Physics 3P70

Homework assignment No. 4

Due December 4, 2018 (Tue)

Questions: **Marks**

1. Prove the famous "(your name) uncertainty principle", relating the uncertainty in position ($\hat{A} = x$) to the uncertainty in energy ($\hat{B} = \hat{H} = \hat{p}^2/2m + V$):

$$\sigma_x \sigma_H \geq \frac{\hbar}{2m} |\langle \hat{p} \rangle|.$$

   For the stationary states this does not tell you much – why not? **(1)**

2. (a) Find $\langle r \rangle$ and $\langle r^2 \rangle$ for an electron in the ground state of hydrogen. Express your answers in terms of the Bohr radius a_0. **(1)**
   (b) Find $\langle x \rangle$ and $\langle x^2 \rangle$ for an electron in the ground state of hydrogen (Hint: This requires no new integration, because of the spherical symmetry of the ground state). **(1)**
   (c) Find $\langle x^2 \rangle$ in the state $n = 3$, $l = 1$, $m = 1$. **(2)**

3. A hydrogen atom is initially in the state

$$\Psi(\mathbf{r}, 0) = A[2\psi_{100}(\mathbf{r}) + \psi_{211}(\mathbf{r})].$$

   (a) Find the normalization coefficient A and the probability density $|\Psi(\mathbf{r}, t)|^2$. **(2)**
   (b) Find the expectation value of the x-component of the electron dipole moment $d_x = ex$. **(2)**

4. An electron in magnetic field $\mathbf{B} = B\hat{z}$ is initially in the spin state

$$\chi(0) = A \begin{pmatrix} 2i \\ -1 \end{pmatrix}.$$

(a) Find the normalized spin wave function $\chi(t)$ at arbitrary t. **(1)**
(b) Find $\langle \mathbf{S}(t) \rangle$. **(1)**

Total mark = 11