Assignment No. 2

Physics 3P36

Due Monday, January 21, 2019

1. In 1879, Edwin H. Hall, then a 24-year-old graduate student at the Johns Hopkins University showed that the drifting conduction electrons in a current carrying copper wire can be deflected by a magnetic field, just like the electric charges moving in vacuum or air. This so-called **Hall effect** allows one to determine whether the charge carriers in a conductor are positively or negatively charged as well as their drift speed and the number density n. The experimental setup is illustrated in Fig. 1.

Figure 1

A current I flows to the right through a rectangular bar of conducting material, in the presence of a uniform magnetic field $\mathbf{B}$ pointing as indicated. The bar has thickness t, width w and length l.

(a) If the moving charges are *positive*, in which direction are they deflected by the magnetic field? This deflection results in accumulation of positive charge on one face of the bar leaving the un-compensated negative charge on the opposite face of the bar. The resulting planes of charges produce electric field $\mathbf{E}$ which acts on the drifting charges in a way that counteracts the force produced by the magnetic field. Equilibrium occurs when the electric force and the magnetic force on the drifting charges exactly cancel.

(b) The electric field $\mathbf{E}$ produces potential difference V (the **Hall voltage**) which is measured. Determine the drift speed v_d of the charges in terms of B, measured V and the relevant dimension of the bar.

(c) If the charge q of the carriers is known, determine the carrier density n in terms of the current I, field B, Hall voltage V, charge q and the relevant dimension of the bar.

(d) How would your analysis change if the moving charges were *negative*? How does the sign of the Hall voltage determine the sign of the charge carriers for the fixed direction of current I and magnetic field $\mathbf{B}$?

2. Consider a rectangular loop with sides a and b centered at the origin and tilted by an angle θ from the z-axis towards the y-axis (Figure 2). The loop caries a current I and is placed in a uniform magnetic field $\mathbf{B}$ along z-axis.

Figure 2

(a) What is the net magnetic force on the loop?

(b) Show that the net torque $\mathbf{N}$ on the loop is

$$\mathbf{N} = \mathbf{m} \times \mathbf{B}\,,$$

where $\mathbf{m}$ is so called magnetic moment of the loop of magnitude

$$m = I \cdot (\text{surface area of the loop})$$

and direction perpendicular to the plane of the loop and such that the current flows counterclockwise when observed from the tip of $\mathbf{m}$.

3. A plane wire loop of irregular shape is partially inserted an a uniform magnetic field tube, with $\mathbf{B}$ perpendicular to the plane of the loop (Figure 3). The loop carries a current I.

Figure 3

Find the force on the loop in terms of I, B, and the distance d between points a and b (*Hint*: Orient the x-axis along the segment $\bar{ab}$ and take the z-axis along $\mathbf{B}$.) What is the direction of the force?

4. Two charges q_1 and q_2 move with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$, respectively, which are much smaller than the speed of light in vacuum.

   (a) Calculate the magnetic forces that these two charges exert on each other?

   (b) Do they satisfy the third Newton's law?

   (c) What is the implication of your finding in (b) on the conservation of the total momentum and the total angular momentum of the system of two moving charges?

5. A long thin trough of radius R, Fig. 4, carries current I. Find the magnetic field along the line in the middle of the gap which is at distance R from the center of the trough (i.e. along the dashed line in Fig. 4). Express your result for the magnitude of the field in terms of I, R and the angle α. What is the the value of the field when $\alpha = \pi$?

Figure 4

6. A long thin cylinder of radius R carries a current I. Its axis is parallel to a thin wire which also carries current I in the same direction, Figure 5. The distance between the wire and the axis of the cylinder is a. Calculate the magnetic force on the cylinder per unit length of the cylinder.

Figure 5