Vectors

$\int_V \vec{\nabla}\cdot\vec{V} d^3r = \oint_S \vec{V}\cdot d\vec{a}$ divergence theorem
$\int_S \vec{\nabla}\times\vec{V}\cdot d\vec{a} = \oint_P \vec{V}\cdot d\vec{l}$ Stoke's theorem
$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$
$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{V}) = 0 \qquad \vec{\nabla}\times\vec{\nabla}\psi = 0$
$\vec{\nabla}\times\vec{\nabla}\times\vec{A} = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A}$
$(\vec{A}\times\vec{B})_i = \epsilon_{ijk}A_jB_k \qquad \vec{A}\cdot\vec{B} = A_iB_i$
$\epsilon_{ijk}\epsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}$
$\vec{\mathscr{r}} = \vec{r} - \vec{r'}$

Maxwell's equations for Electrostatics

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_o} \qquad \vec{\nabla}\times\vec{E} = 0$$

Electrostatics

$$\vec{F} = q\vec{E} \qquad \nabla^2 V = -\frac{\rho}{\epsilon_o}$$

$$\vec{E} = \sum_i \frac{1}{4\pi\epsilon_o}\frac{q_i}{\mathscr{r}_i^2}\hat{\mathscr{r}} \qquad \int\int_S \vec{E}\cdot d\vec{a} = \frac{Q_{inside}}{\epsilon_o}$$

$$\vec{E} = -\vec{\nabla}V \quad V = \sum_i \frac{q_i}{4\pi\epsilon_o\mathscr{r}} \quad V = \int\int\int \frac{\rho}{4\pi\epsilon_o\mathscr{r}}d^3\tau' \qquad \Delta V = -\int_a^b \vec{E}\cdot d\vec{\ell}$$

$$U_{TOT} = \frac{1}{2}\sum_i\sum_j \frac{1}{4\pi\epsilon_o}\frac{q_iq_j}{r_{ij}} \quad U_{TOT} = \int\int\int_{allspace}\frac{1}{2}\epsilon_o E^2 d\tau \quad U_{TOT} = \frac{1}{2}\int\int\int \rho V d\tau$$

$$C = Q/\Delta V$$

Fourier Series and Legendre Polynomials

$\int_0^L \cos(\frac{m\pi x}{L})\cos(\frac{n\pi x}{L})dx = \int_0^L \sin(\frac{m\pi x}{L})\sin(\frac{n\pi x}{L})dx = \frac{L}{2}\delta_{mn}$
$\int_0^L \sin(\frac{m\pi x}{L})\cos(\frac{n\pi x}{L})dx = 0$

$$(1-x^2)P_l''(x) - 2xP_l'(x) + l(l+1)P_l(x) = 0$$

$$g(t,x) = (1-2xt+t^2)^{-1/2} = \sum_{l=0}^{\infty} P_l(x)t^n \qquad \int_{-1}^{1} P_l(x)P_{l'}(x)dx = \delta_{ll'}\frac{2}{2l+1}$$

$P_0(x) = 1 \quad P_1(x) = x \quad P_2(x) = \frac{(3x^2-1)}{2} \quad P_3(x) = \frac{(5x^3-3x)}{2} \quad P_4(x) = \frac{(35x^4-30x^2+3)}{8}$

Multipole Expansions

$V(\vec{r}) \approx \frac{Q_{TOT}}{4\pi\epsilon_o r} + \frac{\vec{p}\cdot\hat{r}}{4\pi\epsilon_o r^2} + \cdots$

$\vec{p} = \sum_i q_i \vec{r_i'} = \int \rho \vec{r'} d^3\tau'$ electric dipole moment

$\vec{E} = \frac{p}{4\pi\epsilon_o r^3}\left[2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right]$ field due to dipole oriented along the z-axis

Materials

$\vec{p} = \alpha\vec{E}$

$E = -\vec{p}\cdot\vec{E}$ $\quad \vec{\tau} = \vec{p}\times\vec{E}$ properties of dipoles in external fields

$\rho = \rho_f + \rho_b$ $\quad \rho_f$ is excess charge placed by experimenter

$-\vec{\nabla}\cdot\vec{P} = \rho_b$ $\quad \vec{P}\cdot\hat{n} = \sigma_b$ polarized material equivalent to ρ_b, σ_b

$\vec{P} = \frac{1}{V}\sum_i \vec{p}_i$

$\vec{D} = \epsilon_o\vec{E} + \vec{P}$ always true

$\vec{\nabla}\cdot\vec{D} = \rho_f$ $\quad \vec{\nabla}\times\vec{D} = \vec{\nabla}\times\vec{P}$

$\vec{P} = \chi_e\epsilon_o\vec{E}$ $\quad \vec{D} = \epsilon_o\epsilon_r\vec{E} = (1+\chi_e)\epsilon_o\vec{E}$ l.i.h dielectrics