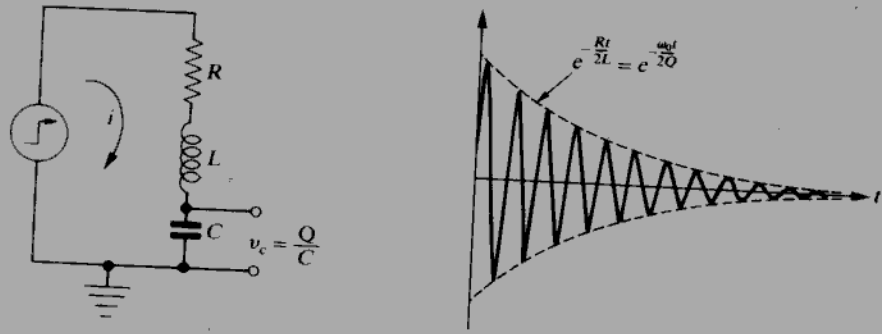


(a) *circuit*

(b) *voltage across c*

FIGURE 2.46 Ringing excited by step input.

By inspection $Q_0 = CV_0$ is a constant particular solution of (2.73). The solution to (2.74) can be shown to be

$$Q_h = Ae^{-Rt/2L}e^{\pm j\omega_0 t} \tag{2.75}$$

where A is a constant, and provided $R^2/4L^2 \ll 1/LC = \omega_0^2$ or $4Q^2 \gg 1$. Using $Q = \omega_0 L/R$, we can write the solution as

$$Q = CV_0(1 - e^{-\omega_0 t/2Q}e^{\pm j\omega_0 t}) \tag{2.76}$$

Thus we see that for a high-Q circuit ($4Q^2 \gg 1$), the charge on the capacitor oscillates at approximately ω_0, and the amplitude of the oscillations decreases exponentially according to

$$e^{-\omega_0 t/2Q} \tag{2.77}$$

In other words, the *envelope* of the oscillations or ringing decreases according to (2.77) as shown in Fig. 2.46(b). The current and voltage also decrease according to (2.77). Thus a measurement of the voltage across the *RLC* circuit versus time can yield the circuit Q.

PROBLEMS

1. Calculate the period in seconds of a 400-Hz sinusoidal voltage. If the zero-to-peak amplitude is 5 V, calculate the maximum instantaneous rate of change of the voltage in volts per second.
2. Prove that the root-mean-square (rms) value of

$$V(t) = V_0 \cos \omega t$$

is equal to $V_0/\sqrt{2}$. What is the rms value of the voltage of Problem 1?

3. Calculate the average power (in watts) dissipated as heat in a device passing a current of $i(t) = 4 \cos \omega t$ if the voltage across the device is (a) $v(t) = 10 \cos(\omega t + 30°)$, (b) $v(t) = 10 \cos \omega t$.

4. Calculate the capacitance in microfarads between two 1-cm^2 conducting plates 1 mm apart in a vacuum. Repeat if the space between the plates is filled with a plastic dielectric with a dielectric constant of 8.

5. Calculate the capacitive reactance in ohms of a 0.01-μF capacitor at (a) 100 Hz, (b) 1 kHz, (c) 100 kHz, (d) 1 MHz.

6. Calculate the inductive reactance in ohms of a 2.5-mH choke at (a) 100 Hz, (b) 1 kHz, (c) 100 kHz, (d) 1 MHz.

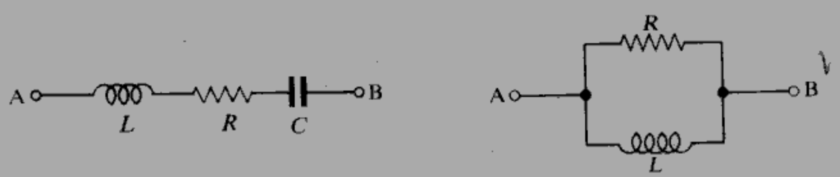
7. Express L dimensionally in terms of ohms and seconds. Repeat for C.

8. Calculate the energy in joules stored in a 2000-μF capacitor charged to 5 V. Physically, how is the energy stored?

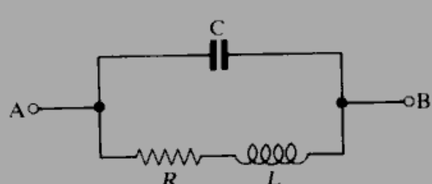
9. A 1-H inductance carries a current of 500 mA. The wire breaks, and in 10^{-3} s the current drops to zero. What would happen?

10. Calculate the impedance Z_{AB} in the form $a + jb$ and $|z|e^{j\theta}$ for

11. Calculate the impedance Z_{AB} in the form $a + jb$ and $|z|e^{j\theta}$ for



12. Calculate the impedance Z_{AB} in the form $a + jb$ and $|z|e^{j\theta}$ for



13. Design a high-pass RC filter with a breakpoint at 100 kHz. Use a 1-kΩ resistance. Explain in words why the high-pass filter attenuates the low frequencies.

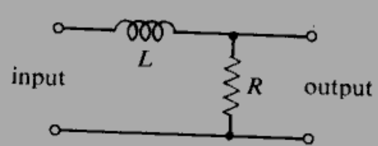
14. Design a low-pass RC filter that will attenuate a 60-Hz sinusoidal voltage by 12 dB relative to the dc gain. Use a 100-Ω resistance. Explain in words why the low-pass RC filter attenuates the high frequencies.

15. For a low-pass RC filter prove that (a) at the frequency $\omega = 1/RC$ the voltage gain equals $0.707 = 1/\sqrt{2}$; (b) the rise time of the output pulse equals $2.2RC$ for a zero rise time input pulse.

16. Calculate the slope of the gain versus frequency curve for a low-pass RC filter in dB per octave and also in dB per decade for high frequencies ($\omega \gg 1/RC$). [One decade frequency change is a factor of ten (e.g., from 10 to 100 Hz, or from 50 to 500 kHz).]

17. Carefully sketch the voltage vector diagram for a high-pass RC filter and calculate the phase of the output voltage relative to the phase of the input voltage.

18. Derive an expression for the voltage gain and phase shift for the following LR circuit.

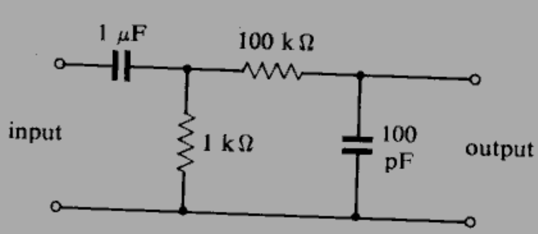


19. Design a parallel LC resonant circuit or tank to resonate at 1 MHz. Assume the inductance $L = 100\ \mu$H and has a dc resistance of 10 Ω. What is the Q of this circuit at resonance?

20. Carefully sketch the rotating voltage vector diagram for a series RLC circuit at resonance. If the circuit has a Q of 100, calculate the voltage ratings L and C must have.

21. Sketch a graph of the magnitude of the impedance versus frequency for series and parallel RLC circuits. State the change in phase of the impedance as the frequency passes through resonance.

22. Sketch the approximate gain-versus-frequency curve for the following circuit. You may treat the circuit as being composed of two independent RC filters.



23. For a high-Q parallel RLC circuit prove that $Q = \omega_0/\Delta\omega$, where ω_0 is the (angular) resonant frequency and $\Delta\omega$ is the width at the half-power points.

24. For a high-Q parallel RLC circuit prove that at resonance the impedance equals the Q times the inductive reactance at resonance. Calculate the im-

pedance at resonance for

$$L = 100\,\mu\text{H} \quad C = 0.001\,\mu\text{F} \quad R = 5\,\Omega$$

25. For a series RLC circuit show that the angular frequency for the charge on C (or the current) is approximately

$$\omega \cong \omega_0\left(1 - \frac{1}{8Q^2}\right)$$

26. Derive (2.75) for the series RLC circuit excited by a step-function voltage. [*Hint*: Solve (2.74) by assuming a solution of the form $Q_h = e^{\lambda t}$ where λ is an unknown constant independent of time but depending upon R, L, and C. Assume $R^2/4L^2 \ll 1/LC$.]

27. A general ac bridge can be made from the Wheatstone bridge of Fig. 1.35 by replacing each of the four resistances by a general impedance Z, that is, $R_1 \rightarrow Z_1$, etc. The battery of Fig. 1.35 is also replaced by a sinusoidal ac source of angular frequency ω, and the null meter is replaced by a sensitive ac voltmeter such as an oscilloscope. Show that the general balance condition is

$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$$

28. With $Z_1 = R_1$, $Z_2 = R_2 + j\omega L_2$, $Z_3 = R_3$, and $Z_4 = R_4 + j\omega L_4$, show that the two balance conditions are

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \quad \text{and} \quad \frac{R_1}{L_2} = \frac{R_3}{L_4}$$

This is an inductance bridge. Also sketch the bridge.

29. With $Z_1 = R_1$, $Z_2 = R_2 + 1/j\omega C_2$, $Z_3 = R_3$, and $Z_4 = R_4 + 1/j\omega C_4$, show that the two balance conditions are

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \quad \text{and} \quad R_1 C_2 = R_3 C_4$$

This is a capacitance bridge. Also sketch the bridge.

30. With $Z_1 = R_1$, $Z_2 = R_2 \| C_2$, $Z_3 = R_3$, and $Z_4 = R_4 + 1/j\omega C_4$, show that the two balance conditions are

$$\frac{R_4}{R_2} + \frac{C_2}{C_4} = \frac{R_3}{R_1} \quad \text{and} \quad \omega^2 R_2 C_2 R_4 C_4 = 1$$

This is the Wien bridge. If $R_2 = R_4 = R$ and $C_2 = C_4 = C$, find the two balance conditions. Also sketch the bridge.