# Physics 3P70

Homework assignment No. 3

Due November 16, 2018 (Fri)

**Questions:** **Marks**

1. Calculate the transmission coefficient $T(E)$ for a particle of $E > 0$ in the potential $V(x) = \alpha[\delta(x+a) + \delta(x-a)]$ $(a, \alpha > 0)$.
   (Note: Transmission coefficient is real, so your final answer should not contain $i = \sqrt{-1}$.) **(3)**

2. An interface between two different metals is sometimes modelled by the potential $V(x) = V_0(x) + \alpha\delta(x)$, where

$$V_0(x) = \begin{cases} 0 & , \text{ at } x < 0 \\ V_0 & , \text{ at } x > 0 \end{cases}$$

   $(V_0, \alpha > 0)$. Calculate the transmission coefficient at $E > V_0$. **(2)**

3. A particle of mass $m$ in the harmonic oscillator potential $V(x) = (1/2)m\omega^2 x^2$ has the initial wave function

$$\Psi(x, 0) = A[\psi_1(x) + \psi_2(x)],$$

   where $A$ is some constant, and $\psi_{1,2}(x)$ are the stationary states corresponding to $n = 1$ and $n = 2$.
   (a) Find the normalized time-dependent wave function $\Psi(x, t)$ and the probability density $|\Psi(x, t)|^2$. **(1)**
   (b) Show that the expectation values of $x$ and $p$ oscillate as functions of $t$. Find the amplitudes and frequencies of the oscillations. **(2)**

4. (a) Calculate the ground state energy of a harmonic oscillator of frequency $\omega$ modified by a small "bump" in the potential energy $\delta V(x) = V_0 e^{-\lambda x^2}$, where $\lambda > 0$, and $V_0 \ll \omega$. **(1)**
   (b) Do the same for the second excited state. **(1)**

**Total mark = 10**