Experiment 4

Frequency response of RC circuits

Before you begin:

- the midterm Lab Test will require you to understand all of the content of this experiment. Review the test by clicking on the "midterm test" link in the Lab Manual section of the course homepage and be sure to read the "How to analyse a circuit" hints included there.
- You will only be allowed the use your lab book during this test. Make careful notes of your setups and observations and include clear and complete calculations that you can refer to during the test.
- Your lab book will be handed in at the end of this lab session and will not be available for review until the start of the test. This content will be graded according to clarity and completeness of content and observance of the stated instructions and outlined procedures.
- Read carefully and observe the following experimental procedures. Do not be sloppy and superficial in your work. Get organized to complete and document the experiment during this lab session.

4.1 The transfer function in AC circuits

A common kind of time-dependent signal is the continuous AC current of a certain frequency, represented by a sine wave. *RC* circuits respond differently to signals of different frequencies and a convenient way to describe this behaviour is in terms of the circuit frequency response, or transfer function.

The simplest AC signal is that of a sine wave of frequency f. Other periodic AC signals can be represented by a combination of this fundamental signal sine frequency f and sine overtone frequencies that are integer multiples of the fundamental signal sine frequency f.

To analyse RC circuits, we use a sine wave since we wish to examine the behaviour of the circuit at a specific frequency f.

As a circuit usually affects both the amplitude and the phase of the output signal relative to that of the input signal, the complete description of the transfer function requires plots of amplitude vs frequency as well as phase vs frequency. To generate these plots:

- begin by determining the theoretical centre frequency f_0 of the circuit, then select a series of frequency values that span from $f \approx f_0/20$ to $f \approx 20f_0$, concentrating on the region near f_0. A dozen or so well chosen values are typically sufficient.
- To generate the amplitude-frequency plot, measure for each of the chosen frequencies the peak-to-peak amplitude $V_{\rm in}(f)$ of the incoming and $V_{\rm out}(f)$ of the outgoing signal. Calculate their ratio $G(f) = V_{\rm out}(f)/V_{\rm in}(f)$, called the *gain*, and plot $\log(G)$ as a function of $\log(f)$.
- To generate the phase-frequency plot, measure for each of the chosen frequencies the amount of time Δt that the output signal is delayed from the input signal, then calculate and plot the corresponding phase shift $\phi(f)$, in degrees, as a function of $\log(f)$.

  With the period $T = 1/f$, $\phi = 360 * \Delta t/T$. The phase shift refers to the shift of the output signal relative to the input signal. If the output leads, or precedes the input in time, then Δt and the phase shift ϕ will be positive. If the output lags, or follows the input in time, then Δt and the phase shift ϕ will be negative. Be careful to give $\phi(f)$ the correct sign.

4.2 Low-pass and high-pass filters

The circuit in Figure 4.1 can be thought of as a frequency filter with centre frequency

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi RC}. \qquad (4.1)$$

In this Section you will measure the transfer function, *i.e.* the relationship between the input and the output voltages of this low-pass filter.

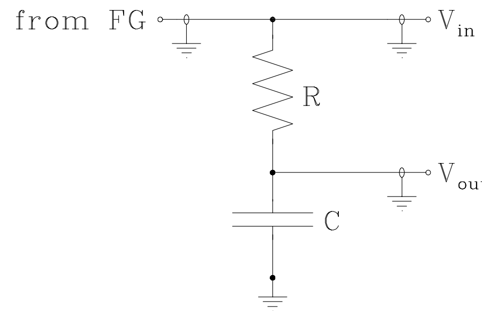

Figure 4.1: Typical RC low-pass filter

- On your breadboarding workstation, assemble the circuit of Figure 4.1. Use $C = 0.01\ \mu$F and $R = 10000\ \Omega$.

- With coaxial cables, make the connection between the breadboard and the function generator `FG`, then connect $\mathrm{V_{in}}$ to the scope `CH1` input and $\mathrm{V_{out}}$ to the scope `CH2` input.

(!) Note that when coaxial cables are used to make BNC connections between instruments and the workstation, the ground connections, shown by the ground symbols at the $\mathrm{V_{in}}$, $\mathrm{V_{out}}$ and `FG` nodes in Figure 4.1, are automatically made to GND on the breadboard. You do not need to duplicate these conenctions.

- Select a sine waveform on the function generator and adjust the signal amplitude to 5 $\mathrm{V_{pp}}$.

- Determine the theoretical centre frequency f_0 for the filter, then make a table of i test frequencies f_i that you will use.

- Set `FG` to each of the test frequencies f_i in turn and for each measure and tabulate $V_{\mathrm{in}}(f_i)$, $V_{\mathrm{out}}(f_i)$ and the time delay between signals Δt_i, then calculate and tabulate $G(f_i)$, T_i and $\phi(f_i)$.

- Use `Physicalab` or some other graphing program to plot $\log G(f) vs. \log(f)$ and $\phi(f) vs. \log(f)$. In `Physicalab` you can check the `X log` and `Y log` boxes to display a log-log graph.

(!) Duplicate all graphs in your lab book. Draw and label the pair of coordinate axes as shown, then sketch the the displayed curve, being careful to properly match the curve to the axes.

Pay particular attention to and identify relevant details such as the f_0 frequency and phase shift points and rolloff slopes and make sure that all the other parameters that were obtained from the computer generated graph can also be determined from your sketch.

- Interchange R and C and repeat the above steps to graph the transfer function for a high-pass filter.

- Compare the low-pass and high-pass transfer functions in terms of their centre frequencies f_0, their gains at f_0 and rolloff rates in dB/octave. What is the order of these filters? How do the phase shifts of the two filters differ? Recall that

$$dB = -20 \log\left(\frac{V_{\mathrm{out}}}{V_{\mathrm{in}}}\right).$$

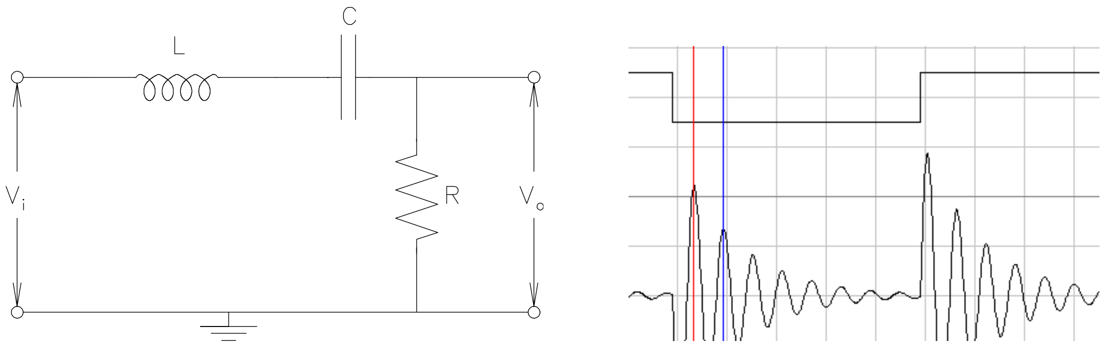

Figure 4.2: Transients in an RCL circuit; measuring the ringing frequency and decay envelope.

4.3 RCL transients (ringing)

Adding an inductor L to an RC circuit produces a circuit capable of resonant oscillations (ringing). The presence of R, an energy-dissipating element, guarantees that the amplitude of the oscillations does not remain constant. Typically, one observes oscillations of frequency

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad [\text{rad} \cdot s^{-1},] \tag{4.2}$$

$$f_0 = \frac{1}{2\pi}\omega_0 \qquad [\text{Hz}] \tag{4.3}$$

The amplitude of these oscillations decays exponentially with a time constant $\tau = L/R$.

- Setup your circuit as shown in Figure 4.2. Use an inductor $L = 2.2\,\text{mH}$, $C = 0.01\ \mu\text{F}$ and $R = 100\ \Omega$.
- Calculate the theoretical value of the natural oscillation frequency, f_0, for this circuit.
- Set the `FG` to output a 5 V_{pp} *square wave* of about $0.1f_0$. Drive the RCL circuit with the square wave and observe the decaying oscillations of the ringing signal.
- Adjust the scope for best resolution, then use the cursors to determine the period of the ringing signal by measuring the time difference between two adjacent peaks.

(!) Duplicate the scope screen in your lab book with coordinate axes scaled according to the set grid divisions, then sketch the the displayed curve, being careful to properly match the curve to the axes.

Pay particular attention to identify the cursor positions and values at their points of intersection with the waveform so that the results obtained from the scope screen can also be determined from your sketch.

- Calculate the experimental value for f_0, including error estimates, then compare the result with the theoretical value and account for any discrepancies.
- Estimate the experimental value of τ assuming an exponential envelope of the ringing signal then compare the result with the theoretical value of τ you obtain from the nominal component values. Include error estimates.

  Note that for an exponential function $V = V_0 e^{-t/\tau}$ with decay time constant τ,

$$\frac{V_1}{V_2} = \frac{V_0 e^{-t_1/\tau}}{V_0 e^{-t_2/\tau}} = e^{-(t_1-t_2)/\tau} \quad \rightsquigarrow \quad \ln V_1 - \ln V_2 = -(t_1 - t_2)/\tau \quad \rightsquigarrow \quad \tau = \frac{t_1 - t_2}{\ln V_2 - \ln V_1} \tag{4.4}$$

  Thus one can use any two points on the exponential envelope, *e.g.*, two adjacent peaks in the ringing signal, to determine τ.