Physics 3P70

Homework assignment No. 2

Due October 19, 2018 (Fri)

Questions: **Marks**

1. A particle of mass m in the infinite square well of width a has as its initial wave function a mixture of the first two stationary states:

$$\Psi(x,0) = A[\psi_1(x) + \psi_2(x)].$$

(a) Normalize $\Psi(x,0)$. Does A depend on time? **(1)**
(b) Find $|\Psi(x,t)|^2$. **(1)**
(c) Show that $\langle x \rangle$ oscillates as a function of time. What is the frequency and amplitude of the oscillations? **(2)**
(d) Find $\langle p \rangle$. **(1)**
(e) Find $\langle H \rangle$. **(1)**

2. A particle of mass m moves in the potential given by

$$V(x) = \begin{cases} \infty & , \text{for } x < 0 \\ -V_0 & , \text{for } 0 < x < a \\ 0 & , \text{for } x > a. \end{cases}$$

Find the smallest possible depth, V_{0c}, of the potential well at which there exists at least one bound state with $E < 0$. **(1)**

3. A free particle of mass m in 1D has the initial wave function

$$\Psi(x,0) = \cos k_0 x + e^{i\phi} \sin 2k_0 x,$$

where ϕ is some constant. Calculate the probability density of finding the particle near x as a function of time. **(2)**

4. A particle of mass m is moving in the potential
$V(x) = -\alpha[\delta(x+a) + \delta(x-a)]$, where $a, \alpha > 0$.

(a) Find the energies $E = -|E| < 0$ of the even bound states (use graphical solution: plot the left and the right sides of the energy equation as functions of $\kappa = \sqrt{2m|E|/\hbar^2}$). When do the even bound states exist? What is the limit of E when $a \to \infty$? **(2)**

(b) Do the same for odd bound states. When do the odd bound states exist? **(2)**

Total mark = 13