

## Useful Information

### Constants

$$\begin{aligned}
 c &= 2.998 \times 10^8 \text{ m/s} \\
 h &= 6.626 \times 10^{-34} \text{ J.s} \\
 k &= 1.381 \times 10^{-23} \text{ J/K} \\
 e &= 1.602 \times 10^{-19} \text{ C} \\
 a_o &= 5.292 \times 10^{-11} \text{ m} \\
 G &= 6.674 \times 10^{-11} \text{ N.m}^2/\text{kg}^2 \\
 m_e &= 9.109 \times 10^{-31} \text{ kg} = 0.511 \frac{\text{MeV}}{c^2} \\
 m_N &= 1.675 \times 10^{-27} \text{ kg} = 939.6 \frac{\text{MeV}}{c^2} \\
 m_p &= 1.673 \times 10^{-27} \text{ kg} = 938.3 \frac{\text{MeV}}{c^2} \\
 m_{\pi^0} &= 135 \frac{\text{MeV}}{c^2} \\
 \sigma &= 5.67 \times 10^{-8} \frac{W}{m^2 K^4}.
 \end{aligned}$$

### Special Relativity

- $\gamma = \frac{1}{\sqrt{1 - \beta^2}}$
- $\beta = \frac{v}{c}$
- Time Dilation  $\Delta t = \gamma \Delta t_0$
- Length Contraction  $L = \frac{L_0}{\gamma}$
- Lorentz transformation for  $S \rightarrow S'$ 

$$\begin{aligned}
 x' &= \gamma(x - vt) \\
 y' &= y \\
 z' &= z \\
 t' &= \gamma(t - \frac{v}{c^2}x)
 \end{aligned}$$

- Lorentz velocity transformation for  $S \rightarrow S'$

$$\begin{aligned}
 u'_x &= \frac{u_x - v}{1 - \frac{v}{c^2}u_x} \\
 u'_y &= \frac{u_y}{\gamma(1 - \frac{v}{c^2}u_x)} \\
 u'_z &= \frac{u_z}{\gamma(1 - \frac{v}{c^2}u_x)}
 \end{aligned}$$

- Doppler Effect for light

$$f = f_0 \sqrt{\frac{1 \pm \beta}{1 \mp \beta}}$$

- relativistic momentum

$$\bar{p} = m\bar{v}\gamma$$

- relativistic energy

$$E = K + mc^2 = \gamma mc^2$$

- $E^2 = (pc)^2 + (mc^2)^2$

- Schwarzchild radius

$$R_s = \frac{2Gm}{c^2}$$

### Particles and Waves

- blackbody radiation

$$P = \sigma AT^4$$

- $\lambda_{\max}T = 2.898 \times 10^{-3} \text{ mK}$

- $R(\lambda) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$

- photoelectric effect

$$eV_s = hf - \Phi$$

- Compton effect

$$\Delta\lambda = \frac{h}{mc}(1 - \cos\phi)$$

- $\theta = \tan^{-1} \left[ \frac{\sin\phi}{\frac{\lambda'}{\lambda} - \cos\phi} \right]$

- photons  $E = hf, p = \frac{h}{\lambda}$

- matter waves  $\lambda = \frac{h}{p}$

- $\Delta x \Delta p_x \geq \frac{\hbar}{2}$

- $\Delta E \Delta t \geq \frac{\hbar}{2}$

### Interference and Diffraction

- Two-slit experiment

$$\Delta y = \frac{\lambda D}{d} \text{ (fringe spacing)}$$

- Bragg condition

$$\Gamma = n\lambda = d \sin\phi$$

- Single-slit diffraction

$$\sin\phi = \frac{\lambda}{d} \text{ (first min)}$$

## Bohr Model

- Radius  $r_n = a_0 n^2$
- Energy  $E_n = \frac{-13.6 eV}{n^2}$

## Quantum Mechanics

- 1-D TISE
- $$\frac{-\hbar^2}{2m} \frac{d^2\Psi(x)}{dx^2} + U\Psi(x) = E\Psi(x)$$
- $$k = \frac{\sqrt{2m(E - U)}}{\hbar} = \frac{2\pi}{\lambda}$$
- Probability to find particle between  $x_1$  and  $x_2$

$$P = \int_{x_1}^{x_2} |\Psi(x)|^2 dx$$

- expectation value of  $x$
- $$x_{av} = \langle x \rangle = \int_{-\infty}^{\infty} |\Psi(x)|^2 x dx$$

- Infinite Square Well

$$\Psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \quad n = 1, 2, 3, \dots$$

- $$E_n = \frac{n^2 \hbar^2 \pi^2}{2m L^2}$$
- $$\lambda_n = \frac{2L}{n}$$

- 1-D step 
$$R = \left(\frac{k_I - k_{II}}{k_I + k_{II}}\right)^2$$

- Potential Barrier 
$$T \sim e^{-2\alpha L}$$

$$\alpha = \frac{\sqrt{2m(V_o - E)}}{\hbar}$$

- Hydrogen Atom 
$$E_n = \frac{-13.6 eV}{n^2}$$

$$\Psi_{100} = \frac{1}{\sqrt{\pi} a_0^{3/2}} e^{-r/a_0}$$

$$P(r)dr = \frac{4r^2}{a_0^3} e^{-2r/a_0} dr \quad \text{in ground state}$$

$$E_n = \left(n + \frac{1}{2}\right) \hbar \sqrt{\frac{k}{m}} \quad n = 0, 1, 2, \dots$$

$$\Psi(x) = \left(\frac{\sqrt{km}}{\hbar\pi}\right)^{\frac{1}{4}} e^{-\frac{\sqrt{km}}{2\hbar}x^2} \quad \text{in ground state}$$