Brock University

Final Examination: EXAMPLE
Course: PHYS 2P50
Date of Examination: December
Time of Examination: XX:00–XX:00

Number of pages: 13
Number of Students: XX
Number of hours: 3
Instructor: M. Reedyk

Students may use a calculator, and are provided with a list of useful information. No examination aids other than those specified are permitted. Use or possession of unauthorized materials will automatically result in the award of a zero grade for this examination.

All questions are to be answered on the examination paper. Please hand this paper in at the completion of the exam.

NAME:

Useful Information

Constants

$c = 2.998 \times 10^8$ m/s
$h = 6.626 \times 10^{-34}$ J.s.
$k = 1.381 \times 10^{-23}$ J/K
$e = 1.602 \times 10^{-19}$ C
$m_e = 9.11 \times 10^{-31}$ kg $= 0.511 \frac{MeV}{c^2}$
$\sigma = 5.67 \times 10^{-8} \frac{W}{m^2K^4}$.

Special Relativity

- $$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$
- $$\beta = \frac{v}{c}$$
- Time Dilation $$\Delta t = \gamma \Delta t_0$$
- Length Contraction $$L = \frac{L_0}{\gamma}$$
- Lorentz transformation for $S \to S'$

$$\begin{aligned} x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma(t - \frac{v}{c^2}x) \end{aligned}$$

- Lorentz velocity transformation for $S \to S'$

$$\begin{aligned} u'_x &= \frac{u_x - v}{1 - \frac{v}{c^2} u_x} \\ u'_y &= \frac{u_y}{\gamma(1 - \frac{v}{c^2} u_x)} \\ u'_z &= \frac{u_z}{\gamma(1 - \frac{v}{c^2} u_x)} \end{aligned}$$

- Doppler Effect for light $$f = f_0 \sqrt{\frac{1 \pm \beta}{1 \mp \beta}}$$
- relativistic momentum $$\bar{p} = m\bar{v}\gamma$$
- relativistic energy $$E = K + mc^2 = \gamma mc^2$$
- $$E^2 = (pc)^2 + (mc^2)^2$$
- blackbody radiation $$P = \sigma A T^4$$
- $$\lambda_{\text{max}} T = 2.898 \times 10^{-3} \text{mK}$$
- $$R(\lambda) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$
- photoelectric effect $$eV_s = hf - \Phi$$
- Compton effect $$\Delta\lambda = \frac{h}{mc}(1 - \cos\phi)$$
- $$\theta = \tan^{-1}\left[\frac{\sin\phi}{\frac{\lambda'}{\lambda} - \cos\phi}\right]$$
- photons $$E = hf, \quad p = \frac{h}{\lambda}$$
- matterwaves $$\lambda = \frac{h}{p}$$
- $$\Delta x \Delta p_x \geq \frac{\hbar}{2}$$
- $$\Delta E \Delta t \geq \frac{\hbar}{2}$$

Interference and Diffraction

- Two-slit experiment $$\Delta y = \frac{\lambda D}{d} (\text{fringe spacing})$$
- Bragg condition $$\Gamma = n\lambda = d\sin\phi$$
- Single-slit diffraction $$\sin\phi = \frac{\lambda}{d} (\text{first min})$$

Quantum Mechanics

- 1-D TISE
$$\frac{-\hbar^2}{2m}\frac{d^2\Psi(x)}{dx^2} + U\Psi(x) = E\Psi(x)$$
- $$k = \frac{\sqrt{2m(E-U)}}{\hbar} = \frac{2\pi}{\lambda}$$
- Probability to find particle between x_1 and x_2
$$P = \int_{x_1}^{x_2} |\Psi(x)|^2 dx$$
- expectation value of x
$$x_{av} = \int_{-\infty}^{\infty} |\Psi(x)|^2 x dx$$
- Infinite Square Well
$$\Psi_n(x) = \sqrt{\frac{2}{L}} \ \sin\frac{n\pi x}{L} \quad n = 1, 2, 3....$$
- $$E_n = \frac{n^2\hbar^2\pi^2}{2mL^2}$$
- $$\lambda_n = \frac{2L}{n}$$
- 1-D step $$R = \left(\frac{k_I - k_{II}}{k_I + k_{II}}\right)^2$$
- Potential Barrier $$T \sim e^{-2\alpha L}$$
- $$\alpha = \frac{\sqrt{2m(V_o - E)}}{\hbar}$$
- Hydrogen Atom $$E_n = \frac{-13.6eV}{n^2}$$
- $$\Psi_{100} = \frac{1}{\sqrt{\pi}a_0^{3/2}} \ e^{-r/a_0}$$
- $$P(r)dr = \frac{4r^2}{a_0^3} \ e^{-2r/a_0} dr \quad \text{in ground state}$$
- Harmonic Oscillator $$E_n = (n + \frac{1}{2})\hbar\sqrt{\frac{k}{m}} \quad n = 0, 1, 2...$$
- $$\Psi(x) = \left(\frac{\sqrt{km}}{\hbar\pi}\right)^{\frac{1}{4}} e^{-\frac{\sqrt{km}}{2\hbar}x^2} \quad \text{in ground state}$$

NOTE: There are a total of 60 marks available.
DO ALL PARTS OF ALL QUESTIONS.

For multiple choice questions 1-10 circle the correct answer.

1. [1 mark] Which of these statements are *postulates* of Einstein's special relativity?
   (1) The speed of light is the same in all inertial reference frames.
   (2) Moving clocks run slow.
   (3) Moving objects are contracted along the direction of motion.
   (4) The laws of nature are the same in all inertial reference frames.
   (5) $E=mc^2$

   (a) 1 only.
   (b) all 5.
   (c) 1 and 4 only.
   (d) 1, 4 and 5.
   (e) 2 and 3 only.

2. [1 mark] An astronaut heading out toward a star at constant high speed can determine that he is in motion by

   (a) the contraction of on-board meter sticks.
   (b) the slowing down of time on his clocks.
   (c) the increase of his mass.
   (d) the speeding up of his heart.
   (e) none of these.

3. [1 mark] You are trying to communicate with a rocket ship that is traveling away from Earth on its way into space at relativistic speed. You send your message at frequency f_o. In order to receive your message, the astronauts should

   (a) tune to a frequency greater than f_o.
   (b) tune to a frequency less than f_o.
   (c) tune to frequency f_o.
   (d) tune to a frequency of $2f_o$.
   (e) tune to a frequency of f_o^2.

4. [1 mark] Black holes

   (a) are holes in space, devoid of matter.
   (b) cannot be detected in binary star systems.
   (c) are the collapsed remnant of stars.
   (d) are predicted by Einstein's special theory of relativity.
   (e) are described by all of the above.

5. [1 mark] A proton and an electron are both accelerated to the same final kinetic energy. If λ_p is the de Broglie wavelength of the proton and λ_e is the de Broglie wavelength of the electron, then

   (a) $\lambda_p > \lambda_e$.

   (b) $\lambda_p = \lambda_e$.

   (c) $\lambda_p < \lambda_e$.

   (d) whether $\lambda_p > \lambda_e$ or $\lambda_e > \lambda_p$ depends on whether the final speeds are relativistic or not.

   (e) one needs to know the accelerating potential in order to determine whether $\lambda_p > \lambda_e$ or $\lambda_e > \lambda_p$.

6. [1 mark] It follows from the de Broglie hypothesis that

   (a) $p = \omega\hbar$.

   (b) $p = \lambda h$,

   (c) $p = k\hbar$.

   (d) $p = \lambda\hbar$.

   (e) None of these.

7. [1 mark]In the Compton effect, a photon of wavelength λ and frequency f hits an electron that is initially at rest. Which of the following occurs as a result of the collision?

   (a) The photon is absorbed completely.

   (b) The photon gains energy, so the final photon has a frequency greater than f.

   (c) The photon gains energy, so the final photon has a wavelength greater than λ.

   (d) The photon loses energy, so the final photon has a frequency less than f.

   (e) The photon loses energy, so the final photon has a wavelength less than λ.

8. [1 mark] A major advantage of an electron microscope over a visible light microscope is that the electron microscope

   (a) has much greater magnification.

   (b) operates with much lower intensity.

   (c) can penetrate opaque samples.

   (d) can have much better resolution.

   (e) requires no lenses for its operation.

9. [1 mark] The square of the wavefunction represents the

   (a) velocity of the particle.

   (b) probability of finding the particle.

   (c) position of the particle as a function of time.

   (d) probability density for finding the particle.

   (e) energy of the particle.

10. [1 mark] The principal quantum number of the sixth excited state of the hydrogen atom is

    (a) 5
    (b) 6
    (c) 7
    (d) 8
    (e) none of these.

11. Two relativistic rockets move toward each other. As seen by an observer on Earth, rocket A travels with a speed of 0.8c, while Rocket B travels with a speed of 0.6c. The proper length of Rocket A is 500 m and that of Rocket B is 1000 m.

    (a) What is the speed of the rockets relative to each other?

[3 marks]

    (b) What is the length of Rocket B according to Rocket A?

[3 marks]

(c) The earthbound observer sets her clock to $t = 0$ when the two noses of the rockets just pass each other (defined as Event 1).

i. In the frame of reference of Rocket A how far does Rocket B travel between Event 1 and Event 2 which is defined to be when the tails of the rockets just pass each other?

[1 mark]

ii. How long does it take Rocket B to cover this distance according to Rocket A? (This is the time between Events 1 and 2 according to Rocket A).

[1 mark]

iii. What is the time between Events 1 and 2 according to the earthbound observer?

[3 marks]

12. The K^o particle decays according to the equation $K^o \rightarrow \pi^+ + \pi^-$. A particular K^o decays while it is at rest in the laboratory. The rest energy of the K^o is 497.7 Mev, while the π^+ and π^- mesons each have a rest energy of 139.6 MeV.

    (a) What is the magnitude of the momentum of each of the two pions?

[3 marks]

    (b) What is the magnitude of the velocity of each of the two pions?

[2 marks]

13. A sodium atom is in a state in one of the lowest excited levels. It remains in that state for an average time of 1.6×10^{-8} s before it makes a transition back to the ground state. The corresponding spectral line is centered at a wavelength of 589.0 nm. What is the linewidth (in nm) of the spectral line?

[3 marks]

14. Consider light shining on a photographic plate. The light will be recorded if it dissociates an AgBr molecule in the plate. The minimum energy to dissociate this molecule is of the order 10^{-19} J. Evaluate the cutoff wavelength greater than which light will not be recorded.

[3 marks]

15. A 40 W incandescent bulb radiates from a tungsten filament operating at 3300 K. Assume that the bulb radiates like a black body.

    (a) What is the frequency, f_M, at the maximum of the spectral distribution?

[2 marks]

    (b) Assuming f_M is a good approximation of the average frequency of photons emitted by the bulb, about how many photons is is radiating per second?

[2 marks]

    (c) If you are looking at the bulb from 5.0 m away, how many photons enter your eye per second. Assume the diameter of the pupil of the eye is 5.0mm.

[3 marks]

16. For a region where the potential is zero, the wavefunction is given by:

$$\Psi(x) = \sqrt{\frac{2}{\alpha}} sin(\frac{3\pi x}{\alpha}). \quad (1)$$

(a) Calculate the energy of this system.

[2 marks]

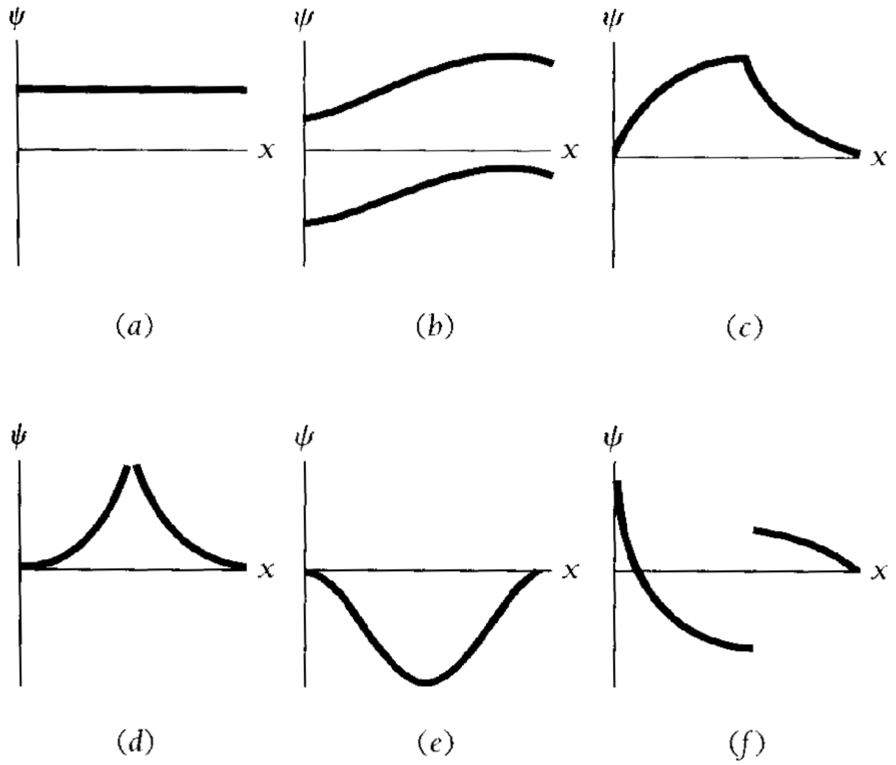
(b) What is the probability of finding the particle in the region from $x = 0$ to $x = 0.25\alpha$? Note that

$$\int sin^2(ax)dx = \frac{x}{2} - \frac{1}{4a} sin(2ax).$$

[3 marks]

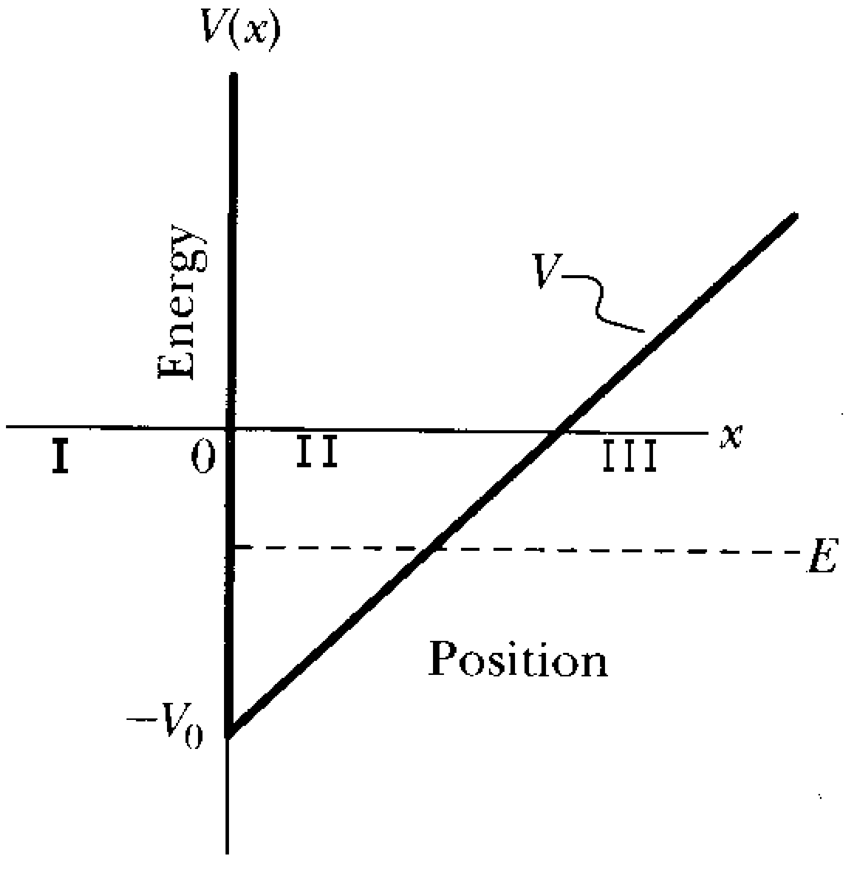
17. Which of the wave functions shown below cannot have physical significance in the interval shown? In each case explain why not.

[4 marks]



18. A particle of energy E is trapped within the potential shown below. The potential is infinite for $x < 0$ and increases linearly from $V = -V_o$ at $x = 0$. Carefully sketch the form of the wavefunction in each region (I, II and III). Pay attention to wavelength, amplitude and curvature. Explain for each region why the wavelength and amplitude vary as they do.

[4 marks]



19. The electron of a hydrogen atom is in the $3d$ state.

(a) The radial probability of finding the electron between r and $r + dr$ in this state is given by:

$$P(r)dr = A^2 \frac{r^6}{a_o^4} e^{-2r/3a_o} dr. \tag{2}$$

Find the most probable radius for the electron in this state.

[4 marks]

(b) Enumerate all sets of quantum numbers corresponding to the hydrogen atom in the $3d$-state.

[2 marks]

(c) Calculate the energy of the Hydrogen atom in the $3d$ state.

[2 marks]