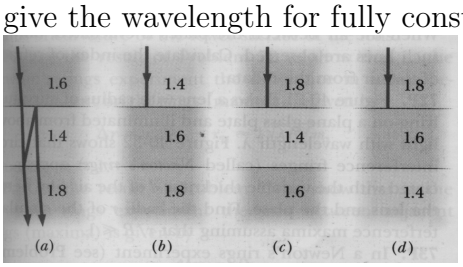
PHYSICS 2P51: Wave Optics

1. One of the slits of a double slit arrangement is wider than the other, so that the amplitude of the electric field reaching the central part of the screen from one slit, acting alone is twice that of the other slit, acting alone. Derive an expression for the intensity I in terms of θ, where θ is defined as usual for two-slit geometry.

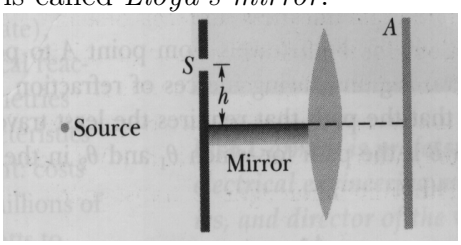
2. In the figure below, light is incident perpendicularly on four thin layers of thickness L. the indices of refraction of the thin layers and of the media above and below these layers are given. let λ represent the wavelength of light in air, and n_2 represent the index of refraction of the thin layer in each situation. Consider only transmission of light that undergoes either (i) no reflection or (ii) two reflections as in the figure. For which of the situations does

$$\lambda = \frac{2Ln_2}{m} \qquad m = 1, 2, 3$$

give the wavelength for fully constructive interference?



3. In the figure below, monochromatic light of wavelength λ diffracts through a narrow slit S in an otherwise opaque screen. On the other side, a plane mirror is perpendicular to the screen and a distance h from the slit. A lens is placed at the far end of the mirror, and a viewing screen A is placed at the focal distance of the lens. Light form the slit that travels directly through the lens to A interferes with light from the slit that reflects from the mirror, through the lens to A. This arrangement for obtaining a double-slit interference pattern from a single slit is called *Lloyd's mirror*.

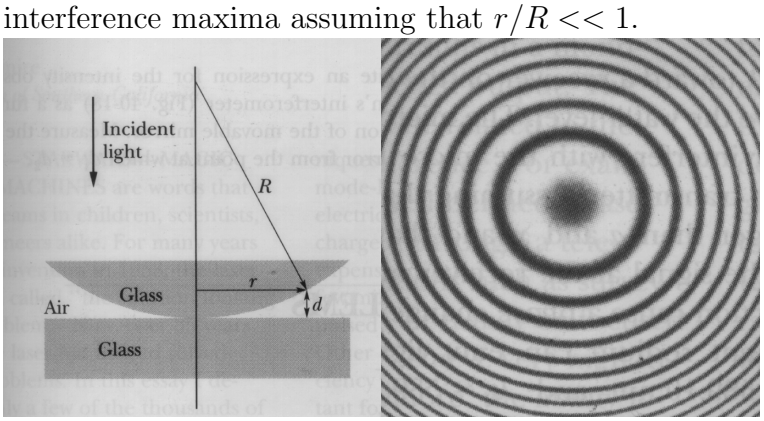


   (a) What, if any, is the phase change due to reflection?

   (b) Is the fringe that corresponds to zero path length difference bright or dark?

   (c) Find expressions that locate the bright and dark fringes by considering the image of S produced by the mirror.

4. The figure below shows a lens with radius of curvature R lying on a plane glass plate and illuminated from above by light with wavelength λ. The adjacent figure shows the circular interference fringes (called *Newton's rings*) appear, associated with the variable thickness d of the air film between the lens and the plate. Find the radii of the circular interference maxima assuming that $r/R << 1$.



5. Show that the values of $\alpha = \frac{\pi a}{\lambda} \sin\theta$ at which the intensity maxima for single slit diffraction can be found by solving the transcendental equation

$$\tan\alpha = \alpha$$

Solutions to transcendental equations are usually impossible to find algebraically. Instead, the solutions can be found either graphically or numerically.

6. The full width at half-maximum (FWHM) of the central maximum is defined as the angle between the two points in the pattern where the intensity is one-half that at the centre of the pattern.

   (a) Show that, for single slit diffraction, the intensity drops to one-half of the maximum when $\sin^2\alpha = \alpha^2/2$.

   (b) By using a graph, show that this transcendental equation has a solution at approximately $\alpha = 1.39$.

   (c) Show that the FWHM is $\Delta\theta = 2\sin^{-1}(0.443\lambda/a)$.

   (d) Calculate the the FWHM of the central maximum for slits whose widths are 1.0, 5.0 and 10 wavelengths.

7. Derive this expression for the intensity pattern for a three-slit "grating":

$$I = \frac{1}{9} I_m \left(I + 4\cos\phi + 4\cos^2\phi \right)$$

where

$$\phi = \frac{2\pi d \sin\theta}{\lambda}$$

Assume that $a << \lambda$.

8. It is desired to have an actual double slit with a spacing such that the third order image does not appear (because it corresponds to a diffraction minimum from the width of the slits). If the separation between the centers of the slits is 0.125 mm and if light with wavelength of 532 nm is used, what should the width of the slit be?

9. You want to design a grating spectrometer to cover the first-order spectrum completely from 360 nm to 675 nm. The image sensor will that will detect the first-order image on one side is 22.5 mm across. The spacing from the diffraction grating is 5.00 cm. What grating spacing in (lines/cm) should be used?

10. A grating has 350 rulings/mm and is illuminated at normal incidence with white light. A spectrum is formed on a screen 30 cm from the grating. If a 10-mm square hole is cut in the screen, its inner edge being 50 mm from the central maximum and parallel to it, what range of wavelengths passes through the hole?

11. Consider a wave disturbance given by the expression

$$\vec{E}(z,t) = [\hat{x}\cos\omega t + \hat{y}\cos(\omega t - \pi/2)]\, E_o \sin kz$$

. What kind of a wave is it? Draw a rough sketch showing its main features.

12. We want to rotate the direction of polarization of polarized light by 90° by sending the beam through one or more polarizing sheets.

    (a) What is the minimum number of sheets?

    (b) What is the minimum number of sheets required if the transmitted intensity is to be more than 70% of the original (linearly polarized) intensity?

    (c) What is the minimum number of sheets required if the transmitted intensity is to be more than 80% of the original (linearly polarized) intensity?

13. Compute the critical angle for total internal reflection for the ordinary ray i.e. the angle for total internal reflection at the calcite - Canada Balsam gum layer of a Nicol prism.

14. A ray of yellow light is incident on a calcite plate at 50°. The plate is cut so that the optic axis is parallel to the front face, and perpendicular to the plane of incidence. Find the angular separation between the two emerging rays.

15. When red light in vacuum is incident at the Brewster angle on a certain glass slab, the angle of refraction is 32°. What are

    (a) the index of refraction of the glass

    (b) the Brewster angle.

16. The calcite crystal in the figure below is shown in the different orientations. Its blunt corner -through which the optical axis (three fold axis of symmetry) passes - is on the left in (a), the lower left in (b) and the bottom in (c). The polaroid's transmission axis is horizontal. Explain each photo, especially (b).