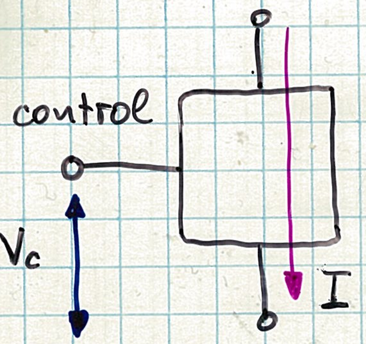
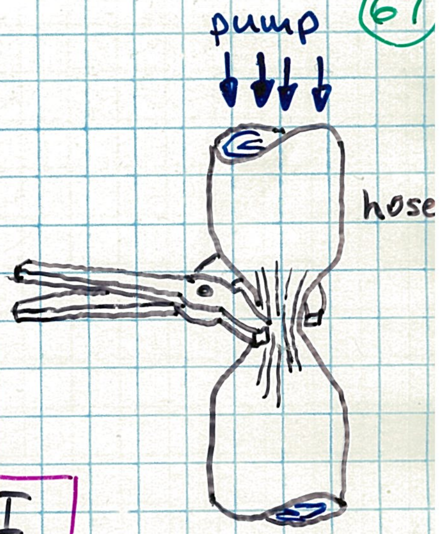
General amplifier theory.



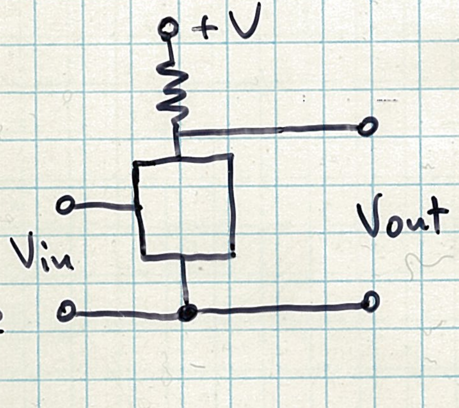
small V_c controls large I (I supplied by an external source)



Mutual transconductance $\boxed{g_m \equiv \dfrac{\Delta I}{\Delta V_c}}$

The control device can be a transistor or a vacuum tube (British: "valve").

Typical use



= Amplifier

$\boxed{G_v = \dfrac{\Delta V_{out}}{\Delta V_{in}}}$

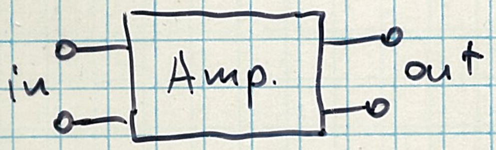
only signals, no biases or offsets

need also:
- coupling C
- bias voltages
- in/out impedance matching

Assuming that we can take care of the technical details – why use amplifiers?

- detect weak signals
- use as controllers (small $\Delta V_{in} \rightarrow$ large ΔV_{out})
- perform mathematical operations (op-amps)

Some common features of all active circuit elements.



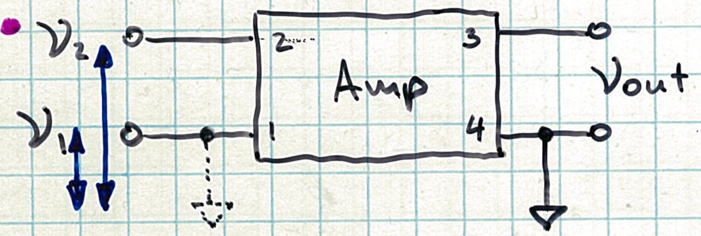
= a "black box" approach

Operational amplifiers

Basic terminology

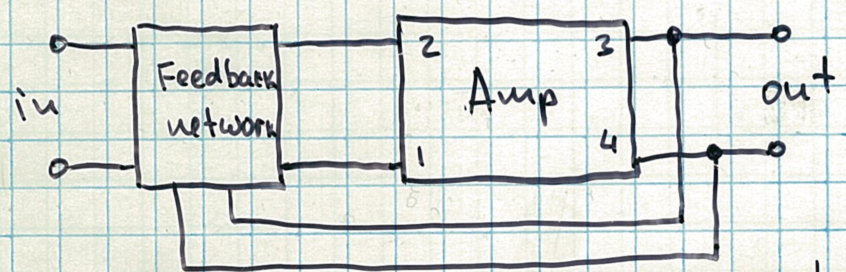
- Gain (voltage or current) $G_v = \frac{V_{out}}{v_{in}}$ ← signals only

  G, in general, is a complex number, i.e. there may be a change in phase as well as amplitude
- input impedance : how loads up the source

  output impedance : $\leftarrow Z = Z_{Th}$ of the "black box"
- $G = G(\omega)$, frequency response of an amp
- $|G|$ may be < 1 ⇒ attenuators
- 

  In general, 1 & 4 are not necessarily the same (not referenced to ground)

  ⇒ $V_{out} \propto (v_2 - v_1)$

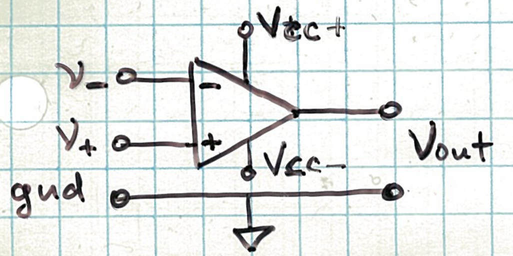
  ⇒ difference amplifier
  differential inputs
- 

  Part of the output diverted back to input — feedback

negative feedback : higher output suppresses further increases

positive feedback : higher output stimulates further increases

Ex. a typical op-amp (741)



$V_+ > V_- \Rightarrow V_{out}$ positive

$V_- > V_+ \Rightarrow V_{out}$ negative

typical $R_{in} \sim 2M\Omega$ $\quad R_{out} \sim 75\Omega$

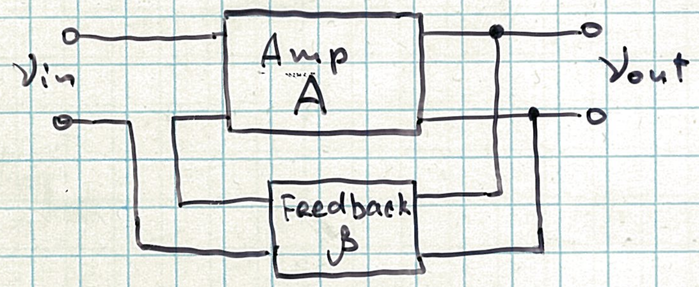
$V_{cc+} \simeq +15V$ $\quad V_{cc-} \simeq -15V$

open loop gain (no feedback) $A \sim 103dB$ (nominal)

frequency response : flat ϕ (DC) — 100 Hz

decreases to $A \sim 1$ at $\sim 1MHz$

The importance of feedback



β = feedback fraction

$$V_{out} = A\left(V_{in} + \beta V_{out}\right)$$

$$\Rightarrow V_{out} = \frac{A}{1-\beta A} V_{in}$$

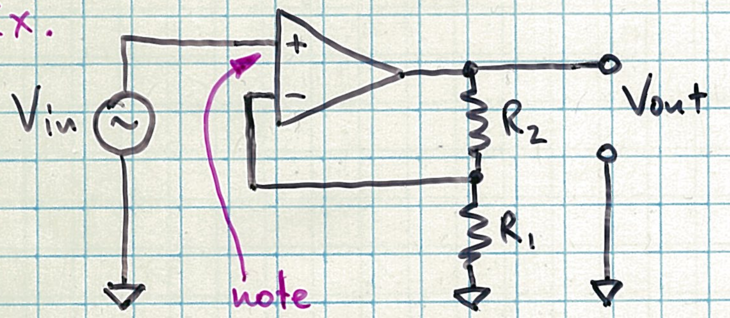
For $\beta A \gg 1$: $\boxed{V_{out} \simeq -\frac{1}{\beta} V_{in}}$ independent of A !

(i.e. large A)

i.e. the gain of the circuit is completely determined by the feedback network, not by op-amp itself.

$\Rightarrow$ closed loop gain

Ex.



Voltage divider provides the feedback voltage

$$V_{out} \frac{R_1}{R_1+R_2}$$

i.e. $\beta = - \frac{R_1}{R_1+R_2}$

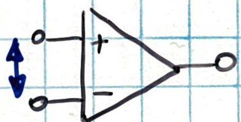
"minus" because connected to the − input of the op-amp

$$\Rightarrow V_{out} = -\frac{1}{\beta} V_{in} = \frac{R_1+R_2}{R_1} V_{in}$$

Ideal and real op-amps

Ideal op-amp value

- **Input offset voltage** — ideal: ∅

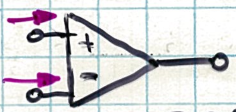


in a real op-amp, the two inputs are not perfectly balanced. I.O.V. is the voltage needed at the input to bring the output to zero.

741: 2mV, slightly temperature-dependent, 3μV/°C

In some op-amps there are extra leads to attach an external balance potentiometers to null the offset voltage

- **Input bias currents** — ideal: ∅



origin: gate leakage currents of FETs, etc.
741: 0.1 μA. I.b.c. are quite sensitive to temperature changes, ~ ×2/10°C. The difference of the two i.b.c. is input offset current

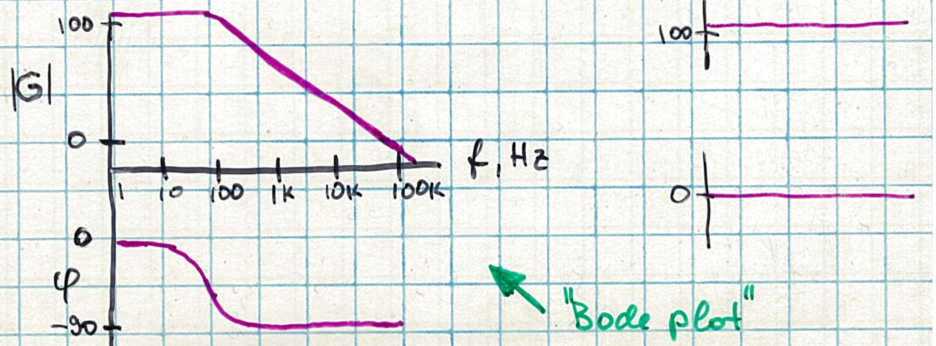
- **Input resistance** $= \frac{i.o.v.}{i.o.c}$ can be $\lesssim 10^{14}\,\Omega$ — ideal: ∞

- **Output resistance**: limits the max power that an op-amp can dissipate ($= V_{out}^2 / R_{out}$) — ideal: ∅

741: 75Ω; some can be $\lesssim 1\Omega$ (high-power)

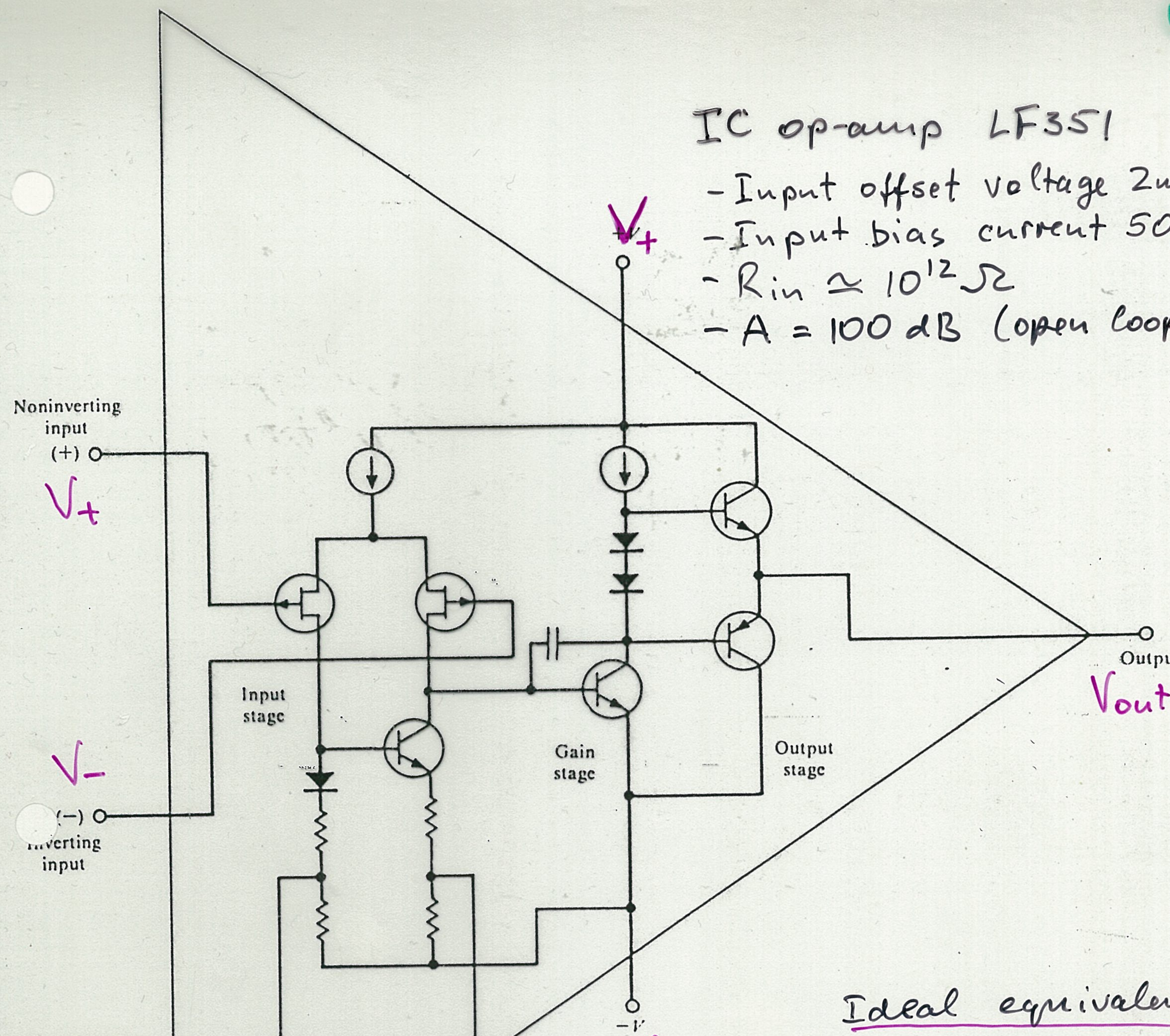
- **Frequency response**

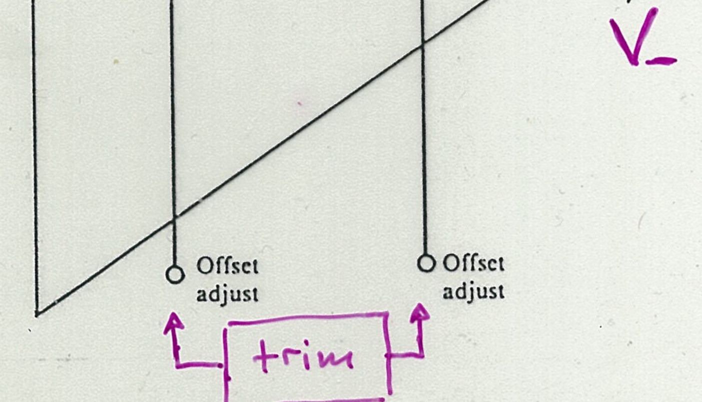
not very good!
for an op-amp in an open-loop config.



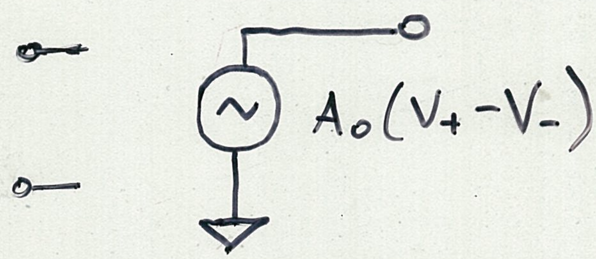
IC op-amp LF351

- Input offset voltage 2mV
- Input bias current 50pA
- $R_{in} \simeq 10^{12}\,\Omega$
- $A = 100$ dB (open loop)

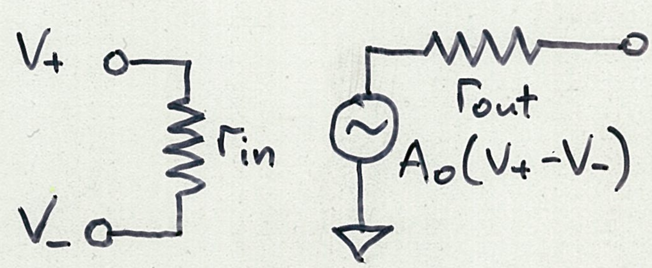




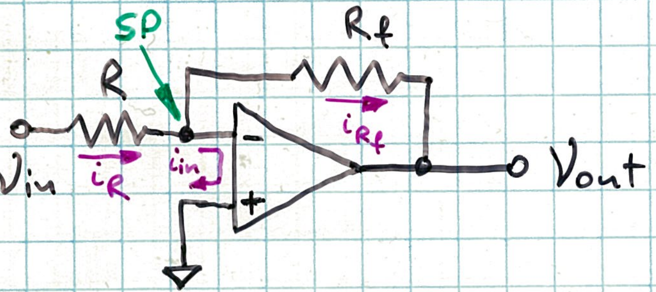
Ideal equivalent



Real equivalent



Analyzing op-amp circuits



Principle: make simplifying assumptions & treat an op-amp as an ideal device. Deal with imperfections later if interested.

① Assume $A = \infty$.

For a circuit with feedback, $\dfrac{v_{out}}{v_{in}} = \dfrac{A}{1-\beta A}$

E.g. $A \simeq 10^{+5}$, $\beta = -0.01 \Rightarrow \dfrac{v_{out}}{v_{in}} \simeq -99.9$

+10%: $A \simeq 1.1 \cdot 10^{+5}$, $\beta = -0.01 \Rightarrow \dfrac{v_{out}}{v_{in}} \simeq -99.89$ (−0.1%)

Closed loop gain is determined mostly by β, not A.

If $A = \infty$ then $V_+ - V_- = \dfrac{v_{out}}{A} = \emptyset$, i.e.

$\boxed{V_+ = V_-}$ ← virtual ground approximation

(−ve input is at ground, see above)

! op-amp with a negative feedback tries to maintain zero voltage across its + & − inputs.

② $i_{in} = \emptyset \Rightarrow R_{in} = \infty$

Input bias current is zero. ⇒ ! op-amp draws no current.

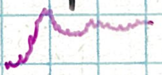
Apply KCL at the summing point (SP):

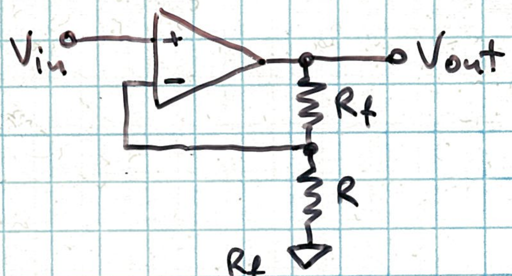
$i_R - i_{in} = i_{Rf}$, with $i_{in} = \emptyset \Rightarrow i_R = i_{Rf}$

Since SP is at virtual ground, $\begin{cases} i_R = v_{in}/R \\ i_{Rf} = -v_{out}/R_f \end{cases}$

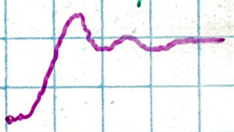
$\Rightarrow \dfrac{v_{out}}{v_{in}} = \boxed{G_\infty = -\dfrac{R_f}{R}}$ ← inverting amp.

Non-inverting amplifier

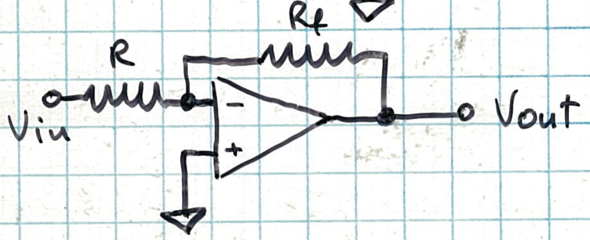




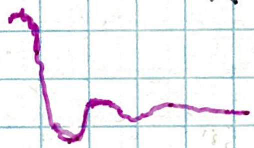
$$G_V = \frac{R_f+R}{R} = 1 + \frac{R_f}{R}$$



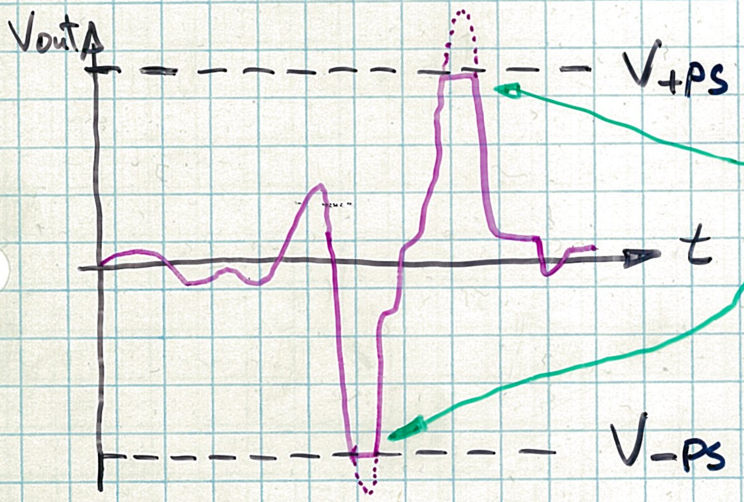
Inverting amplifier



$$G_V = -\frac{R_f}{R}$$



- These two assumptions may break down when the desired Vout exceeds the power supply voltages:



Vout is clipped
Amplifier is saturated

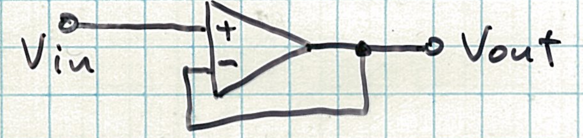
- Another restriction is the max values of R's that are practical: as we reach ~10's MΩ, the value of the resistor becomes comparable to the leakage resistance of the surrounding insulators. So, to get gains of ~100 we need to keep R down

$$100 \sim \frac{R_f}{R}, \quad R_f \le 10\,M\Omega \;\Rightarrow\; R \le 100\,k\Omega$$

And R = input resistance of the above inverting amp circuit (more complex in non-inverting case, $R_{in} \propto \frac{R}{R+R_f}$) i.e. in practice input resistance $< \infty$

A catalog of op-amp circuits

- voltage follower



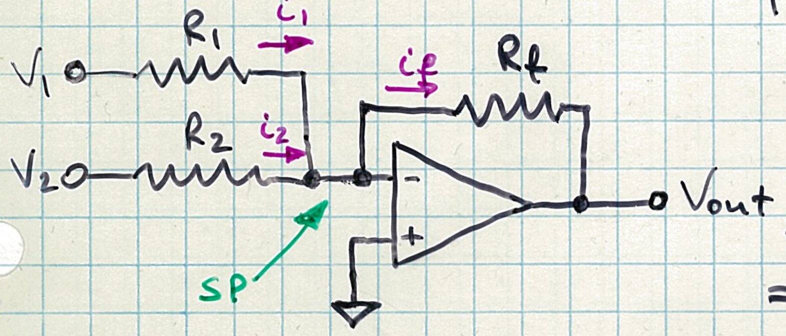
Non-inv. amp with $R_f = \emptyset$

$\Rightarrow \quad G_V = 1 + \frac{R_f}{R} = 1$

$\Rightarrow \quad \boxed{V_{in} = V_{out}}$

useful as impedance transformer. Note that the power gain need not be 1 $\Rightarrow$ a buffer

- summing amplifier



1) KCL at Summing Point (SP)

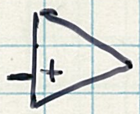
$\Rightarrow \quad i_1 + i_2 = i_f$ (op-amp draws no current!)

2) Virtual ground: $V_+ = V_- = \emptyset$

$\Rightarrow \quad i_1 = \frac{V_1}{R_1} \qquad i_2 = \frac{V_2}{R_2} \qquad i_f = \frac{-V_{out}}{R_f}$

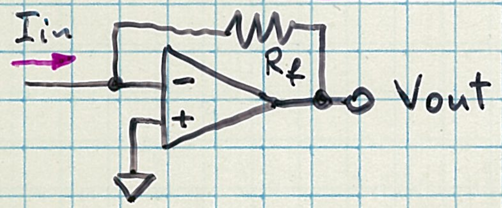
$\Rightarrow \quad V_{out} = -\frac{R_f}{R_1} V_1 - \frac{R_f}{R_2} V_2$ ← a weighted sum

$= -(V_1 + V_2)$ ← for $R_1 = R_2 = R_f$

! virtual ground at  $\Rightarrow$ inputs do not interact

N.B. only works in an <u>inverting</u> configuration!

- current-to-voltage converter

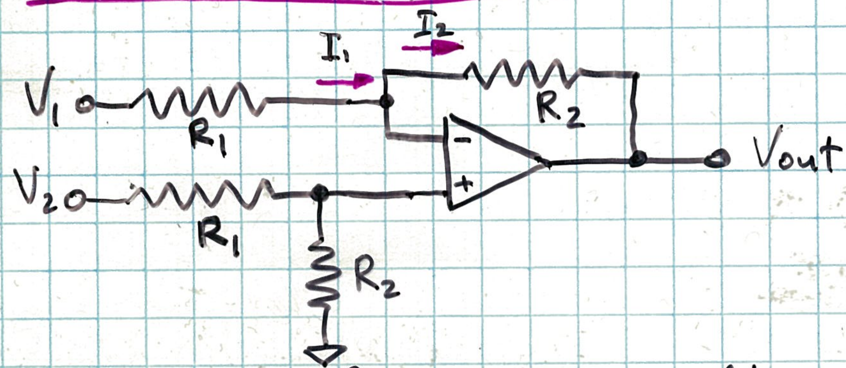


At summing pt: $I_{in} + \frac{V_{out}}{R_f} = \emptyset$

$\Rightarrow \quad V_{out} = -R_f I_{in}$

N.B. the above summing amplifier can be viewed as an adder of currents (just omit the R's)

• difference amplifier



$$\boxed{V_{out} = \frac{R_2}{R_1}(V_2 - V_1)}$$

1. $V_+ = V_2 \dfrac{R_2}{R_1 + R_2}$ (*) (a voltage divider)

2. virtual ground approximation : $V_- = V_+$

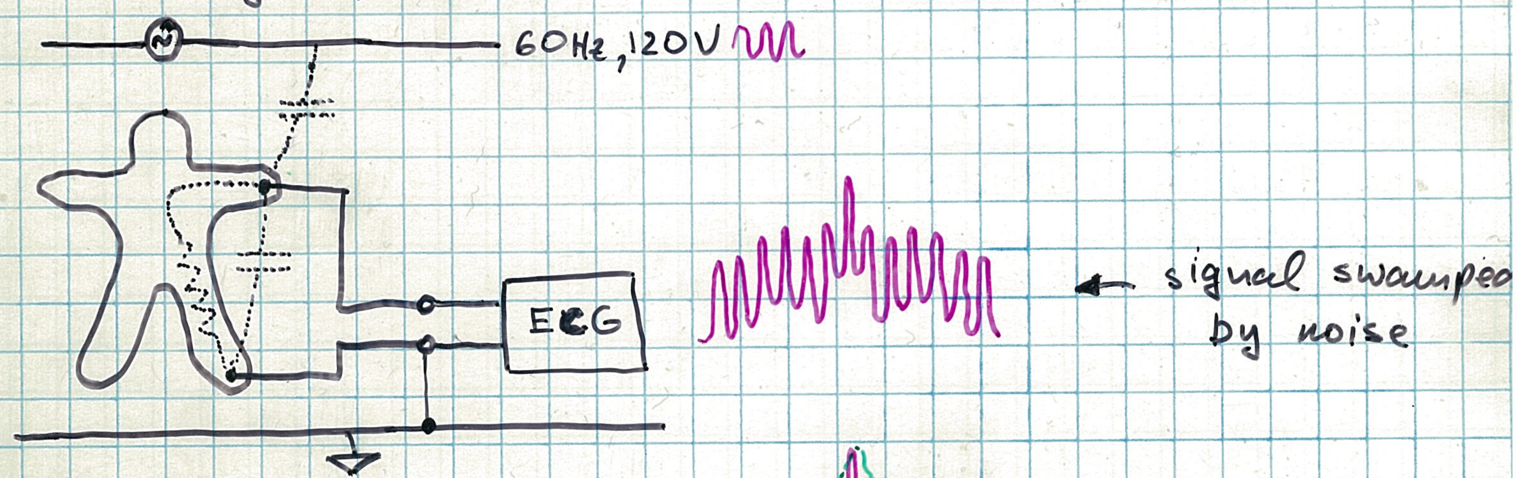
3. $\Rightarrow I_1 = I_2 \Rightarrow \dfrac{V_1 - V_-}{R_1} = \dfrac{V_- - V_{out}}{R_2}$

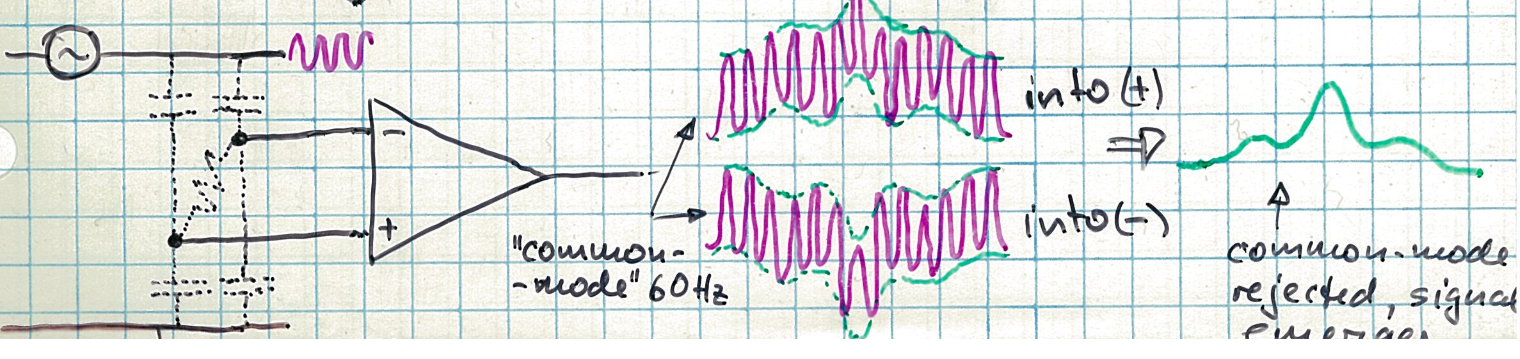
$\Rightarrow \dfrac{V_1}{R_1} - \dfrac{V_+}{R_1} = \dfrac{V_+}{R_2} - \dfrac{V_{out}}{R_2}$ (**)

4. Solve (*) & (**) to eliminate V_+ and get the above result

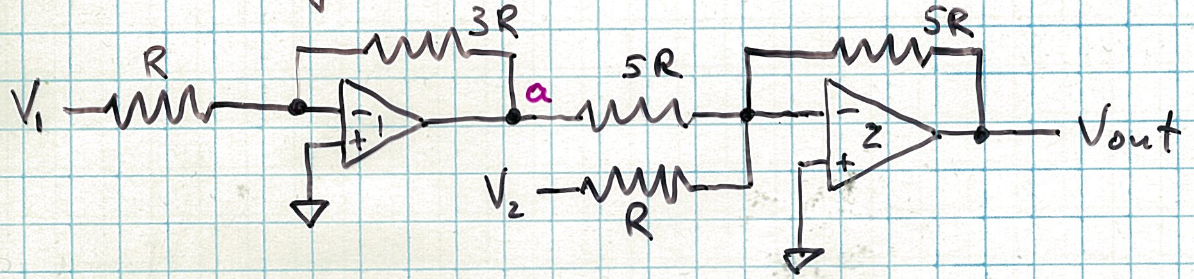
In practice: careful matching of the resistors is needed

Ex.1. using differential amplifiers to eliminate noise.





Ex 2. Analog arithmetics



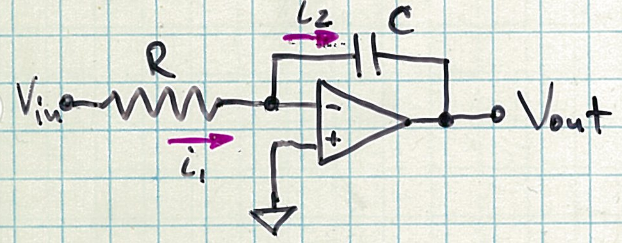
1. Amp 1: inverting amplifier, gain = 3

$\Rightarrow \quad V_a = -3V_1$

2. Amp 2: summing amplifier

$$\Rightarrow \quad \underline{V_{out}} = -\left(\frac{5R}{5R} V_a + \frac{5R}{R} V_2\right) = -V_a - 5V_2 = \underline{3V_1 - 5V_2}$$

- the integrator



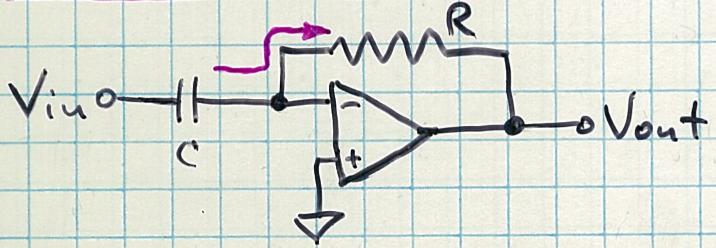
KCL at S.P. : $i_1 = i_2$

$$\Rightarrow \quad \frac{V_{in}}{R} = -C \frac{dV_{out}}{dt}$$

$$\Rightarrow \quad \boxed{V_{out} = -\frac{1}{RC}\int_0^t V_{in}\, dt} \quad (+\text{const})$$

We assume const = $V_{out}(t=\emptyset) = \emptyset$

- the differentiator



Again, at S.P. :

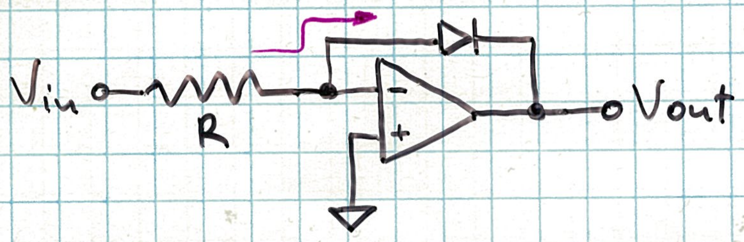
$$-\frac{V_{out}}{R} = C\frac{dV_{in}}{dt}$$

$$\Rightarrow \quad \boxed{V_{out} = -RC\frac{dV_{in}}{dt}}$$

Note: for a sine wave, $\int$ or $\frac{d}{dt}$ brings out a frequency dependence:

- integrator: $V_{out} = V_{in}\frac{j}{R\omega C}$ → low-pass filter
- differentiator: $V_{out} = V_{in}\, jR\omega C$ → high-pass filter

- logarithmic amplifier



Diode has an exponential voltage-current dependence

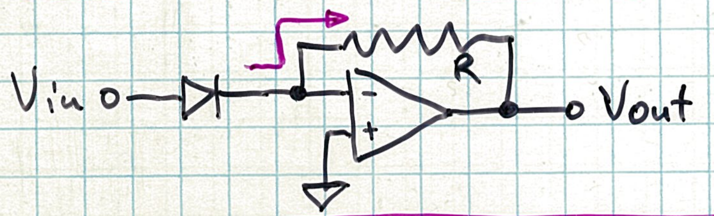
$$\Rightarrow \frac{V_{in}}{R} = B\, e^{V_{out}/\alpha}$$

$B, \alpha = \text{const.}$

$[\alpha] = V \quad [B] = A$

$$\Rightarrow \boxed{V_{out} = \alpha \ln \frac{V_{in}}{RB}}$$

- exponential amplifier



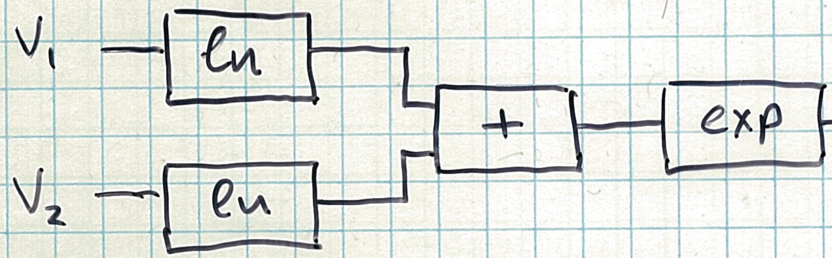
interchange R and diode:

$$B\, e^{V_{in}/\alpha} = -\frac{V_{out}}{R}$$

$$\Rightarrow \boxed{V_{out} = -RB\, e^{V_{in}/\alpha}}$$

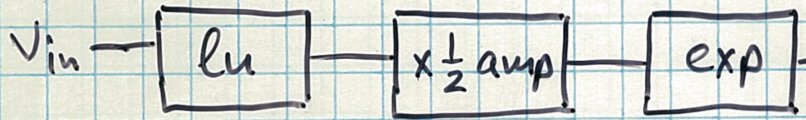
Combining op-amp "building blocks" – analog computing

- multiplication



$V_{out} = V_1 V_2$

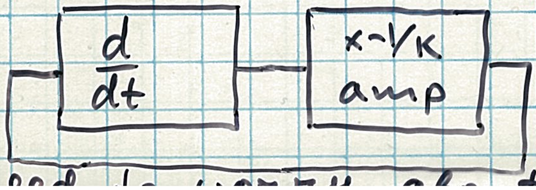
- square root



$V_{out} = \sqrt{V_{in}}$

- differential equations

in principle:



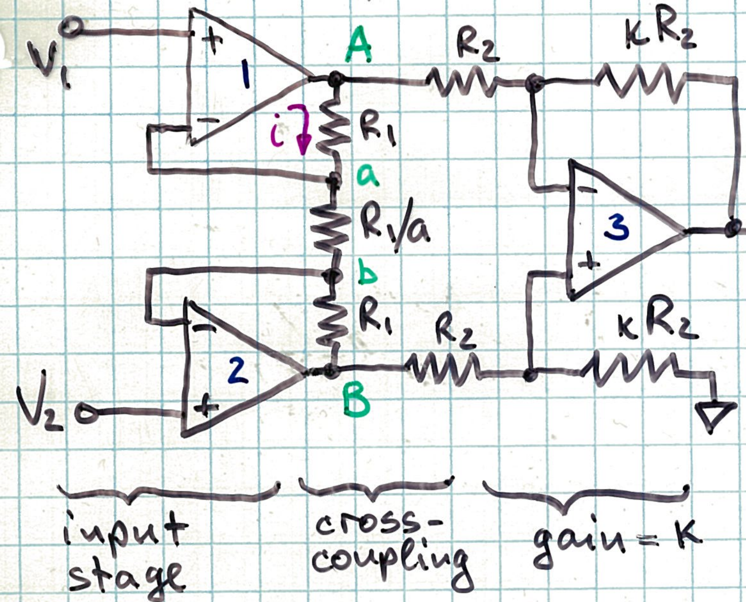
$$V = -\frac{1}{K}\frac{dV}{dt}$$

or $\dot{V} + KV = \emptyset$

in practice: need to worry about setting the IC's

· instrumentation amplifier

useful in many measuring instruments



① input stage is a couple of voltage followers: buffer stage to get high input impedance

② cross-coupling: a clever way to make the two input channels track each other:

$$V_a = V_1, \quad V_b = V_2 \Rightarrow i = a(V_1 - V_2)/R_1$$

(op-amps themselves draw no current)

$$\Rightarrow \begin{cases} V_A = V_1 + a(V_1 - V_2) \\ V_B = V_2 - a(V_1 - V_2) \end{cases}$$

$$\Rightarrow \left.\begin{aligned} A_{difference} &= \frac{V_A - V_B}{V_1 - V_2} = 1 + 2a \\ A_{common\text{-}mode} &= \frac{\tfrac{1}{2}(V_A + V_B)}{\tfrac{1}{2}(V_1 + V_2)} = 1 \end{aligned}\right\}$$

Not sensitive to precision matching of R_1's

⇒ with reasonable R's can easily obtain

common-mode rejection ratio

$$CMRR \equiv \frac{A_{diff}}{A_{c.m.}} = \frac{1+2a}{1} \sim 10 \text{ to } 1000$$

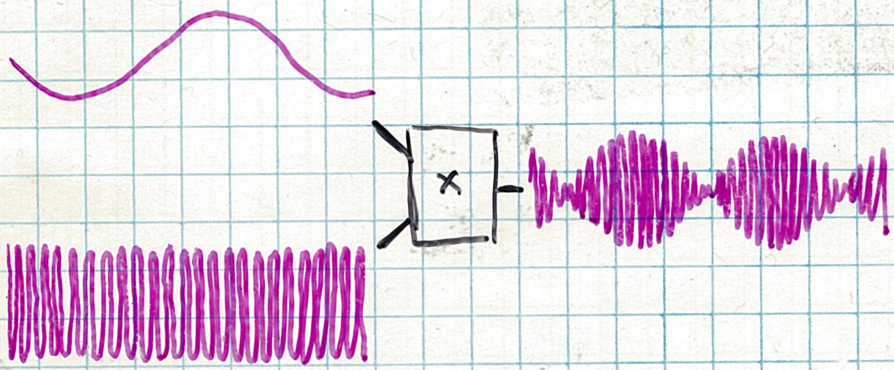
in the input stage alone! (× CMRR of op-amp 3)

! ‖ the above circuit as a whole is a much better op-amp: high R_{in}, high CMRR.

Mathematical aside: signal modulation

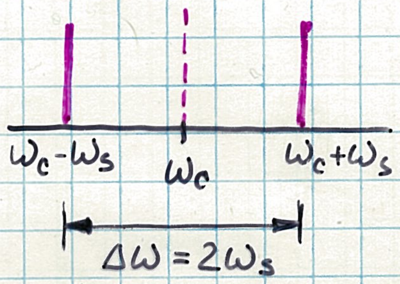
Signal:
$A_s \cos \omega_s t$

Carrier:
$A_c \cos \omega_c t$



$$A_c \cos \omega_c t * A_s \cos \omega_s t = \frac{A_c A_s}{2}\left[\cos(\omega_c - \omega_s)t + \cos(\omega_c + \omega_s)t\right]$$

In frequency domain:

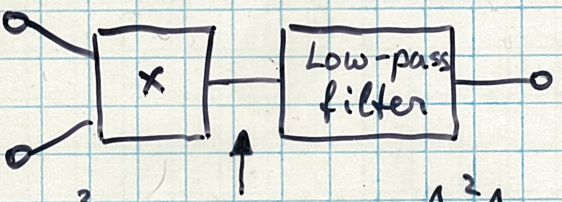


Must be reversible to be useful!

⇒ Demodulation:

Modulated carrier
$A_c \cos \omega_c t * A_s \cos \omega_s t$

Reference
$A_c \cos \omega_c t$



$\frac{A_c^2 A_s}{2} \cos \omega_s t$

$\frac{A_c^2 A_s}{2} \cos \omega_s t + \frac{A_c^2 A_s}{2} \cos \omega_s t \cos \underline{\underline{2\omega_c t}}$

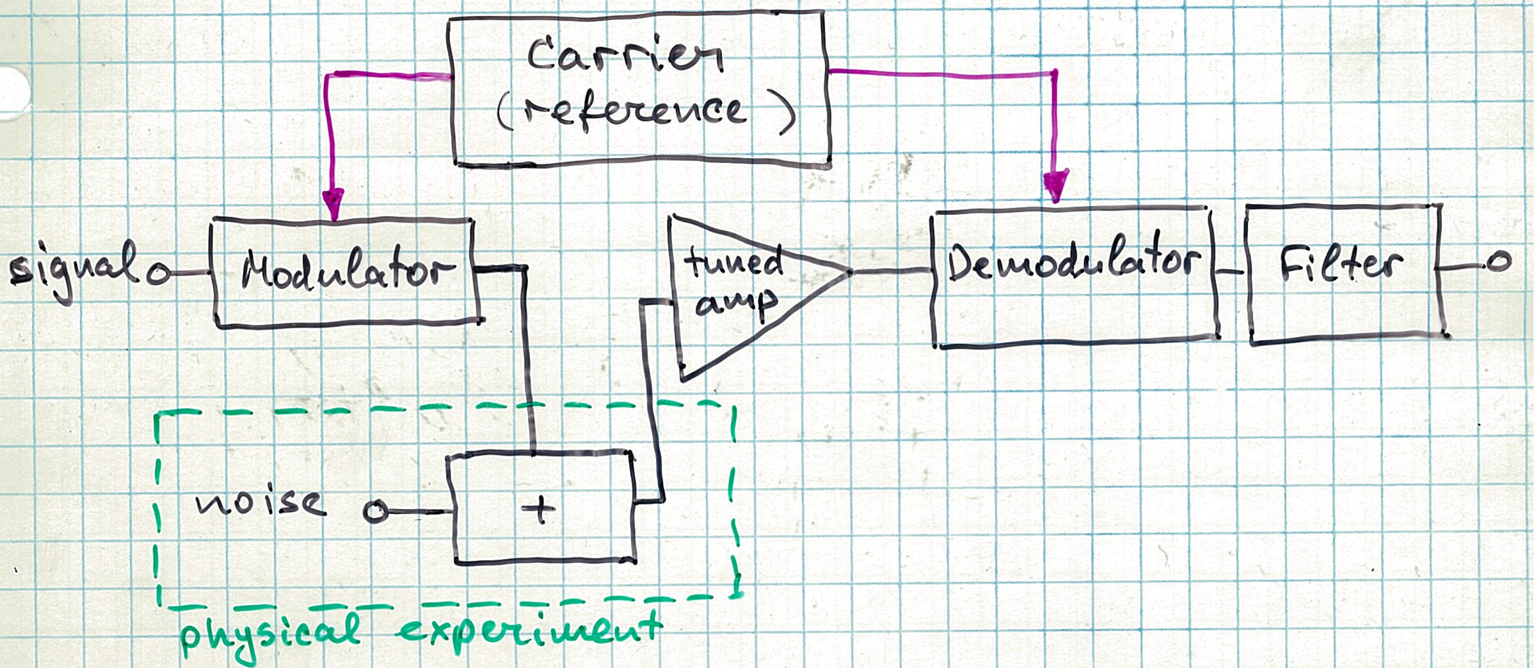
high freq.

⇒ can encode and recover the original signal

Ex. 1. AM radio

Ex. 2. heterodyne amplification
(amplify at a frequency where it is easier!)

• lock-in amplifier



- "tag" the signal by carrier
- put the modulated signal to use (e.g. send light to probe optical properties)
- the signal emerges "buried" in the noise
- use a "phase-sensitive detector", i.e. treat signal during the +ve half of the carrier wave differently from the signal during the –ve half. ⇒ de-modulate.

Those components that do not vary with carrier (noise) will get averaged to zero; those that vary in-phase with carrier, will add up.

⇒ extract the "buried" signal

Note: the carrier need not be a sine wave!
E.g. ⇒ lends itself to "digital" (on/off) versions of lock-in amplifiers

Note: distinguished history of lock-in detection in physics

See: Am. J. Phys
59: 569-572, 1991

Note: "signal" & "noise" have same period ⇒ overlapping frequency spectra

Note: can modulate by a square wave, too! i.e. "chop" the signal

"signal"

"noise"

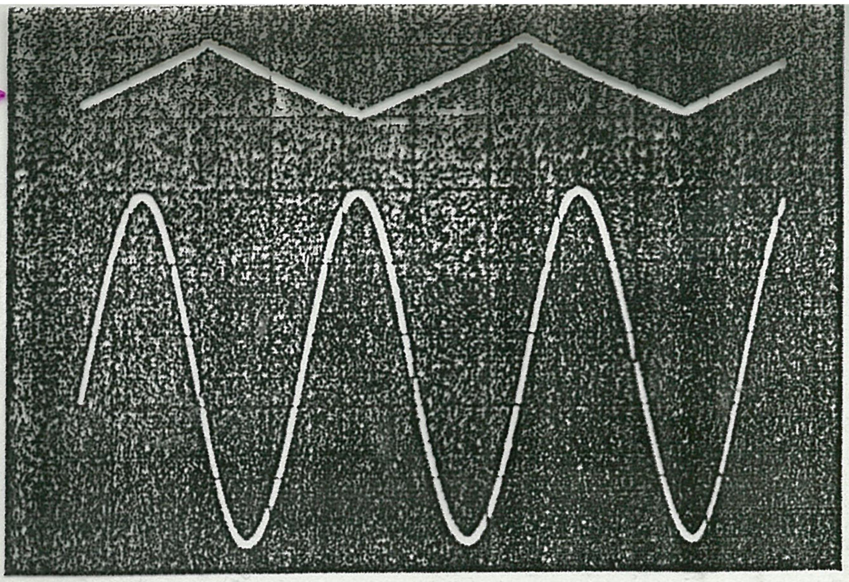


Fig. 4. Signal (upper trace) and "noise" (lower trace), shown at the inputs to the circuit of Fig. 2. Both traces are displayed at 1 V/division, with the sweep rate at 5 ms/division. The signal-to-noise ratio is 1:5.

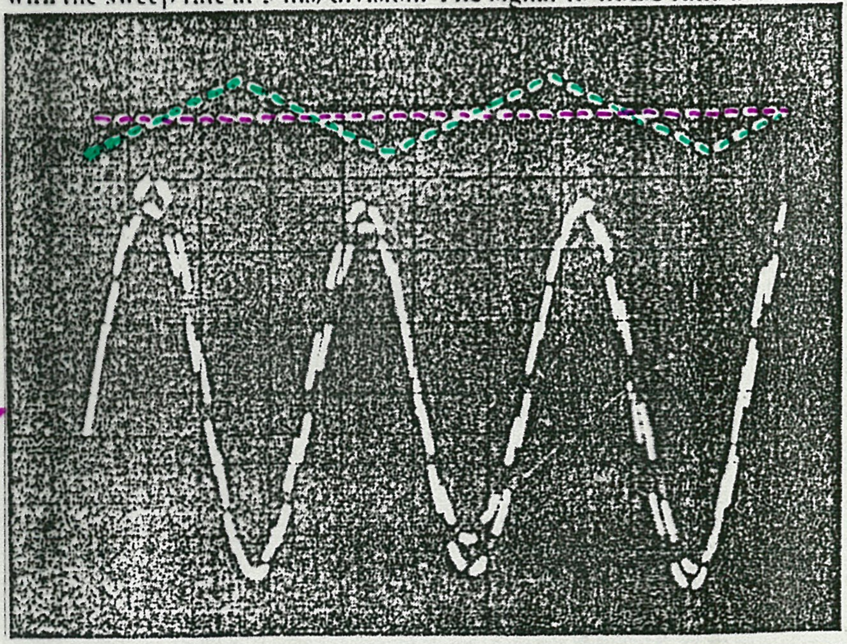
signal modulated with

1

0

Note: signal is buried by noise; little evidence of (triangular wave)

signal + noise modulated with



ignore

demodulated signal