PHYSICS 2P51: Assignment 8 - Due March 15, 2017

1. **Wave Equation and Travelling Waves**

   (a) Demonstrate, via partial differentiation, that a traveling gaussian wave pulse, represented by the function $y = \sin[kx - \omega t]$ satisfies the one-dimensional wave equation. Find an expression for the velocity in terms of ω and k.

   (b) Which of the following functions could represent travelling waves ? If the function does represent a travelling wave, determine the velocity of the wave where x is measured in m and t is measured in s.

      i. $y = \cos[(x - 2t)^2]$
      ii. $y = x^3 - 4t^3$
      iii. $y = \sin(x^2 - 10xt + 25t^2)$

   (c) A traveling wave pulse is represented by the gaussian function

   $$y = 2e^{-(x-4t-5)^2}$$

   where x and t are measured in m and s, respectively.

      i. Where is the center of the pulse at t=0 ?
      ii. When will the center of the pulse pass through x= 25 m?

2. Consider an electromagnetic wave that whose electric field can be represented by

   $$\vec{E} = 500\hat{x}\sin(\frac{\pi}{250}z - \omega t)$$

   (a) If z is measured in nm, what is the wavelength of the light ?

   (b) If the light is travelling through vacuum, what is the frequency of the light in Hz?

   (c) What is the equation for the magnetic field?

   (d) What is the intensity of the light?

   (e) Determine the photon energy in eV and the photon flux in $m^{-2}s^{-1}$

3. Two narrow slits, 0.8 mm apart from each other, are used to produce interference. If, on a screen 80 cm away the distance between the two second order maxima is 2 mm, what is the wavelength of the light.

4. When one of the slits in a Young's experiment is covered by a film of transparent material, the zeroth order fringe is seen to shift by 1.2 fringes. If the refractive index of the material is 1.5, and the wavelength of the light is 600 nm, how thick is the film?

5. **Complex Numbers** Express the following numbers in the $x+iy$ form. Try to visualize each complex number using the argand plane. The first four problems are quite easy and should be able to be done without calculators.

   (a) $9e^{3\pi i/2}$

   (b) $e^{-2\pi i} - e^{-4\pi i} + e^{-6\pi i}$

   (c) $e^{i\pi} + e^{-i\pi}$

   (d) $e^{i\pi/2} - e^{-i\pi/2}$

   (e) $2e^{5\pi i/6}$

   (f) $\frac{1}{(1+i)^3}$

   (g) $(1+i)^2 + (1+i)^4$

   (h) $5e^{i\pi/4} + 3e^{-i\pi/6}$

6. **Phasors** Consider three coherent sources of 600 nm radiation. The first is 30 μm from point P while the second and third are 30.075 μm and 30.15 μm from point P, respectively. The amplitudes of the electric fields of the three sources at point P are 300, 200 and 100 N/C, respectively.

   (a) If we represent the electric field of the first source as $Im(300e^{i\omega t})$, determine the amplitude and phase of the resultant field at point P.

   (b) Assuming that these fields are interfering in vacuum, determine the time average intensity of the light at point P.

   (c) Would you characterize the interference as constructive, partially constructive, partially destructive or destructive? Explain.