Assignment No. 3

Physics 3P36

Due Monday, February 4, 2019

1. A cylinder of radius b has a cylindrical hole of radius a ($a < b$). The axis of the hole is parallel to the axis of the cylinder and their distance is d (Fig. 1). The electric current I flows through the cylinder. What is the magnetic field on the axis of the hole?

Figure 1

2. A long coaxial cable consists of two concentric conductors whose dimensions are given in Figure 2. Assume that the current densities in the conductors are uniform.

Figure 2

   (a) Find the magnetic field $\mathbf{B}$ at a point inside the inner conductor at a distance r from its axis ($r < a$).

   (b) Find the magnetic field $\mathbf{B}$ at a point in the space between the conductors which is at a distance r from the axis ($a < r < b$).

(c) Find the magnetic field **B** at a point inside the outer conductor which is at a distance r from the axis ($b < r < c$).

(d) Find the magnetic field **B** at a point outside the cable which is at a distance r from the axis $r > c$).

3. Use the Ampere's law to show that the magnetic field **B** cannot drop abruptly to zero, as suggested just to the right of point a in Figure 3, as one moves perpendicular to **B**, say along the horizontal arrow in the figure. (*Hint*: Use the Ampere's loop given by the dashed line in the figure.)

Figure 3

In actual magnets "fringing" of the magnetic field lines always occurs, which means that **B** approaches zero in a gradual manner. Modify the field lines in the figure to indicate a more realistic situation.

4. Show that the Lorentz force on charge q given in terms of the scalar potential $\phi(\mathbf{r}, t)$ and the vector potential $\mathbf{A}(\mathbf{r}, t)$,

$$\mathbf{F} = q\left[-\nabla_{\mathbf{r}}\phi - \frac{\partial \mathbf{A}}{\partial t} + \dot{\mathbf{r}} \times (\nabla_{\mathbf{r}} \times \mathbf{A})\right] ,$$

can also be written as

$$\mathbf{F} = -\nabla_{\mathbf{r}} U + \frac{d}{dt}\nabla_{\dot{\mathbf{r}}} U ,$$

where U is velocity dependent generalized potential

$$U = q\phi - q\dot{\mathbf{r}} \cdot \mathbf{A} .$$

Here

$$\begin{aligned} \nabla_{\mathbf{r}} &= \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z} , \\ \nabla_{\dot{\mathbf{r}}} &= \hat{x}\frac{\partial}{\partial \dot{x}} + \hat{y}\frac{\partial}{\partial \dot{y}}\hat{z}\frac{\partial}{\partial \dot{z}} , \end{aligned}$$

and $x, y, z, \dot{x}, \dot{y}, \dot{z}$ are independent variables.

5. An uniform magnetic field $\mathbf{B}$ is along z-axis. Find the corresponding vector potential $\mathbf{A}$ which has

   (a) only the x-component.

   (b) only the y-component.

   (c) both x- and y-components and write your result in terms of $\mathbf{r} \times \mathbf{B}$.

   (d) Do your results in (a), (b) and (c) satisfy Coulomb/London gauge $\nabla \cdot \mathbf{A}=0$?

   The gauge (a) or (b) is called the Landau gauge and the gauge in (c) is called symmetric gauge.

6. A thin ring of radius R is carrying current I.

   (a) Calculate the vector potential very close to the symmetry axis perpendicular to the ring, taken to be the z-axis, i. e. $x, y \ll R$.

   (b) Use your result in (b) to compute the magnetic field along the z-axis and compare the result with the one found in the Example 5.6 in the textbook.