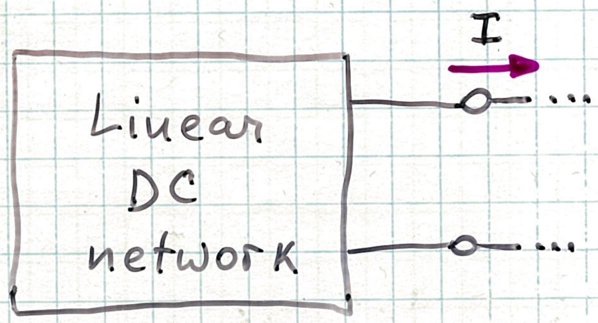
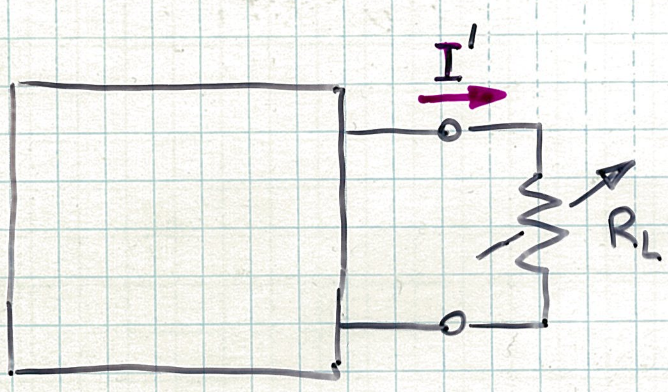
Equivalent Circuits



Consider a "black box" containing a combination of many linear elements (resistors, sources)

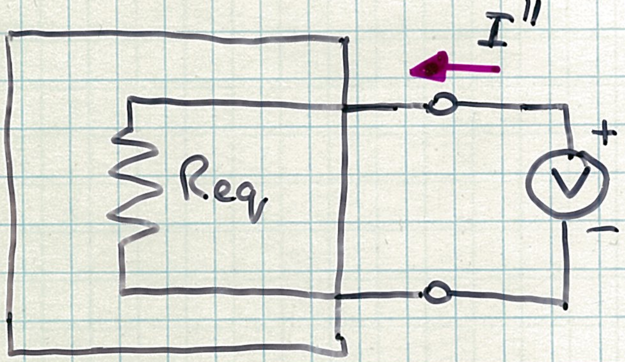


Connect to a load: Current I' is due to internal sources only

As $R_L \to \emptyset$, $I' \to I_{sc}$

Open up the box, turn off all the sources. What is left is a combination of R's only

$\Rightarrow$ use our circuit reduction rules to find R_{eq}



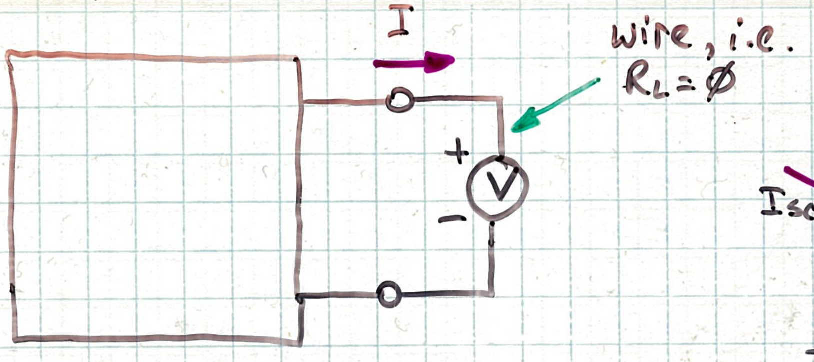
Connect to a V source: Current I'' is due to external source(s) only

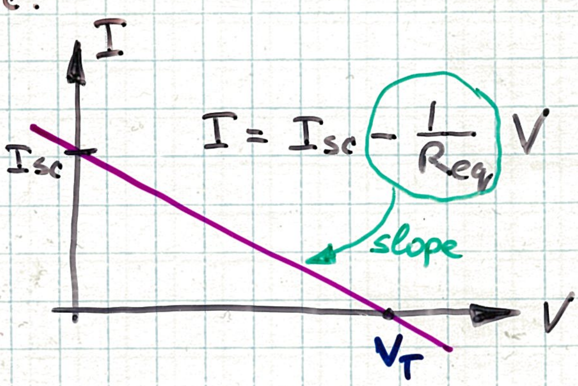
$$I'' = -\frac{V}{R_{eq}}$$

Turn internal sources back on. By principle of superposition:

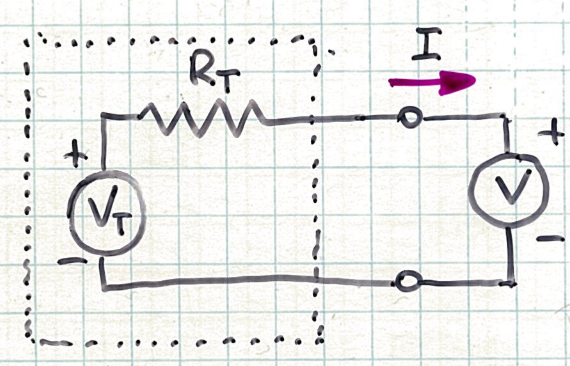
$$I = I_{sc} - \frac{V}{R_{eq}}$$

due to: internal & external sources





Design a circuit that has exactly this VI behaviour:



- $V = V_T \longleftrightarrow I = \emptyset$
- $V = \emptyset \longleftrightarrow I = \dfrac{V_T}{R_T}$
  (short circuit)

Choose V_T, R_T so that : $\begin{cases} R_T = R_{eq} \\ I = \dfrac{V_T}{R_T} = I_{sc} \end{cases}$ from before

If we now replace the "black box" (many components!) with the above $V_T + R_T$ circuit,

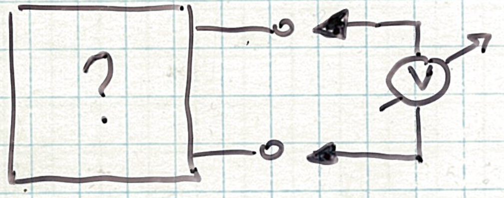
there is no way to distinguish the two!

$\boxed{\text{Thévenin's theorem}}$

Any linear, two-terminal, DC network can be represented by a single voltage source in series with a resistor

In practice:

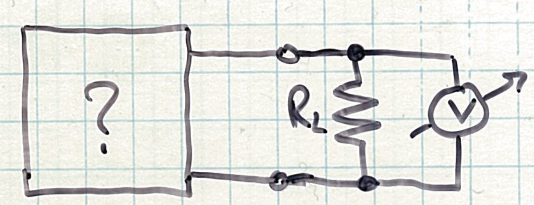
- open-circuit voltage, V_{oc}:



ideal voltmeter, $I \simeq \emptyset$:

$$V_{oc} = V_T - IR_T \simeq V_T$$

- instead of short-circuit current, I_{sc}:

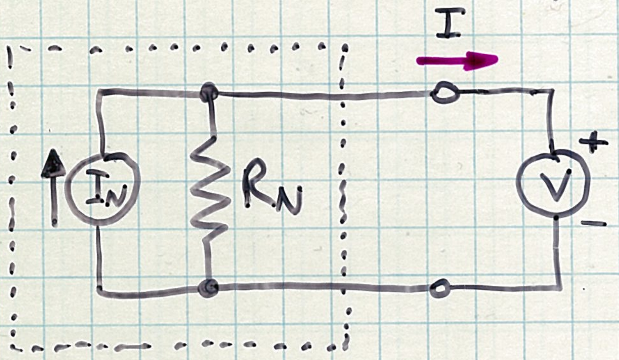


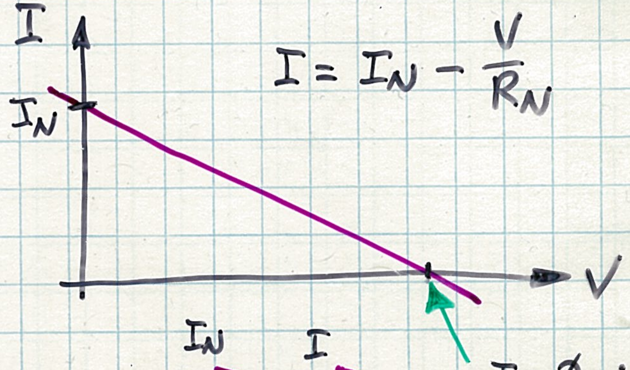
vary R_L until $V = \frac{1}{2} V_{oc}$:

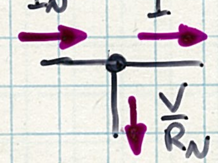
$$\underbrace{\frac{1}{2} V_{oc}}_{V_T} = IR_L = \frac{V_T}{R_T + R_L} R_L$$

$$\Rightarrow \quad R_T + R_L = 2R_L \quad \Rightarrow \quad R_T = R_L$$

Another circuit is possible:







$I = \emptyset$, i.e. open circuit

$I_N = \frac{V_{oc}}{R_N}$

Use KCL: $I_N - I - \frac{V}{R_N} = \emptyset$

Choose I_N, R_N: $\begin{cases} R_N = R_{eq} \\ I_N = \frac{V_{oc}}{R_N} \end{cases}$

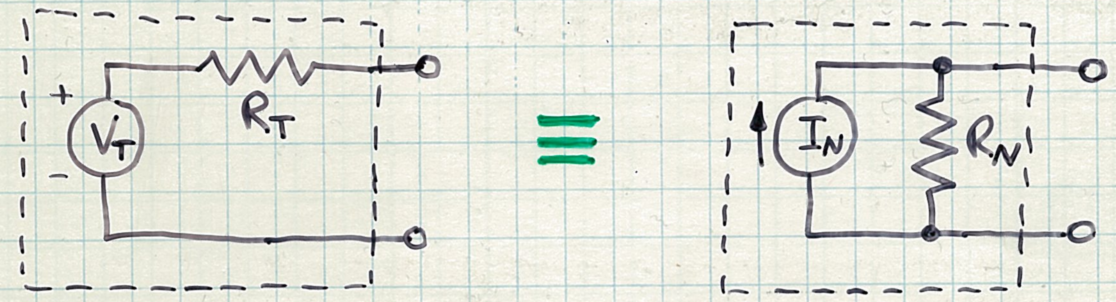
Again, our $I_N + R_N$ circuit behaves exactly like the original "black box"

Norton's Theorem

Any linear, two-terminal, DC network can be represented by a single current source in parallel with a resistor.

The two theorems are obviously related!

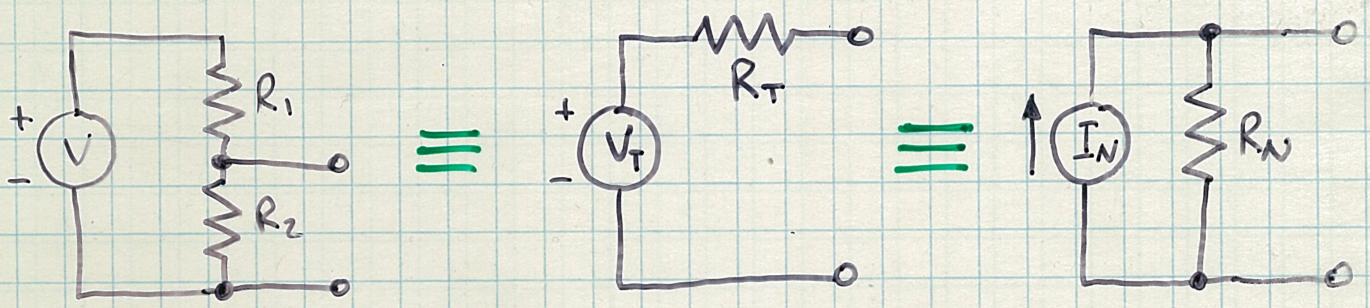
EFTS : apply Norton's theorem to Thévenin's equivalent circuit



with : − Norton equivalent current, $I_N = I_{sc}$

− Norton equivalent resistance, $R_N = V_{oc}/I_{sc} = R_T$

Ex. Voltage divider

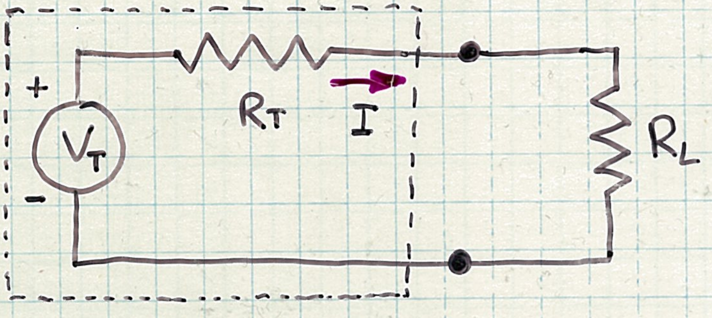


- oc voltage $\quad V_T = V_{oc} = V \dfrac{R_2}{R_1+R_2} \qquad V_{oc} = I_N R_N$
- sc current ($R_2 = \emptyset$) $\quad I_{sc} = \dfrac{V}{R_1} \qquad I_{sc} = I_N$

$\Rightarrow \quad R_T = V_T / I_{sc} = \dfrac{R_1 R_2}{R_1+R_2} = R_N$

! • power $\quad P_{oc} = \dfrac{V^2}{R_1+R_2} \qquad P_{oc} = \emptyset \qquad P_{oc} = \dfrac{V^2 R_2}{R_1(R_1+R_2)}$

Ex. Maximum power transfer



Power delivered to load R_L:

$$P = I^2 R_L = \left(\frac{V_T}{R_T + R_L}\right)^2 R_L$$

$$\Rightarrow \quad P = \frac{V_T^2/R_L}{(1 + R_T/R_L)^2}$$

$\rightarrow \emptyset$ as $R_L \to \infty$, $R_L \gg R_T$

$\rightarrow \emptyset$ as $R_L \to \emptyset$, $R_L \ll R_T$

Maximum in between:

$$\frac{dP}{dR_L} = \emptyset = \frac{V_T^2}{R_L} \frac{-2(-R_T/R_L^2)}{(1 + R_T/R_L)^3} + \frac{V_T^2}{(1 + R_T/R_L)^2} \frac{-1}{R_L^2}$$

$$\Rightarrow \quad \emptyset = \frac{V_T^2}{R_L^2 (1 + R_T/R_L)^3} \left[\frac{2R_T}{R_L} - \left(1 + \frac{R_T}{R_L}\right) \right]$$

$$\Rightarrow \quad 2\frac{R_T}{R_L} = 1 + \frac{R_T}{R_L}$$

$$\Rightarrow \quad \frac{R_T}{R_L} = 1$$

$$\boxed{R_L = R_T}$$

"Impedance matching" condition

- maximum power delivered to the load when the load resistance matches the internal resistance of the network.
- the best power transfer: $P = \frac{V_T^2}{4R_L} = P_{internal}$

$\Rightarrow$ transfer efficiency $\varepsilon = 50\%$