Physics 3P41/5P11 Assignment 4

Due: December 6, 2019 in drop box across from MC B210a by 12:00 noon.

1. Use thermodynamic arguments to obtain the general result that, for any gas at temperature T, the pressure is given by:

$$P = T\left(\frac{\partial P}{\partial T}\right)_V - \left(\frac{\partial U}{\partial V}\right)_T \tag{1}$$

where U is the total energy of the gas.

2. A soap bubble of radius R_1 and surface tension γ is expanded at constant temperature by forcing in air by driving a piston containing volume V_{piston} fully home. Show that the work needed to increase the bubble's radius to R_2 is:

$$\Delta W = P_2 V_2 \ln(\frac{P_2}{P_1}) + 8\pi\gamma(R_2^2 - R_1^2) + P_o(V_2 - V_1 - V_{piston}) \tag{2}$$

where P_1 and P_2 are the initial and final pressures in the bubble, P_o is the pressure of the atmosphere and V_1 and V_2 are initial and final volumes of the bubble (assumed to be spherical).

3. Find U, C_V, F and S *at high temperature* such that $k_B T >> \hbar\omega$ for a diatomic molecule with rotational levels excited.

4. Find an expression for the single-particle partition function of a 2-D gas confined to an area A. Write your answer in terms of the thermal wavelength.

5. Consider a system consisting of a single Hydrogen atom/ion, which has two possible states: unoccupied (no electron present) and occupied (one electron present in the ground state).

   (a) Find an expression for the ratio of the probabilities of these two states. Let the ionization energy be I. Treat the electrons as a monatomic ideal gas for the purpose of obtaining the chemical potential. Neglect the fact that the electron has two independent spin states (it would cancel in the ratio).

   (b) Find the ratio of ionized hydrogen to un-ionized hydrogen at the surface of the sun which has a temperature of 5800 K. Assume the electron concentration is 2.0×10^{19} m^{-3}. The ionization energy for hydrogen is 13.6 eV.