Experiment 6

Speed of Light

Introduction

This experimental apparatus determines the speed of light in air/media by measuring the phase difference between two optical signals of short and long traveling paths. The experiment can be expanded to measure the refractive index of other media such as organic glass or synthetic quartz, and various liquids by using a media tube.

The electronic unit is an integrated module that uses a GaAs laser diode with a central wavelength of 650 nm as the light emitter. The beam exits the device and is reflected back by a prism to a light receiving Silicon photodetector. With the reference and measurement signals represented by the light waves at the light-emitter and photodetector respectively, as shown in Figure 6.1, the phase difference between the two signals is proportional to the path length of the apparatus, as set by the reflecting prism.

The measured signal has a delay, $\phi = \omega t$ (t is the time interval of light traveling the distance between transmitter and receiver), relative to the reference signal. If the path length of the reflected signal changes by one wavelength, the phase between the signals will change by 2π radians. By moving the reflector along the rail and recording the change in phase with change in path length, the speed of light can be determined.

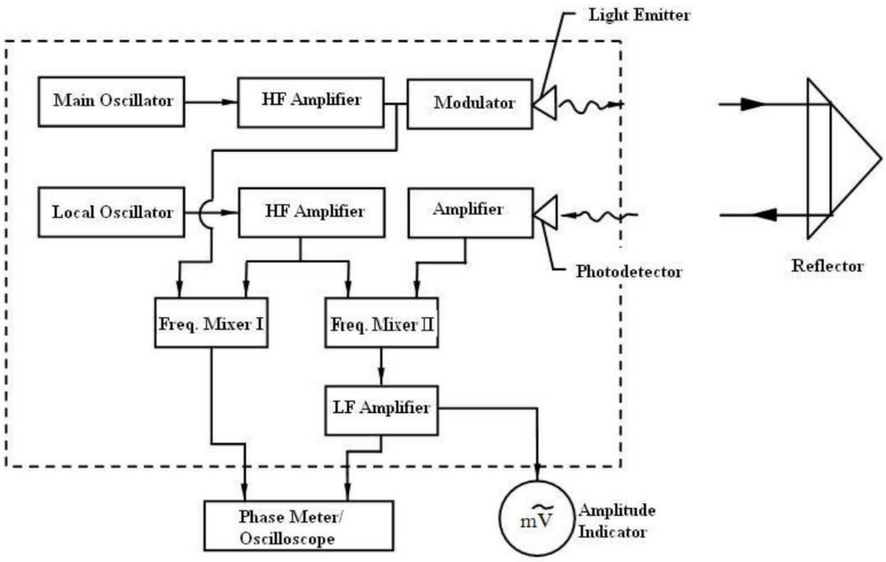

Figure 6.1: Internal schematic of the Speed of Light apparatus

Theory

The frequency f, wavelength λ and speed c of an electromagnetic wave are related by $f = c/\lambda$. By knowing any two parameters, the third parameter can be determined. It is difficult to directly measure these parameters for the light wave used in this apparatus since $\lambda = 650$ nm and $f \approx 10^{14}$ Hz.

However, by modulating the intensity of the laser beam with a relatively low frequency such as 100 MHz, the phase shift of the modulated signal can be measured instead since the modulated light wave still propagates at the speed c of the laser beam.

In practice, it is also difficult to directly measure the phase of this 100 MHz signal at high accuracy. Instead, a phase differential method is used where a local oscillator is mixed with two 100 MHz signals that are phase shifted by ϕ radians, resulting in two low-frequency signals that are also phase-shifted by ϕ radians, as follows.

When two sinusoidal signals of different frequency are input to a nonlinear element such as a diode or a transistor, the output signal contains a component at the frequency differential of the two input signals. In general, the output signal of a nonlinear element to an input signal x can be expressed as:

$$y(x) = A_0 + A_1 x + A_2 x^2 + \dots \tag{6.1}$$

By ignoring the higher-order terms, the second order term generates a frequency mixing effect. We begin by defining the high frequency reference and measurement signals u_r and u_m as

$$\begin{aligned} u_r &= U_r cos(\omega t + \phi_r) && (6.2) \\ u_m &= U_m cos(\omega t + \phi_r + \phi) && (6.3) \end{aligned}$$

and introduce a local oscillator u_o where

$$u_o = U_o cos(\omega_o t + \phi_o). \tag{6.4}$$

The initial phase of the reference and local oscillator signals are ϕ_r and ϕ_o respectively and ϕ is the phase shift amount of the modulated light wave traversing the optical path on the rail. By substituting 6.3 and 6.4 into 6.1 while omitting higher-order terms, we have:

$$y(u_m + u_o) \approx A_0 + A_1 u_m + A_1 u_o + A_2 u_m^2 + A_2 u_o^2 + 2A_2 u_m u_o. \tag{6.5}$$

Expanding the cross term using the identity $cos(A)cos(B) = \frac{1}{2}[cos(A+B) + cos(A-B)]$ yields:

$$\begin{aligned} 2A_2 u_m u_o &= 2A_2 U_m U_o cos(\omega t + \phi_r + \phi) cos(\omega_0 t + \phi_o) && (6.6) \\ &= 2A_2 U_m U_o (cos[(\omega + \omega_o)t + (\phi_r + \phi_o) + \phi] + cos[(\omega - \omega_o) + (\phi_r - \phi_o) + \phi]) && (6.7) \end{aligned}$$

Apart from the components of the two original frequencies, higher-order harmonic frequencies, and summing frequency, there is a component with the differential frequency of the two original signals as:

$$A_2 U_m U_o cos[(\omega - \omega_o)t + (\phi_r - \phi_o) + \phi] \tag{6.8}$$

Similarly, the differential term for the reference signal and local oscillator is:

$$A_2 U_r U_o cos[(\omega - \omega_o)t + (\phi_r - \phi_o)] \tag{6.9}$$

Comparing 6.8 and 6.9, the two differential signals down-converted with the local oscillator still have the same phase difference ϕ as that of the modulated 100 MHz reference and measured beams.

This experimental apparatus employs frequency differential and phase discrimination method by mixing the 100 MHz high frequency reference and measurement signals u_r and u_m with a local oscillator u_o at 100.455 MHz, respectively. As a result, two low frequency beat signals of 455 kHz with a phase difference of ϕ are generated.

Figure 6.1 shows the block diagram of the experimental setup, in which `Freq.Mixer I` generates the 455 KHz low frequency between reference signal and local oscillator at `Ref`$\sim$ while `Freq.Mixer II` generates the 455 KHz low frequency between the measurement signal and local oscillator at `P-Meas`$\sim$.

The oscilloscope

An oscilloscope displays on a two-dimensional grid the variation in voltage (y) with time (x) of one or more input signals. A digital scope includes a lot of features intended to simplify the measurement and analysis of these signals. You will use a small subset of these. The basic controls are grouped into the following functional blocks:

1. the VERTICAL controls set the voltage gain for each channel. Select a channel by pressing the CH1 or CH2 keys, then use the VOLTS/DIV knob to adjust the vertical resolution, or gain, of the waveform. The voltage gain per grid division is shown on the bottom of the display.

   By default, the display gain occurs about the ground level (V=0) of the signal. You can zoom in on any part of the waveform by setting the Expand menu selection to center; the zoom then occurs about the vertical center of the display. As you change the gain setting, you will likely need to re-center the region of interest using the vertical position knob.

2. the HORIZONTAL control similarly adjusts the common time scale for the two input channels.

3. the TRIGGER controls set the starting position (level) and direction (slope) of a signal relative to the origin ($t = 0$) on the x-axis. The trigger source can be either CH1 or CH2. The voltage level is set with the LEVEL knob and is monitored by an arrow on the right edge of the display. The trigger slope can be set to $\uparrow$ (positive) or $\downarrow$ (negative).

   The SINGLE capture mode causes the scope to wait for and record a single screen and then stop; this is useful when monitoring non-periodic events.

As with all things, familiarity comes with practice. Try out the various settings, if the signal disappears, press the Autoset button to display a best fit of the signals to the scope screen.

Procedure

CAUTION: Do not look directly at the laser beam emitted from the apparatus!

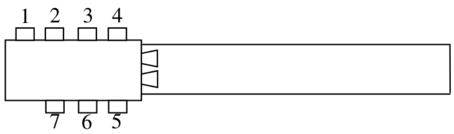

Figure 2 Assignment of BNC ports on electronic unit

1	F-Meas. – test port of 100 MHz high frequency signal
2	Mod. – Modulation signal input port (for simulation communication experiment)
3	Ref. ( ) – test port of 455 kHz square-wave signal (original reference)
4	Ref. ( ) – test port of 455 kHz sine-wave signal (original reference)
5&6	P-Meas. – test ports of 455 KHz sine-wave signal (differential signal for phase measurement)
7	Level – test port of receiving signal level (0.4 ~ 0.6 V)

Figure 6.2: Assignment of BNC ports on electronic unit

- Switch on the power to the oscilloscope and the Speed of Light apparatus. Ideally, the apparatus should be warmed up for at least 10 minutes before taking measurements to allow for the temperature of the electronic components to stabilize. Referring to Figure 6.2, verify that the scope `ch1` input is connected to `Ref`∼ (Port 4) and that the `ch2` input is connected to `P-Meas`∼ (Port 6) on the apparatus.

- Press the Autoset button; two 455KHz sine waveforms should appear on the screen. Try moving the reflector along the rail, the phase relationship between the two waveforms should change. The amplitude of the measured signal (ch2) should remain constant, otherwise the prism has to be adjusted so that the reflected beam will be aligned with the detector.

- Move the zero voltage markers at the left of the scope screen to overlap at the centre of the vertical axis, then adjust the gain of the two waves to maximize their vertical resolution. A horizontal resulution that displays around two periods of the waveforms, as shown in Figure 6 should yield the best result for your subsequent fit of these waveforms.

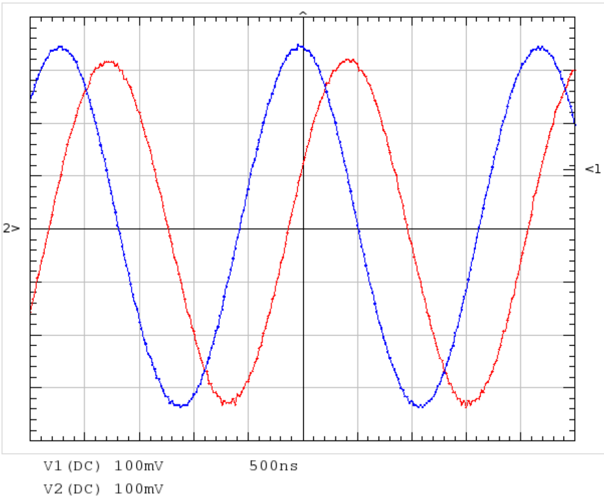

Figure 6.3: Properly scaled oscilloscope 455KHz reference and measured waveforms

6.1 Determination of the speed of light in air

You will now record and analyse the phase delay between the emitted and reflected waveforms as a function of the rail path length. For a series of $i = 1 \ldots 10$ reasonably spaced positions x_i along the whole length of the rail:

1. use the index mark on the prism cradle to carefully set an x_i position on the scale.

2. With the Physicalab data acquisition and analysis software connected to the Instek digital oscilloscope (Options menu), click $\boxed{\textbf{Get data}}$ to acquire the current waveforms. A three-column table of `time`, `ch1` and `ch2` amplitudes should appear in the data window. Click `File/Save` to save the data set. Note that the scope acquires and uploads data for sixteen time divisions (4000 points) but displays only the central ten on the scope screen. All of the data is used by the fitting routine.

3. Above the data window, check `ch1` to select that data set. Click $\boxed{\textbf{Draw}}$ to display the `(time,ch1)` graph, then select from the fitting equation drop-down list the generic equation of a sine curve, **A*sin(B*x+C)+D**. Check the `fit to:` box and click $\boxed{\textbf{Draw}}$ to perfor a fit of your `ch1` data.

   If the data fit fails, review the sine wave scatter plot to estimate better values for the fit parameters, where **A** is the wave amplitude and the angular velocity ω in radians/s is given by the fit parameter **B**$=2\pi/T$, with T being the wave period.

The initial phase angle (in radians) at time=0 is **C**, the quantity of interest. If **C**$> \pm 2\pi$, replace the initial guess values with those that resulted from the fit; this should minimize the value of **C**. In a spreadsheet, record this initial phase angle ϕ_r of the emitted wave.

4. Select `ch2` above the data table to display the `(time,ch2)` data set, then fit this data to determine the initial phase angle ϕ_m of the measured wave.

5. Calculate the phase difference $\phi = \phi_m - \phi_r$ at the scale setting of x_i.

You can now plot the series of points (ϕ_i, x_i) and fit this data to a straight line to determine the speed of light c from the slope of the graph. Note that since c is obtained from the *change in phase shift* with path length, the absolute path length is not important. The path length introduced by the reflecting prism as well as the double 180° phase inversion at the prism reflecting surfaces affects each measurement equally, as do any phase delays introduced by the electronic circuits in the apparatus.

1. Select `File/New` to clear the data window, then enter the points (ϕ_i, x_i) and click **Draw**. The points should exhibit a linear realtionship. Obviously misplaced points should be redone or removed from the data set.

2. Fit the data to y=**A+B*x** and record the value of the slope **B**.

With a phase shift $\Delta\phi$ corresponding to a change in path length $2\Delta x$, a phase shift of 2π corresponds to a change in path length of λ. With the $f = 100$ MHz modulated beam propagating at the speed of light c, the speed of light in air is thus determined:

$$\frac{\Delta\phi}{2\pi} = \frac{2\Delta x}{\lambda} \rightarrow \lambda = 2\pi\left(\frac{2\Delta x}{\Delta\phi}\right) = 4\pi\mathbf{B}; \qquad c = \lambda f = 4\pi\mathbf{B}f \tag{6.10}$$

6.2 Refractive index of test material

Position the reflector at the right end of the rail, setting an arbitrary path length of $2x_0$ for this experiment. With the resulting phase shift of this unobstructed wave given by

$$\phi_0 = \frac{4\pi x_0}{\lambda}, \tag{6.11}$$

determine and record the initial phase angle of the unobstructed waveform.

Place the sample material on the rail so that it intersects one or both of the optical paths. Adjust the orientation of the sample so that the beam falls on the detector. The sample decreases the propagating speed and thus shortens the wavelength of the light wave, effectively increasing the optical path length and causing a further phase shift in the measured signal. This phase shift is given by

$$\phi_1 = \frac{4\pi[x_0 + d(n-1)]}{\lambda}, \tag{6.12}$$

where n is the reflective index and d is the length along the optical path of the test material. Determine and record the initial phase angle of this waveform.

By subtracting 6.11 from 6.12, we get the phase change due to the insertion of the test material into the optical path:

$$\Delta\phi = |\phi_1 - \phi_0| = \frac{4\pi d(n-1)]}{\lambda} \tag{6.13}$$

Using the wavelength $\lambda = c/f$ of the modulating wave and these measured phase shifts, the refractive index of the liquid in the tube can be derived from 6.13. Compare the result for $\Delta\phi$ using λ obtained from your previously derived value for c and by using the accepted value for the speed of light in a vacuum.