

Vectors

$$\int_V \vec{\nabla} \cdot \vec{V} d^3r = \oint_S \vec{V} \cdot d\vec{a} \quad \text{divergence theorem}$$

$$\int_S \vec{\nabla} \times \vec{V} \cdot d\vec{a} = \oint_P \vec{V} \cdot d\vec{l} \quad \text{Stoke's theorem}$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{V}) = 0 \quad \vec{\nabla} \times \vec{\nabla} \psi = 0$$

$$\vec{\nabla} \times \vec{\nabla} \times \vec{A} = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A}$$

$$(\vec{A} \times \vec{B})_i = \epsilon_{ijk} A_j B_k \quad \vec{A} \cdot \vec{B} = A_i B_i$$

$$\epsilon_{ijk} \epsilon_{ilm} = \delta_{jl} \delta_{km} - \delta_{jm} \delta_{kl}$$

$$\vec{r} = \vec{r} - \vec{r}'$$

Maxwell's equations for Electrostatics

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad \vec{\nabla} \times \vec{E} = 0$$

Electrostatics

$$\vec{F} = q\vec{E} \quad \nabla^2 V = -\frac{\rho}{\epsilon_0}$$

$$\vec{E} = \sum_i \frac{1}{4\pi\epsilon_0} \frac{q_i}{r_i^2} \hat{r}_i \quad \int \int_S \vec{E} \cdot d\vec{a} = \frac{Q_{inside}}{\epsilon_0}$$

$$\vec{E} = -\vec{\nabla} V \quad V = \sum_i \frac{q_i}{4\pi\epsilon_0 r_i} \quad V = \int \int \int \frac{\rho}{4\pi\epsilon_0 r} d^3\tau'$$

$$U_{TOT} = \frac{1}{2} \sum_i \sum_j \frac{1}{4\pi\epsilon_0} \frac{q_i q_j}{r_{ij}} \quad U_{TOT} = \int \int \int_{all\ space} \frac{1}{2} \epsilon_0 E^2 d\tau \quad U_{TOT} = \int \int \int \rho V d\tau$$

$$C = Q/V$$

Fourier Series and Legendre Polynomials

$$\int_0^L \cos\left(\frac{m\pi x}{L}\right) \cos\left(\frac{n\pi x}{L}\right) dx = \int_0^L \sin\left(\frac{m\pi x}{L}\right) \sin\left(\frac{n\pi x}{L}\right) dx = \frac{L}{2} \delta_{mn}$$

$$\int_0^L \sin\left(\frac{m\pi x}{L}\right) \cos\left(\frac{n\pi x}{L}\right) dx = 0$$

$$(1-x^2)P_l''(x) - 2xP_l'(x) + l(l+1)P_l(x) = 0$$

$$g(t, x) = (1-2xt+t^2)^{-1/2} = \sum_{l=0}^{\infty} P_l(x)t^l \quad \int_{-1}^1 P_l(x)P_l'(x)dx = \delta_{ll'} \frac{2}{2l+1}$$

$$P_0(x) = 1 \quad P_1(x) = x \quad P_2(x) = \frac{(3x^2-1)}{2} \quad P_3(x) = \frac{(5x^3-3x)}{2} \quad P_4(x) = \frac{(35x^4-30x^2+3)}{8}$$

Multipole Expansions

$$V(\vec{r}) \approx \frac{Q_{TOT}}{4\pi\epsilon_0 r} + \frac{\vec{p} \cdot \hat{r}}{4\pi\epsilon_0 r^2} + \dots$$

$$\vec{p} = \sum_i q_i \vec{r}'_i = \int \rho \vec{r}' d^3\tau' \quad \text{electric dipole moment}$$

$$\vec{E} = \frac{p}{4\pi\epsilon_0 r^3} [2 \cos \theta \hat{r} + \sin \theta \hat{\theta}] \quad \text{field due to dipole oriented along the z-axis}$$

Materials

$$\vec{p} = \alpha \vec{E}$$

$$E = -\vec{p} \cdot \vec{E} \quad \vec{\tau} = \vec{p} \times \vec{E} \quad \text{properties of dipoles in external fields}$$

$$\rho = \rho_f + \rho_b \quad \rho_f \text{ is excess charge placed by experimenter}$$

$$-\vec{\nabla} \cdot \vec{P} = \rho_b \quad \vec{P} \cdot \hat{n} = \sigma_b \quad \text{polarized material equivalent to } \rho_b, \sigma_b$$

$$\vec{P} = \frac{1}{V} \sum_i \vec{p}_i$$

$$\vec{D} = \epsilon_o \vec{E} + \vec{P} \quad \text{always true}$$

$$\vec{\nabla} \cdot \vec{D} = \rho_f \quad \vec{\nabla} \times \vec{D} = \vec{\nabla} \times \vec{P}$$

$$\vec{P} = \chi_e \epsilon_o \vec{E} \quad \vec{D} = \epsilon_o \epsilon_r \vec{E} = (1 + \chi_e) \epsilon_o \vec{E} \quad \text{l.i.h dielectrics}$$