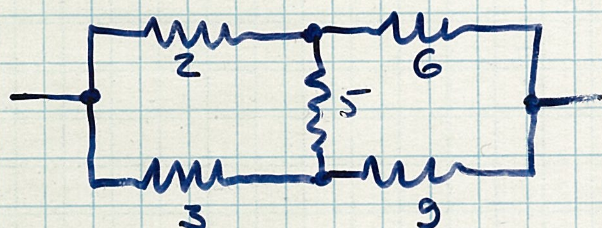
$I_c R_3 = I_D R_4$

or $V_A - V_C = V_A - V_D$

$I_c R_1 = I_D R_2$

$\Rightarrow \dfrac{R_1}{R_2} = \dfrac{R_3}{R_4}$ unknown.

Ex.



what is the current in the 5Ω resistor

Ex Can we