PHYSICS 2P51: Assignment 11 - Due Fri. April 7, 2017

1. **Diffraction Gratings**

   (a) A grating, used in second order, diffracts light of 400 nm wavelength thrown an angle of 32°. How many lines per mm does the grating have?

   (b) A grating has 6000 slits ruled across a width of 5 cm. What is the color of the light whose two fifth order maxima are 40^0 apart?

   (c) Prove that the intensity at the maxima of diffraction approaches $N^2 I_o$ where $\delta = 2\pi \frac{d \sin\theta}{\lambda}$ is the phase difference between light originating from adjacent slits of the grating and the observed intensity satisfies

   $$I = I_o \frac{\sin^2(N\delta/2)}{\sin^2(\delta/2)}$$

   (d) Red light of wavelength 630 nm, diffracted by a grating in a given order, overlaps yellow light of 540 nm wavelength, diffracted by the same grating in the next higher order. What are the two orders?

   (e) Consider a grating monochromator whose light source is an incandescent tungsten filament which operates approximately as a black body at 2700 K.

      i. At what wavelength does the light produce maximum intensity?

      ii. The tungsten lamp is a broadband source of radiation. For what wavelength range can a user be certain to have monochromatic output using a Silicon filter if the grating is operated in first order ?

2. **Coherence**

   (a) If light of $\lambda = 636nm$ has wavetrains that are 30λ long, what is its

      i. coherence length?

      ii. coherence time?

   (b) If the contrast in an interference pattern is 50%, and if the maxima receive 20 units of light, much light do the minima receive?

   (c) Two wavetrains overlap to 37% of their length. If the maxima in the resulting interference patters receive 30 units of light, how much do the minima receive?

   (d) If we put a red filter (with a peak transmission of 600 nm) over a slit that is 1 mm wide and mounted in front of a light source, how close to the source could we place a double slit (with a slit separation of of 1.2 mm) to observe fringes of satisfactory contrast?

3. **Polarization**

   (a) Describe completely the state of polarization of each of the following waves:

      i. $\vec{E} = \hat{x}E_o \cos(kz - \omega t) - \hat{y}E_o \cos(kz - \omega t)$

      ii. $\vec{E} = \hat{x}E_o \sin 2\pi(z/\lambda - ft) - \hat{y}E_o \sin 2\pi(z/\lambda - ft)$

      iii. $\vec{E} = \hat{x}E_o \sin(\omega t - kz) - \hat{y}E_o \sin(\omega t - kz + \pi/2)$

      iv. $\vec{E} = \hat{x}E_o \sin(\omega t - kz) - \hat{y}E_o \sin(\omega t - kz - \pi/4)$

   (b) Determine the Brewster's angle for light travelling from air and reflecting of glass of index of refraction 1.52.

   (c) Consider unpolarized light of intensity 5mW/m^2 that is incident on seven linear polarizers. The transmission axis of each polarizer is rotated $15°$ with respect to the previous polarizer.

      i. Determine the intensity of light passing through the seven polarizers.

      ii. If one of middle five polarizers is removed, determine the intensity of light passing through the six remaining polarizers.

(d) Suppose that an ideal polarizer is rotated at a rate ω between a pair of stationary crossed polarizers. Show that the emergent flux density will be modulated at four times the rotational frequency. In other words, show that

$$I = \frac{I_1}{8}\left(1 - \cos 4\omega t\right)$$

(e) The two indices of refraction of a birefrigent crystal are 1.501 and 1.512 at $\lambda = 400$ nm. How thick does the crystal have to be in order to serve as a quarter wave plate for this wavelength?