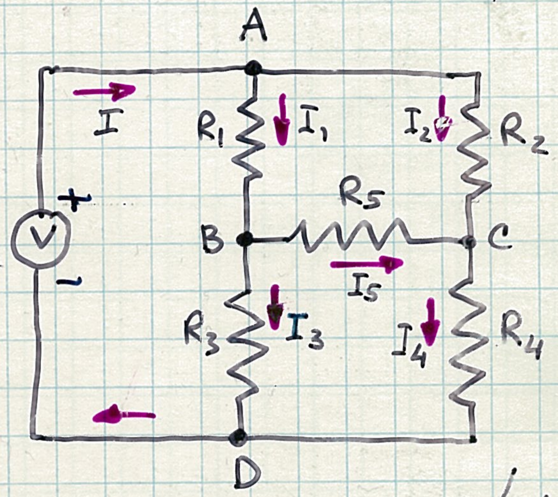
Kirchhoff's Laws

- Kirchhoff's Rules: the underlying physical Laws are conservation of energy and charge.
- need KL for circuits that cannot be reduced by simple parallel/series pairing.
- will apply, but reduce to trivial cases in those circuits.

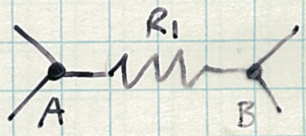
Wheatstone bridge:

! cannot be reduced !



NODE: ≥ 3 circuit elements connected

BRANCH: an element that connects two adjacent nodes



LOOP: a path that begins at one node, passes one or more nodes, and ends at the node it started

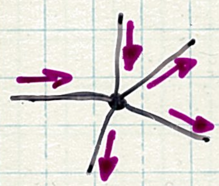
ABDCA
ABCA

MESH: a loop containing no branches in its interior (= elementary loop)

ABCA
BCDB

- above:
  4 nodes
  6 branches
  7 loops; of these:
  3 meshes

$\boxed{\text{KCL}}$ Σ currents flowing into a node is zero

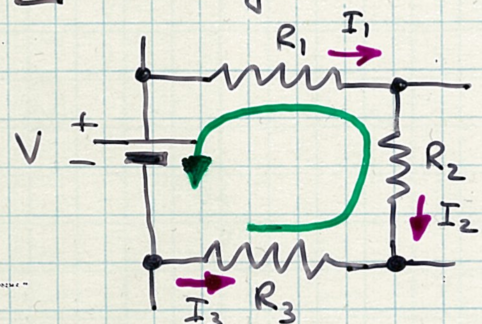


in : +ve
out: −ve

$$\sum_i I_i = 0.$$

- expresses the conservation of charge
- water-flow analogy : no leaks (cons. of mass)

$\boxed{\text{KVL}}$ Σ voltage drops around a loop is zero



$$+I_1R_1 + I_2R_2 - I_3R_3 - V = 0$$

watch the signs !

- expresses the conservation of energy
- a form of the Ohm's Law ; reproduces $V = IR$

Ex. The Wheatstone bridge circuit

- KCL → 4 equations , one for each node

$\underline{A}$: $I = I_1 + I_2$ (1) $\underline{B}$: $I_1 = I_3 + I_5$ (2) $\underline{C}$: $I_2 + I_5 = I_4$ (3) $\underline{D}$: $I_3 + I_4 = I$

Note: not independent, add the first 3 to get the 4th

- KVL → 3 independent equation, one for each mesh

$$I_1R_1 + I_3R_3 - V = 0 \qquad (4)$$
$$I_1R_1 + I_5R_5 - I_2R_2 = 0 \qquad (5)$$
$$I_3R_3 - I_4R_4 - I_5R_5 = 0 \qquad (6)$$

Note: other loops generate equations that can be obtained by combining these three; e.g.

ABCDA : $I_1 R_1 + I_5 R_5 + I_4 R_4 - V = \emptyset$

is (4) − (6)

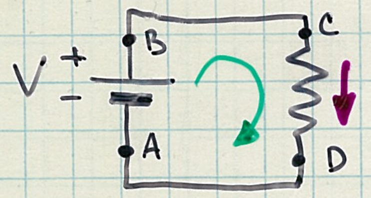
- # of unknowns = # of branches

$\Rightarrow$ 6 equations, 6 unknowns

- Always : # equations = (# nodes − 1) + (# meshes)

Getting the signs right

- for a source, $V_+ > V_-$
- for a resistor, the current flows from higher to lower potential.



① start at A. $V_B > V_A \Rightarrow$ the source will contribute $+V$ as we go from A to B.

② $V_B = V_C \Rightarrow$ contribution $= \emptyset$

③ $V_D < V_C \Rightarrow$ contribution is $-IR$

④ $V_D = V_A \Rightarrow$ contribution is $= \emptyset$

KVL: $\sum_i V_i = \emptyset$

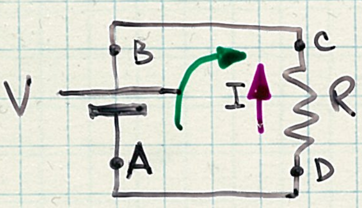
$+V - IR = \emptyset$

- could have chosen to go around the loop in the opposite direction, against the current:

AD: $\emptyset$ DC: $+IR$ CB: $\emptyset$ BA: $-V$ $\Rightarrow +IR - V = \emptyset$

$\Rightarrow$ identical result! $IR = V$, Ohm's law.

"Wrong" current direction

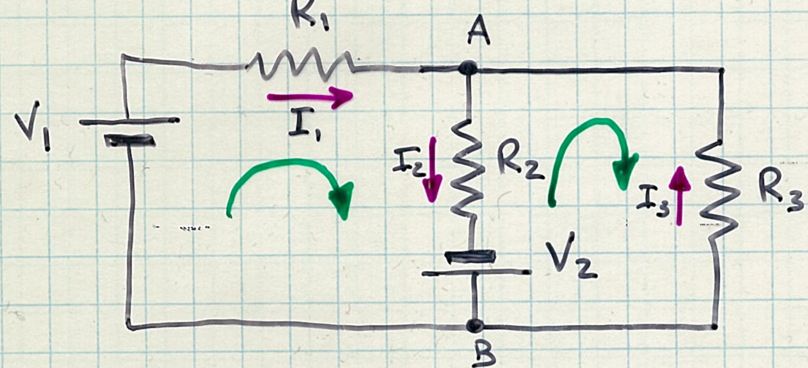


AB : $+V$
BC : $\emptyset$
CD : $+IR$, since $V_C < V_D$ according to our choice of direction of I
DA : $\emptyset$

KVL: $\sum_i V_i = \emptyset = +V + IR \Rightarrow I = -\frac{V}{R}$!

The minus sign means that the actual direction of I is opposite to the one chosen.

Ex.



See also: DH. Ex. 1.3

KCL @ A: $+I_1 - I_2 + I_3 = \emptyset$
KCL @ B: $-I_1 + I_2 - I_3 = \emptyset$
} the same eq-n! (1)

KVL @ LHS loop: $+V_1 - I_1 R_1 - I_2 R_2 + V_2 = \emptyset$ (2)

KVL @ RHS loop: $-V_2 + I_2 R_2 + I_3 R_3 = \emptyset$ (3)

$\Rightarrow$ 3 eq-ns, 3 unknowns (I_1, I_2, I_3)

Note: (2) + (3) yields

$+V_1 - I_1 R_1 - \cancel{I_2 R_2} + \cancel{V_2} - \cancel{V_2} + \cancel{I_2 R_2} + I_3 R_3 = 0$

$+V_1 - I_1 R_1 + I_3 R_3 = 0$ the big loop!

• N ... one current

fl ...

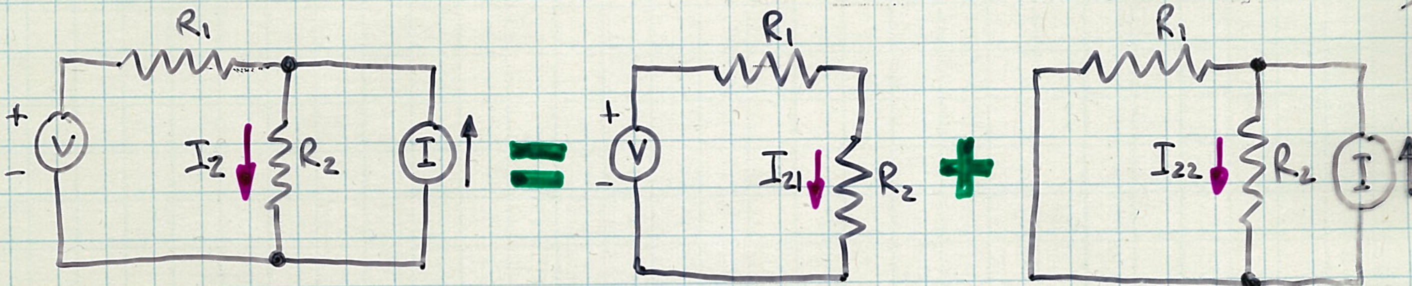
] cf. the answer given in F.

He ...

The principle of SUPERPOSITION

A current in a branch of a linear circuit is equal to the sum of the currents produced by each source individually, i.e. with other sources set to zero.

- ideal sources, R (linear VI-characteristics)
  Kirchhoff's Laws → linear equations
- setting sources to zero:
  Voltage source → short circuit (NO voltage)
  current source → open circuit (NO current)

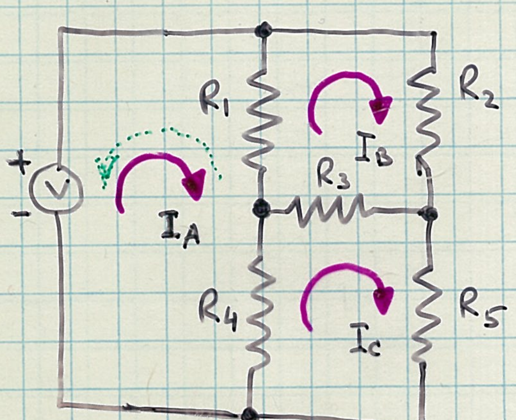


$$I_2 = I_{21} + I_{22}$$

DH.1.10

Loop current technique

Maxwell's method
Mesh equation method



- Closed loops: KCL is automatically satisfied (every loop current into a node also flows out of that node)
- apply KVL to each mesh

WATCH SIGNS!

Loop A: $(I_A - I_B)R_1 + (I_A - I_C)R_4 - V = \phi$

B: $(-I_A + I_B)R_1 + I_B R_2 + (I_B - I_C)R_3 = \phi$

C: $(-I_A + I_C)R_4 + (I_C - I_B)R_3 + I_C R_5 = \phi$

In matrix form:

$$\begin{pmatrix} R_1 + R_4 & -R_1 & -R_4 \\ -R_1 & R_1 + R_2 + R_3 & -R_3 \\ -R_4 & -R_3 & R_3 + R_4 + R_5 \end{pmatrix} \begin{pmatrix} I_A \\ I_B \\ I_C \end{pmatrix} = \begin{pmatrix} V \\ \phi \\ \phi \end{pmatrix}$$

General (vector) form of the Ohm's Law:

$$\mathbb{R}\,\vec{I} = \vec{V} \Leftrightarrow \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

with

- $R_{11} = \Sigma$ of all resistances in which mesh current I_A flow

  $R_{12} = \Sigma$ — " — — " —— " —— " —— I_A & I_B flo

  R_{12} is +ve if I_1, I_2 in the same direction
  -ve if I_1, I_2 in the opposite directions.

  $R_{13} = \Sigma$ — " —— " —— " —— " —— " — I_A & I_C — "

  etc.

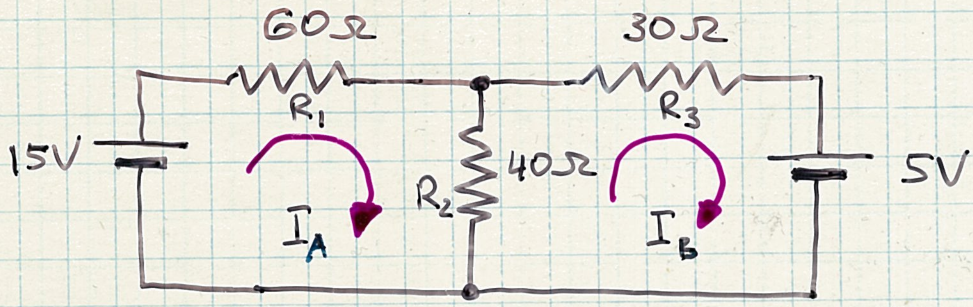
- Resistance matrix $\mathbb{R}$ is symmetric: $R_{ij} = R_{ji}$

- $V_i = \Sigma$ of all voltages driving I_i;

  V_i is +ve if in direction of I_i, -ve if opposite

- unknowns: $\vec{I} = \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}$ solve by matrix methods

Ex.



Mesh A : $15 = 60 I_A + 40 (I_A - I_B) = 100 I_A - 40 I_B$ (1)

B : $-5 = 30 I_B + 40 (I_B - I_A) = -40 I_A + 70 I_B$ (2)

(1)×0.4 +(2): $(40 I_A - 16 I_B) + (-40 I_A + 70 I_B) = 15 \times 0.4 - 5$

Through R_3 : $54 I_B = 1 \Rightarrow I_B = 0.0185 A$

Through R_1 : $40 I_A = 5 + 70 I_B \Rightarrow I_A = 0.157 A$

Through R_2 : $I_A - I_B = 0.139 A$

- Note : $I_A - I_B > 0$ determines the true direction of the current in R_2.

• "straight" KL method	Mesh method
- KCL: (# of nodes) − 1 eq/s KVL: (# of meshes) eqs.	− (# of meshes) eq-s
- Directly obtain currents in each branch.	− Obtain loop currents; branch currents are linear combinations
More equations, straightforward answers	Fewer equations, answers need some interpreting