Assignment No. 1

Physics 3P36

Due Monday, January 15, 2018

1. Figure 1 shows the essentials of a mass spectrometer which is used to measure the masses of ions.

   Figure 1

   An ion of mass m and charge $q > 0$ is produced in source S, a chamber in which a gas discharge is taking place. The initially stationary ion leaves S, is accelerated by potential difference V, and then enters a separator chamber in which there is magnetic field $\mathbf{B}$. In the field it moves in a semicircle, striking a photographic plate a distance s from the entry slit.

   (a) Express m in terms of known q, V, B, and the measured s.

   (b) Two types of singly ionized atoms with masses m and $m + \Delta m$, respectively, strike the photographic plate at spots separated by Δs. Express Δm in terms of the ionic charge, V, B, m, and Δs. Calculate Δs for a beam of singly ionized chlorine atoms of masses 35 and 37 u (1 u$= 1.661\times10^{-27}$ kg) if $V = 7.3$ kV and $B = 0.50$ T.

(c) Uranium atoms of mass 3.92×10^{-25} kg and charge 3.20×10^{-19} C are separated from related species by a mass spectrometer. The ions are first accelerated through a potential difference of 100 kV and then they pass into a magnetic field, where they are bent into a path of radius 1.00 m. After traveling through 180°, they are collected in a cup after passing through a slit of width 1.00 mm and height 1.00 cm.

i. What is the magnitude of the magnetic field in the separator?

ii. If the machine is designed to separate out 100 mg of material per hour, calculate the current of the desired ions in the machine and the thermal energy dissipated in the cup in 1.00 h.

2. An electron (charge -e) moves in a constant electric field **E** along the x-axis and a constant magnetic field **B** along the z-axis. The initial velocity $\mathbf{v}_0$ of the electron is in the xy-plane z =0.

(a) Write down the equations of motion in Cartesian coordinates.

(b) Solve the equations of motion.

(c) By eliminating time from the solution in b) show that the trajectory of the electron could be viewed as a circle of radius

$$R = \frac{1}{\omega}\sqrt{\dot{x}^2(0) + \left(\dot{y}(0) + \frac{E}{B}\right)^2}\,, \quad \omega = \frac{eB}{m}\,,$$

whose center drifts in the negative y-direction with a constant speed E/B.

(d) Use your result in (b) to show that the charges with the same charge/mass ratio (i.e. the same ω) and with the identical initial position $x(0)$, $y(0)$, $z(0)$ will end up at the same spot in the xy-plane after one period $T = 2\pi/\omega$ regardless of their initial velocities in the xy-plane, thereby demonstrating the charge-focusing ability of crossed constant electric and magnetic field.

(e) Assuming $x(0)$=0, $y(0)$=0, $\dot{x}(0)$=0, and $\dot{y}(0)$ >0, use some graphics package to plot the trajectory of the charge from t =0 to $t = 2\pi/\omega$. (You might want to take that $\dot{y}(0)/\omega$ =1 and $E/(\omega B)$ =1.)

3. In systems with axial symmetry it is convenient to use cylindrical coordinates ρ, ϕ and z (Fig.2).

Figure 2

(a) Show that the unit vectors $\hat{\rho}$ and $\hat{\phi}$ are related to Cartesian unit vectors $\hat{x}$ and $\hat{y}$ by

$$\hat{\rho} = \cos\phi\hat{x} + \sin\phi\hat{y}\ , \quad \hat{\phi} = -\sin\phi\hat{x} + \cos\phi\hat{y}\ ,$$

and that a position vector $\mathbf{r}$ can be written as

$$\mathbf{r} = \rho\hat{\rho} + z\hat{z}\ .$$

(b) As the particle moves the unit vectors $\hat{\rho}$ and $\hat{\phi}$ change in time while $\hat{x}$, $\hat{y}$ and $\hat{z}$ are time-independent. Show that

$$\begin{aligned}
\dot{\hat{\rho}} &= \dot{\phi}\hat{\phi}\ , \\
\dot{\hat{\phi}} &= -\dot{\phi}\hat{\rho}\ , \\
\dot{\mathbf{r}} &= \dot{\rho}\hat{\rho} + \rho\dot{\phi}\hat{\phi} + \dot{z}\hat{z}\ , \\
\ddot{\mathbf{r}} &= (\ddot{\rho} - \rho\dot{\phi}^2)\hat{\rho} + \frac{1}{\rho}\frac{d}{dt}(\rho^2\dot{\phi})\hat{\phi} + \ddot{z}\hat{z}\ .
\end{aligned}$$

4. The magnetic field inside a toroid, which is a solenoid bent into the shape of a doughnut (Fig. 3),

Figure 3

is given by

$$\mathbf{B} = -\frac{\mu_0 N I}{2\pi\rho}\hat{\phi}\ ,$$

where ρ is the distance from the symmetry axis of the toroid (z-axis), I is the current in the toroid windings and N is the total number of turns. This position-dependence of magnetic field causes the charges inside the toroid with the velocities parallel to the symmetry axis to drift up or down, depending on the sign of the charge. As a result there is the charge separation in plasma (ionized gas) inside the torus. The charges of opposite sign accumulate on the opposite sides of the toroid and produce an electric field $\mathbf{E}$ causing an additional drift of charges along the direction of $\mathbf{E} \times \mathbf{B}$ as illustrated in Problem 2. Thus a purely toroidal field cannot be used to confine a hot dense plasma of deuterium, i.e. prevent it from touching the walls of the toroidal chamber, for the purpose of thermonuclear fusion.

(a) Write down the equations of motion of charge q with mass m in toroidal magnetic field using the cylindrical coordinates (use the results of Problem 3).

(b) Show that if the initial value of $\dot{\phi}$ is zero, the motion takes place in the plane $\phi = \phi_0$ – the initial value of the azimuthal angle.

(c) Find the z-component of velocity, $\dot{z}$, as a function of ρ for given $\dot{z}_0$ and ρ_0.

(d) Combine your results in (b) and (c) with the equation for $\ddot{\rho}$ to find $\ddot{\rho}$ as a function of ρ. If $\dot{z}_0 > 0$ will $\dot{\rho}$ be increasing or decreasing immediately after $t = 0$ if $q < 0$?

(e) Assuming $\dot{\rho}_0 = 0$ (the initial velocity is along z-axis) find $\dot{\rho}$ as a function of ρ (*Hint*: $\ddot{\rho} = d(\dot{\rho})/dt = (d(\dot{\rho})/d\rho)\dot{\rho} = d(\dot{\rho}^2/2)/d\rho$, where it is assumed that $\dot{\rho}$ depends on t via ρ, $\dot{\rho} = \dot{\rho}(\rho(t))$). At what value of ρ other than ρ_0 does $\dot{\rho}$ vanish?