Experiment 5

The Prism Spectrometer

In this experiment you will determine the refractive index \( n(\lambda) \) of a glass prism by measuring the minimum deviation angle \( D(\lambda) \) with the spectrometer.

The Spectrometer

The spectrometer consists of:

- a light source \( L \), in this case a helium (He) discharge tube
- a slit \( S \) with one adjustable edge
- a mask \( M \) which determines the vertical height of the slit
- a collimator \( C \) which produces a parallel beam of light from the illuminated slit as a source
- a prism \( P \) on a table (the table can tilted with three adjusting screws)
- a thin glass plate \( G \), with a set of crosshairs etched into it
- a telescope \( T \) which forms, from the incident parallel beam, a sharp image of the slit on \( G \)
- an eyepiece \( E \), which acts as a magnifier

5.1 Setting-up the spectrometer

5.1.1 Focusing the telescope

First adjust the optical components of the spectrometer as follows:

1. Adjust the eyepiece position until the crosshairs are seen sharply with the un-accommodated eye.

2. Direct the telescope to an infinitely far-away object, and adjust the distance between \( G \) and the telescope objective lens until the object is seen sharply on the crosshairs. The telescope is now focused for a parallel incident beam.

3. Direct the telescope towards the collimator and adjust the distance between the collimator and slit \( S \) until \( S \) is seen sharply on the crosshairs. Then the collimator is focused to produce a parallel beam from the illuminated slit.
5.1.2 Levelling the table

Put the prism on the prism table, its apex facing the collimator. Swing the telescope around until you see the image of the slit, formed by reflection on one of the polished prism faces. Adjust the mask $M$ until the slit image is small. If the prism table is “level” (strictly speaking, if the plane of the table is exactly parallel to the plane formed by the axes of the collimator and telescope), then this image will be centered on the crosshairs. If this is not the case, adjust the tip-and-tilt of the table with the three adjusting screws. After “levelling” is obtained, swing the telescope around until the slit is seen reflected from the other polished prism face. “Level” this image. By successively observing the slit image reflected from both prism faces, complete the “levelling” of the prism table. Repeat the “levelling” procedure with the prism apex turned 60° away from the collimator.

5.2 Measuring the apex angle

The relative angular position of the prism table and telescope can be read on the circular scale. The scale has two angular verniers labeled I and II, approximately 180° apart, which can be read to 1' (one minute of angle) with a magnifier. Teach yourself how to properly use these verniers. When you know how to do this, measure the prism apex angle $A$ as follows:

1. Set the prism on the table with its apex facing the collimator.
2. Clamp the prism table in place.
3. Swing the telescope around until the slit image, reflected from one prism face, is exactly on the crosshairs.
4. Read the verniers. Let the angles be $\theta_1(I)$ and $\theta_1(II)$.
5. Swing the telescope around until the slit image, reflected from the other prism face, is exactly on the crosshairs.
6. Again read the verniers and let the angles be $\theta_2(I)$ and $\theta_2(II)$. The apex angle can be calculated from the following equations:

$$2A = \theta_1(I) - \theta_2(I) \quad \text{and} \quad 2A = \theta_1(II) - \theta_2(II)$$

Derive these equations and determine the value of $A$ (and its error) from your data.

### 5.3 The Index of Refraction

You now should have adjusted the spectrometer and determined the prism apex angle $A$. The spectrometer can now be used to determine the dispersion of the prism, that is, the variation of its refractive index $n$ with wavelength $\lambda$. The deviation angle $\theta$ for a light ray passing through a prism depends on: apex angle $A$, angle of incidence $i$ and refractive index $n$, which in turn depends on the wavelength $\lambda$. The angle $\theta$ is a function of these other quantities, i.e. $\theta = f(i, A, n(\lambda))$. For a given value of $n(\lambda)$ and $A$, there is a value of $i$ for which $\theta$ is a minimum (see your textbook); this angle is called the minimum deviation angle $D$, and is given by

$$n(\lambda) = \frac{\sin \frac{1}{2}(A + D(\lambda))}{\sin \frac{1}{2}A} \quad (5.1)$$

From measurement of $D(\lambda)$ for a given prism we can therefore determine the dispersion $n(\lambda)$ of the prism.

### 5.4 Procedure

Illuminate the spectrometer slit with light from the He discharge tube. The tube emits a line spectrum at the wavelengths listed in Table 5.1.

1. Set the prism on the prism table so that the light from the collimator fills one polished face.
2. Rotate the prism table and telescope until they are approximately in the minimum deviation position. You should then see the spectrum of the He discharge tube.
3. To find the minimum deviation position for a certain wavelength, rotate the prism table until the slit image (for that wavelength) just reverses its direction of rotation.
4. Clamp the table in this position, and rotate the telescope until the slit image is exactly on the crosshairs. Read both verniers, $\theta_1(I)$ and $\theta_1(II)$. 

Colour | Wavelength (nm) | Intensity
---|---|---
dark red | 706.5 | weak
red | 667.8 | strong
yellow | 587.6 | strong
green | 501.6 | medium-strong
green blue | 492.2 | medium
blue | 471.3 | medium
blue purple | 447.1 | medium-strong

Table 5.1: Wavelengths and intensities of lines in the He spectrum

5. Remove the prism and rotate the telescope until the “straight-through” slit image is again on the crosshairs. Let the vernier readings be \( \theta_2(I) \) and \( \theta_2(II) \).

Calculate \( D \) for this wavelength from \( D = \theta_1(I) - \theta_2(I) \) and \( D = \theta_1(II) - \theta_2(II) \). Repeat the measurements for all wavelengths, and calculate from it \( n(\lambda) \). Graph \( n(\lambda) \) versus \( \lambda \).

5.5 The Cauchy Relation

For most transparent materials \( n(\lambda) \) can be expressed quite well by the relation

\[
n(\lambda) = n_0 + \frac{B}{\lambda^2}
\]  

From your \( n(\lambda) \) data determine the constants \( n_0 \) and \( B \) for the prism glass by plotting \( n(\lambda) \) versus \( 1/\lambda^2 \).

In your report derive equation (5.1). Do proper error analysis; determine \( \sigma n \) from the errors \( \sigma A \) and \( \sigma D \). Since the angles \( A \) and \( D \) can be read with great accuracy using the verniers, \( n(\lambda) \) can be determined very accurately.