

# PHYS 3P91 Laboratory Manual

Physics Department

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# Experiment 1

## X-Ray Diffraction

*Before starting this experiment, you must be familiar with the concepts of: crystal lattice, unit cell, lattice spacing, Miller indices, X-ray spectrum, crystal diffraction and Bragg diffraction (references 1-4).*

### Introduction

When X-rays (electromagnetic waves, wavelength order-of-magnitude  $1 \text{ \AA}$ ) encounter atoms, they are diffracted (changed in direction of propagation, with a different amplitude and phase, but unchanged in wavelength) by the electron clouds of the atoms. If the atoms are arranged in a crystalline lattice (an array which is periodic in three dimensions), and the incident X-ray wave is a monochromatic parallel beam, then the waves diffracted by the  $\cong 10^{23}$  atoms in the crystal will interfere in such a way that only in a few directions constructive interference will take place, and a non-zero intensity diffracted beam will result. The directions of the diffracted beam can be constructed as follows:

- Slice the crystal into planes of atoms, called lattice planes. Each set of lattice planes (two are sketched in figure 1.1) is characterized by its orientation in the crystal (given by the Miller indices  $h, k$  and  $l$  and its lattice spacing  $d_\ell$ ).
- Each lattice plane, when hit by an incoming X-ray beam (direction  $\theta$ ) acts as a mirror, producing a reflected beam as well as a straight-through beam which impinges on the next lattice plane.
- Only when the “reflected” beams from successive lattice planes interfere constructively, that is, when

$$AB + BC - AD = n\lambda \quad (1.1)$$

then the crystal as a whole will produce a diffracted beam with non-zero intensity. The angle between the incident and diffracted beams is then  $2\theta$ . This is called Bragg diffraction, and from the geometry of figure 1.1 it can be seen that it will occur when:

$$2d_\ell \sin \theta = n\lambda \quad (1.2)$$

Often the order number  $n$  and  $d_\ell$  are combined in a quantity  $d$ , given by  $d = d_\ell/n$ , so that Bragg’s Law is written as:

$$2d \sin \theta = \lambda \quad (1.3)$$

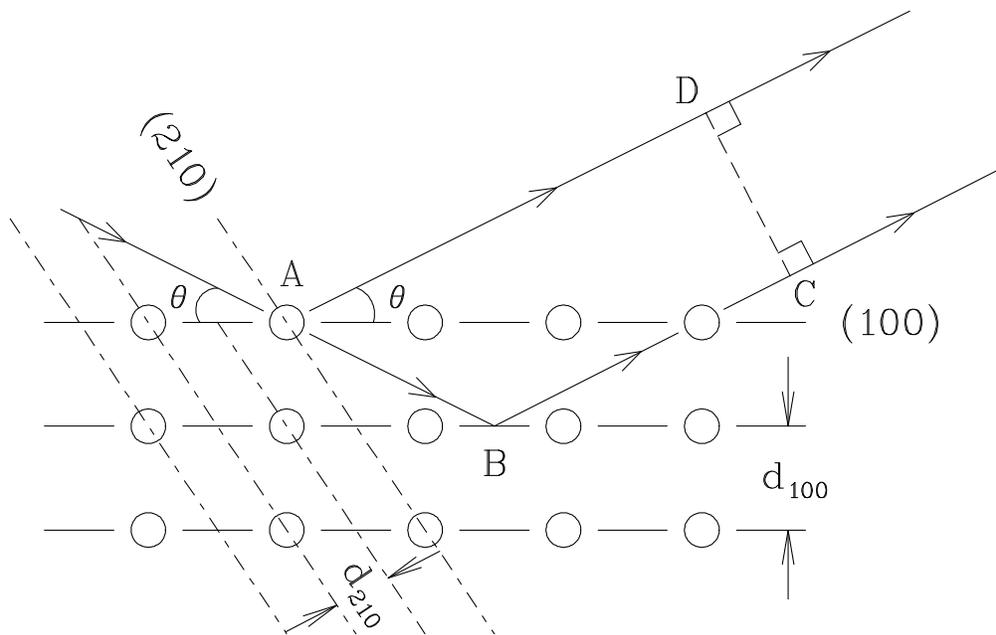


Figure 1.1: X-rays striking a crystal lattice

## 1.1 The X-Ray Emission Spectrum of Copper

You will determine the wavelength of some of the sharp lines of the Cu X-ray emission spectrum. The diffractometer geometry is shown in figure 1.2. The X-rays emitted by the copper target in the X-ray source are filtered, so that only the spectral lines labelled Cu- $K_{\alpha 1}$ , Cu- $K_{\alpha 2}$  and Cu- $K_{\beta 1}$  (with very low intensity) are present in the X-ray beam. The beam is incident on a single crystal of MgO. Its  $(h, k, \ell) = (1, 0, 0)$  lattice planes will be used; they are parallel to the cleaved front face of the crystal. The intensity  $I$  of the diffracted beams (at a direction  $2\theta$  with respect to the incident beam) is measured by the X-ray counter; the chart output is a graph of  $I$  as a function of  $2\theta$ .

*CAUTION: The instructor will demonstrate the diffractometer. Under no circumstances is the apparatus to be used, or adjusted, without prior permission of the instructor. X-rays are a radiation hazard, so you must observe all safety instructions listed next to the experiment.*

Mount the MgO crystal in the sample holder, so that its (100) face is accurately parallel to the sample plane. For the (100) lattice planes the  $d_{\ell}$  spacing is

$$d_{(100)} = 4.205 \text{ \AA}$$

The 2nd order ( $n = 2$ ) Bragg reflection for the Cu $K_{\alpha}$  doublet occurs in the  $2\theta$  range  $42 \rightarrow 44^{\circ}$ ; the  $n = 4$  reflection occurs in the range  $93 \rightarrow 95^{\circ}$ . The  $n = 2$  reflection of the  $K_{\beta 1}$  line can be seen around  $2\theta \cong 38 \rightarrow 39^{\circ}$  with low intensity.

Record the  $I(2\theta)$  curves in these ranges, and calculate the wavelength of the observed spectral lines. Compare the experimental results with the literature values.

The wavelength separation of the  $K_{\alpha 1}$  -  $K_{\alpha 2}$  doublet can be determined quite accurately, in the following way: Let the wavelength of the two lines in the doublet be  $\lambda_1$  and  $\lambda_2$  and  $\Delta\lambda = \lambda_2 - \lambda_1$ , the separation of the recorded lines on the  $I(2\theta)$  chart is measured as  $\Delta_{2\theta}$  (in radians). By differentiating Bragg's Law show that:

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta_{2\theta}}{2 \tan\theta}.$$

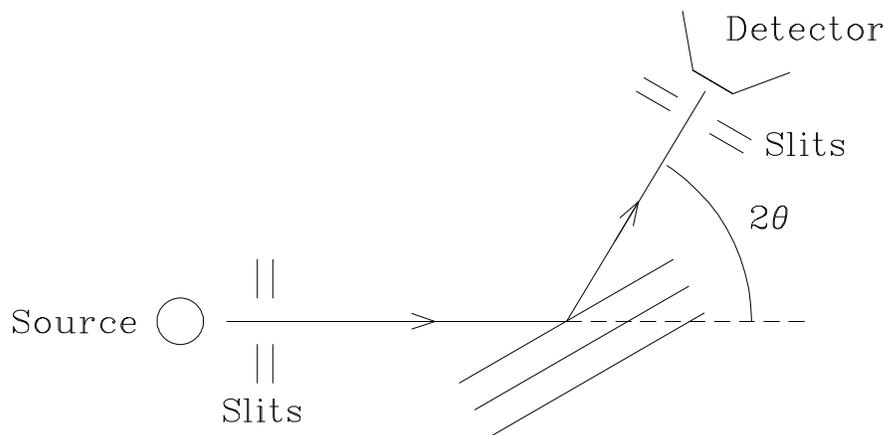


Figure 1.2: Diffractometer geometry

Calculate  $\Delta\lambda/\lambda$  for the  $\text{CuK}_\alpha$  doublet directly from the measured  $\Delta_{2\theta}$  at  $2\theta \cong 43^\circ$  and  $2\theta \cong 94^\circ$ ; compare your experimental results with the calculated values of

$$\frac{\Delta\lambda}{\lambda} = \frac{2(\lambda_2 - \lambda_1)}{(\lambda_2 + \lambda_1)},$$

where  $\lambda_1$  and  $\lambda_2$  are taken from one of the references, or from the *Handbook of Chemistry and Physics*.

## 1.2 The X-ray diffraction pattern of MgO

Replace the single crystal MgO plate by a sample consisting of a thin layer of MgO powder. In the powder the individual crystals (grains of approximately  $10^{-3}$  cm size) are oriented totally at random. Of the millions of grains only a few will, accidentally, lie such that their (100) planes are parallel to the plane of the sample holder. When the sample holder is therefore at an angle  $\theta \cong 21.5^\circ$  with respect to the incident beam, and the detector therefore at an angle  $2\theta \cong 43^\circ$ , these few grains can produce a Bragg reflection. All the other grains will not give a Bragg reflection, and therefore the  $I(2\theta)$  graph will have a (relatively) weak peak at  $2\theta \cong 43^\circ$ . Similarly, there will be in the powder sample, grains with a different set of lattice planes, say with Miller indices  $(h, k, \ell) = (311)$ , parallel to the sample plane; these grains will give a Bragg reflection at an angle  $2\theta$  satisfying Bragg's Law with  $d_{(311)}$ . The diffraction pattern  $I(2\theta)$  of a MgO powder will therefore consist of a set of peaks, each of which is characterized by its own value of  $d$  and of  $(h, k, \ell)$ .

Record the  $I(2\theta)$  pattern of MgO powder in the range  $28^\circ \leq 2\theta \leq 64^\circ$ . Calculate the  $d$  values of the reflection, use an "average"  $\lambda$  value of  $\lambda = 1.5405 \text{ \AA}$ . The possible sets of lattice planes have Miller indices: (111), (311), (331), (200) and (222). Calculate the  $d$ -value for each of these planes according to:

$$d = \frac{a_0}{\sqrt{h^2 + k^2 + \ell^2}}$$

with the lattice constant  $a_0 = 4.205 \text{ \AA}$  and compare with the experimental values.

### 1.3 The XRD pattern of an unknown substance

Each solid crystalline material has its own  $I(2\theta)$  diffraction pattern. In general, the position ( $2\theta$ ) of the diffracted beams is determined by the size and shape of the unit cell of the crystalline lattice; the intensity  $I$  of a given beam is determined by the kind of atoms and their position inside the unit cell. The XRD pattern is therefore like a fingerprint for each crystalline substance, and can be used for identification.

You will be given a powder of an oxide copper. Two copper oxides are known, with formulae  $\text{Cu}_2\text{O}$  and  $\text{CuO}$ . Record the XRD pattern of the powder in the range  $28^\circ \leq 2\theta \leq 64^\circ$ , and calculate  $d$ -spacings and relative intensities for each diffraction peak. Compare your results with the patterns for  $\text{CuO}$  and  $\text{Cu}_2\text{O}$  (patterns 5.0661 and 5.0667 of ref. 5) and determine which copper oxide you have.

### References

1. B.D. Cullity, *Elements of X-Ray Diffraction*, Chapters 1, 2, 3, 7.
2. E.W. Nuffield, *X-Ray Diffraction Methods*, Chapters 1, 2, 3, 5.2.
3. D.L. Livesey, *Atomic and Nuclear Physics*, Chapters 4.6 - 4.9.
4. R.T. Weidner and R.L. Sills, *Elementary Modern Physics*, Chapter 5.
5. *Powder Diffraction File*, (Joint Committee on Powder Diffraction Standards, 1968). In library as REF QC 482 P68.

## Experiment 2

# X-ray fluorescence

*X-ray fluorescence (XRF) and X-ray diffraction (XRD) may be used to determine the constituent elements and the crystalline structure of materials.*

### Introduction

#### 2.1 X-ray fluorescence

XRF refers to the absorption of incident X-rays ( $h\nu_0$ ) resulting in the production of a set of X-rays whose energies ( $h\nu_j$ ) are characteristic of the absorber atom(s) ( $Z$ ). The characteristic X-rays can therefore be used to identify the atom(s) present in the sample.

The incident X-ray photon ( $h\nu_0$ ) undergoes a photoelectric interaction with a target atom ( $Z$ ) resulting in the annihilation of the photon and ionization of the target atom. The energy equation is

$$K = h\nu_0 - B$$

where  $K$  is the kinetic energy of the photoelectron liberated from the atom and  $B$  is its binding energy to the atom. Subsequently, the ionized atom combines with an electron and undergoes a de-excitation to its electrically neutral ground state. X-ray photons, whose energies ( $h\nu_j$ ) are characteristic of the target atom ( $Z$ ), are produced in the de-excitation. An energy level diagram is shown in figure 2.1. Each level corresponds to the excitation of the atom when a single electron is missing from the otherwise full subshell ( $nlj$ ). Zero energy corresponds to the ground state of the neutral atom. Level  $K$ , corresponds to the binding energy ( $B_K$ ) of an electron in the  $K$  shell. There are three closely spaced L shell levels labelled  $L_I, L_{II}, L_{III}$  corresponding to slightly different L shell electron binding energies. If

$$h\nu_0 > B_K$$

then all characteristic X-rays (K series, L series, etc.) are produced. If

$$B_L < h\nu_0 < B_K,$$

then the K series X-rays are not produced.

#### 2.2 X-ray Diffraction

X-ray diffraction is used extensively to determine the structure of crystalline and also of non-crystalline and amorphous materials. For crystalline materials, the directions ( $\theta$ ) and wavelength( $\lambda$ )

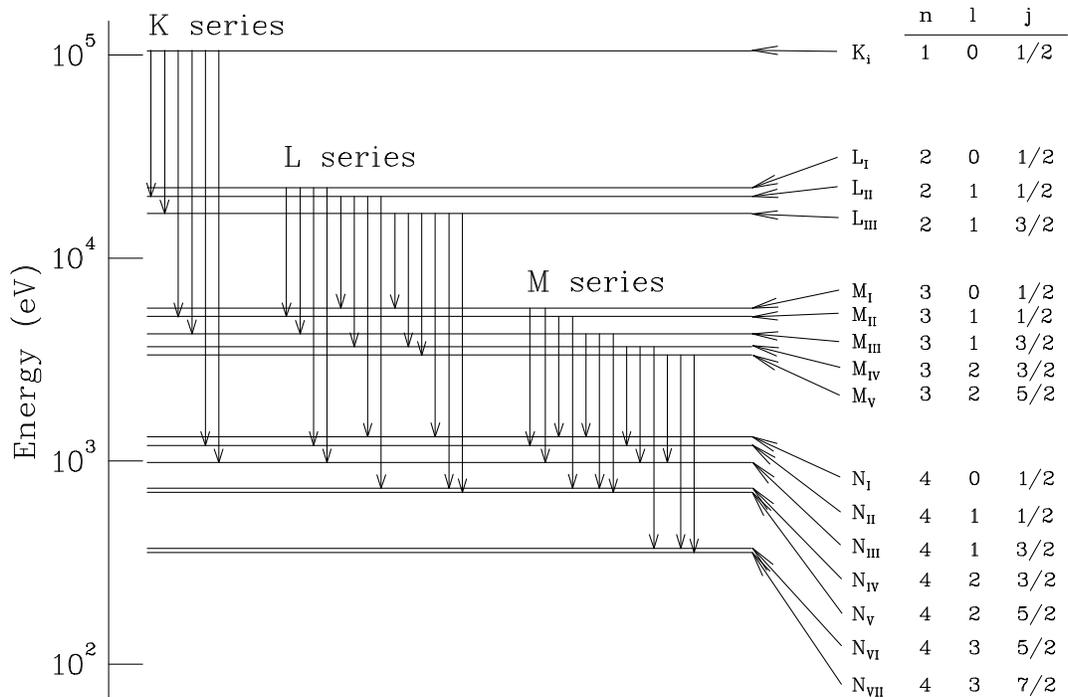


Figure 2.1: A schematic diagram of the X-ray energy levels.

of the diffracted beams are given by Bragg's Law; with reference to figure 2.2, this is

$$2d \sin \theta = n\lambda$$

where  $n$  is the order of diffraction from crystalline lattice planes of spacing  $d$ . The equation may be rewritten as

$$2d_n \sin \theta = \lambda$$

where  $d_n = d/n$  and the equation now represents first order diffraction from planes  $d/n$ . Using Bragg's Law, a measurement of  $\theta$  and  $\lambda$  for a diffracted beam enables the spacing  $d_n$  to be determined. For cubic lattices,

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

where  $a$  is the side length of the cubic lattice cell (the lattice constant), and  $h, k$  and  $l$  are the Miller indices of the lattice planes.

## 2.3 Apparatus

A diagram of the apparatus is shown in figure 2.3. The incident beam produced by the X-ray machine has its intensity continuously distributed over a range of X-ray photon energy up to a maximum value =  $eV$ , where  $V$  is the X-ray tube voltage.

The incident beam strikes a crystalline powder; the X-rays diffracted from the crystals and those fluoresced from the target atoms are recorded by a Si semiconductor junction photon detector (which is sensitive to photons with energies in the range  $\approx 1 \rightarrow 50$  keV) positioned at an angle  $2\theta$  relative to the direction of the incident beam. The Si detector produces a single voltage pulse

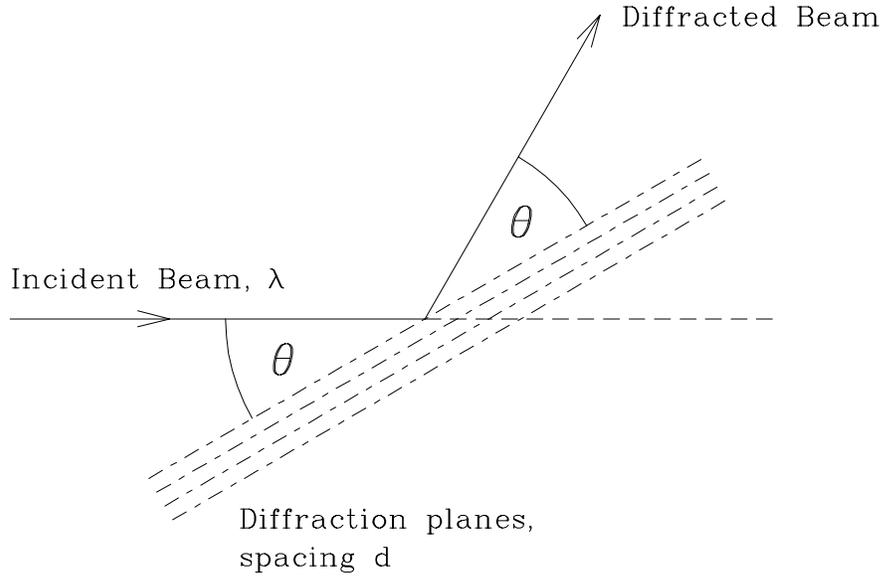


Figure 2.2: Diffraction geometry.

for each photon detected; its amplitude is proportional to the energy of the photon detected. The energy (voltage amplitude) spectrum of X-ray photons produced from the target is recorded by a multichannel analyser (MCA). The spectrum  $I(E)$  which is the variation of the number of photons detected,  $I$ , with photon energy,  $E$ , consists of a series of relatively narrow peaks superimposed on a smoothly varying background. The peaks arise from

- X-rays fluoresced from the target atoms,
- X-rays diffracted from the crystal lattice.

The position of the peak centre on the MCA energy scale equals the corresponding photon energy.

## 2.4 Energy Dispersive X-ray Diffraction

In this apparatus, the diffraction is observed by measuring  $I(E)$  keeping  $\theta$  constant. Bragg's Law may be rewritten in terms of the diffracted beam photon energy,  $E$ , i.e.,

$$2d_n \sin \theta = \lambda = \frac{hc}{E}$$

or,

$$E_{hkl} = \frac{hc}{2 \sin \theta} \cdot \frac{1}{d_{hkl}}. \quad (2.1)$$

$hkl$  are the Miller indices of the planes which produce diffraction of photons of energy  $E_{hkl}$ . For a particular value of the diffraction angle  $\theta$  (the detector is positioned at an angle  $2\theta$  relative to the incident beam) diffraction from planes with different  $d_{hkl}$  values produce diffraction peaks at

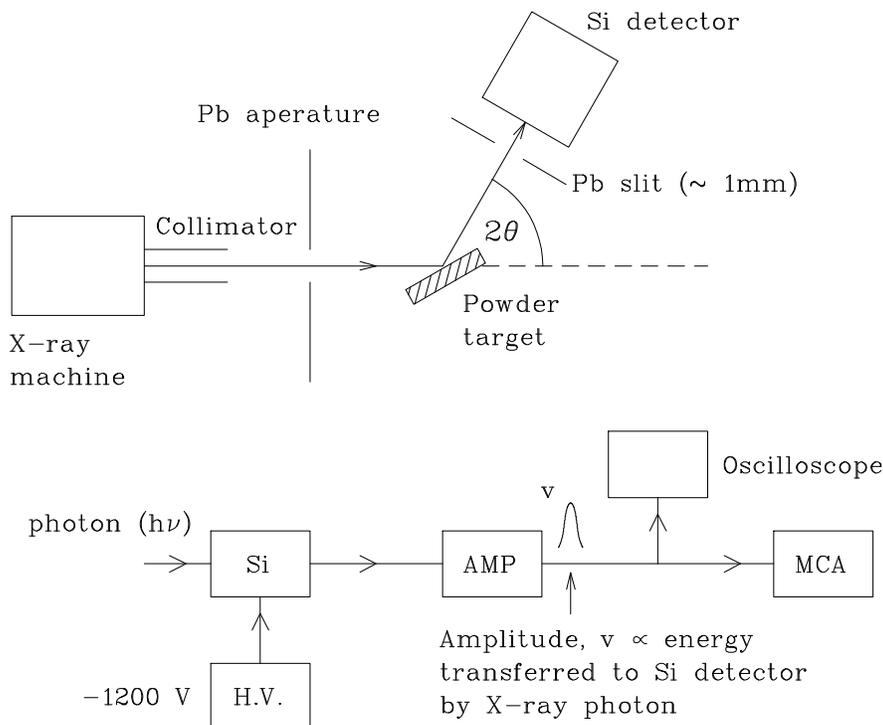


Figure 2.3: Schematic diagram of apparatus.

energies  $E_{hkl}$ , given by equation (2.1). Diffraction peak energies  $E_{hkl}$  are a function of  $\theta$  (Eq. 2.1); XRF characteristic peak energies are independent of  $\theta$ . In addition, the diffraction peak intensities are normally much weaker than those produced by the characteristic X-rays fluoresced from the target. Either of the above features may be used to distinguish the characteristic X-ray and the diffraction peaks.

## Experiment

### CAUTION

*The instructor will demonstrate the operation of the X-ray machine, the Si photon detector and the multichannel analyser.*

Under no circumstances is the apparatus to be operated prior to instruction.

Under no circumstances are adjustments to be made to the target arrangement with the X-ray beam on. The X-ray beam is a radiation hazard.

## 2.5 Objective

The objective is to identify the constituent elements (using XRF) of the powdered samples.

## 2.6 Operation

- Si Photon Detector

The detector is operated at liquid nitrogen temperature. *The detector reverse bias must only be applied to a cold detector, otherwise irreversible damage to the detector will result.*

Normally, the reverse bias ( $\approx -1200$  V) and the amplifier are already switched on and preset, and no adjustments are necessary. However, if the reverse bias has to be switched ON or OFF, this must be done with the bias voltage potentiometer turned to zero. Do not make any adjustment without first consulting the instructor.

- X-ray Machine

The machine is normally operated at a tube voltage in the range, 20-30 kV, and a current up to a maximum of approximately 15 mA. These values are not at all critical; the voltage range provides an incident X-ray beam spectrum which is reasonably well-matched to that required both for XRF and diffraction.

Note: The X-ray beam port and shutter must be *closed* and the X-ray tube high voltage switched OFF prior to making any beam-target alignment changes. *This requirement is MANDATORY.*

- MCA

*The multichannel analyser (MCA) is normally left switched ON, ready for use.* The computer monitor is switched OFF when not in use. The operation of the MCA is by selection from a menu which is displayed, together with the spectrum, on the monitor.

## Procedure

### 2.6.1 Energy Calibration

1. Alignment (*X-ray HV OFF; beam shutter CLOSED*) Position, visually, the energy calibration target into the X-ray beam path (which is determined by the axis of the X-ray beam collimator) and set the Si detector rotatable platform somewhere in the range  $2\theta = 45 \rightarrow 60^\circ$ , relative to the incident X-ray beam path.

Check visually, but carefully, that the X-ray beam path strikes the target surface centrally and also that the target surface can be seen by the Si detector. The precise value of  $2\theta$  is not required; the approximate value ( $\pm$  a few degrees) may be helpful in identifying features of the diffraction spectrum. For the calibration, use samples of Zn, Mo and KBr.

2. Spectrum Accumulation (*X-ray HV ON; beam shutter OPEN*)

Accumulate a “smooth” spectrum in which the peaks are clearly defined (that is, the peak centres can be determined by inspection to  $\pm 1$  channel). The XRF lines due to the main elements in the layer are normally significantly stronger than any diffraction lines. When the accumulation is completed, *CLOSE the beam shutter and turn the X-ray current HV OFF.*

3. Calibration analysis

Record the channel number ( $n$ ) for the centre of each XRF peak and identify the peaks with the characteristic energies ( $E_n$ ) corresponding to the known elements in the calibration target. The energy calibration of the MCA is accurately given by

$$E_n = an + b.$$

The calibration constants ( $a$  and  $b$ ) should be determined by a linear regression analysis and also the data should be plotted. This calibration may be used, without further check, throughout the experiment providing the gain of the Si detector amplifier and the “set-up” of the MCS remain unchanged. Using the calibration, the energies of XRF and diffraction peaks obtained in subsequent spectra may be determined.

#### 4. Diffraction peaks

Use the  $d$  values, with largest intensity, of the calibration samples and an estimated  $2\theta$  value, to calculate the  $E_{hkl}$  values from Eq. (2.1) where a diffraction peak might be located. Inspect the  $I(E)$  spectrum for such (weak) peaks. If one (or more) are found, redo a spectrum at a different ( $\pm$  a few degrees)  $2\theta$  value and check that indeed the peak energy is a function of  $\theta$ .

## 2.7 Analysis of powdered samples

### 1. Alignment (*X-ray HV OFF; X-ray beam shutter CLOSED*)

Install the powdered crystalline target and repeat the beam-target-detector alignment described in section 2.6.1, item 1.

### 2. Spectrum accumulation (*X-ray HV ON, X-ray beam shutter OPEN*)

Accumulate data and store the resulting spectrum; there should be strong XRF lines present and (maybe) much weaker diffraction lines visible. Change the detector angle by a few degrees and repeat the accumulation; compare with the previous spectrum. The diffraction peaks shift position in the spectrum, the XRF peaks do not. After this provisional classification into XRF and diffraction lines, keep the detector angle fixed and accumulate until a “smooth” spectrum is obtained. A “smooth” spectrum is a very qualitative term which simply means that the centres of the peaks of interest may be determined by visual inspection to approximately  $\pm 1$  channel. It is useful to keep an eye on the X-ray machine current and voltage readings during accumulation *since occasionally the machine trips OFF; if this occurs, restart; if trips persist, call the instructor*. When the accumulation is completed, CLOSE the X-ray beam shutter and turn the X-ray current and HV OFF.

### 3. Data Analysis

- (a) Determine the energies of the XRF and diffraction peaks (if any).
- (b) Using the characteristics X-ray energy tables, identify the elements in the powdered sample.

## Experiment 3

# Electron Diffraction

### Introduction

In 1924, de Broglie proposed that the motion of particles was described by a wave equation. He associated with each particle a wave, the de Broglie wave, whose wavelength  $\lambda$  depended on the momentum  $p$  of the particle according to

$$\lambda = \frac{h}{p} \quad (3.1)$$

where  $h$  is Planck's constant. In this experiment you will measure the de Broglie wave-length for fast moving electrons, and show that it is related to its momentum according to equation (3.1).

### Theory

If an electron leaves a cathode with zero velocity, then its kinetic energy after traversing an accelerating potential  $V$  is given by:

$$\frac{1}{2}mv^2 = eV \quad (3.2)$$

where  $m$  is the electron mass,  $v$  is its speed after traversing the potential  $V$ , and  $e$  is its charge. The momentum  $p$  of the electron is then given by

$$p = mv = \sqrt{2meV}$$

and its de Broglie wavelength

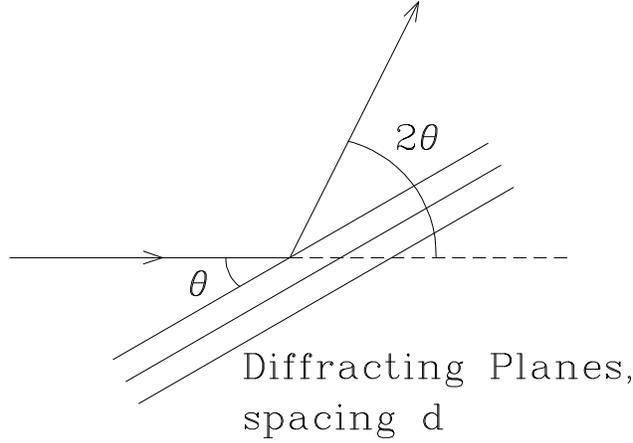
$$\lambda = \frac{h}{\sqrt{2meV}}. \quad (3.3)$$

If this electron strikes a crystal, its de Broglie wave will be diffracted by the 3-dimensional periodic structure of the crystal. The electron will pass through the crystal only in those directions in which the de Broglie wave is diffracted strongly by the crystal. These directions are given by Bragg's Law:

$$2d \sin \theta = n\lambda \quad (3.4)$$

Therefore, if we know the  $d$ -values for a crystal (and we can calculate these if the crystal structure is known), then the wavelength  $\lambda$  can be found by measuring the directions ( $2\theta$ ) in which the waves are strongly diffracted.

Consider now the situation where a parallel beam of waves is incident on many crystals, which are oriented in a completely random fashion with respect to the beam. In such a random aggregate of crystals there are always crystals with the same set of lattice planes in an orientation satisfying



Bragg's Law. All these crystals will diffract the incident beam strongly over the same angle  $2\theta$ : the result is a diffracted beam in the shape of a cone with top angle  $4\theta$ .

This is the powder method of diffraction (Cullity, Chapter 3, p.96) and in effect is the technique used in this experiment to measure the wavelength  $\lambda$ . Our beam of electrons is incident on a polycrystalline foil of aluminum. The cones of diffracted electrons strike a fluorescent screen at a distance  $D$ , forming a set of fluorescent rings of radius  $r$ . It is evident that diffraction angle  $2\theta$  is given by

$$\tan 2\theta = \frac{r}{D}. \quad (3.5)$$

The Bragg's Law

$$2d \sin \theta = n\lambda \quad (3.6)$$

may be rewritten as

$$2 \left( \frac{d}{n} \right) \sin \theta = \lambda \quad (3.7)$$

In equation (3.4) we refer to  $\theta$  as the direction of the  $n_{th}$  order beam diffracted from the set of atomic planes with spacing  $d$ . In the alternative formulation (equation (3.6)) we may refer to  $\theta$  as the direction of the 1st order beam diffracted from a set of planes with spacing  $d/n$ . For cubic crystals the distance  $d$  between successive lattice planes in the set specified by the Miller indices  $hkl$  is

$$d = \frac{a}{\sqrt{h^2 + k^2 + \ell^2}}$$

where  $a$  is the unit cell side length. Hence we may write

$$\frac{d}{n} = \frac{a}{n\sqrt{h^2 + k^2 + \ell^2}} = \frac{a}{\sqrt{H^2 + K^2 + L^2}} \quad (3.8)$$

where  $H = nh, K = nk, L = n\ell$  are the Miller indices of a set of planes with spacing  $d/n$ .

In the present experiment, the diffraction angles are sufficiently small that  $\tan 2\theta \cong 2\theta$  and  $\sin \theta \cong \theta$ . Hence, combining equations (3.6), (3.7), and (3.8) gives an expression for the electron beam wavelength in terms of the radii of the diffraction rings on the fluorescent screen, namely

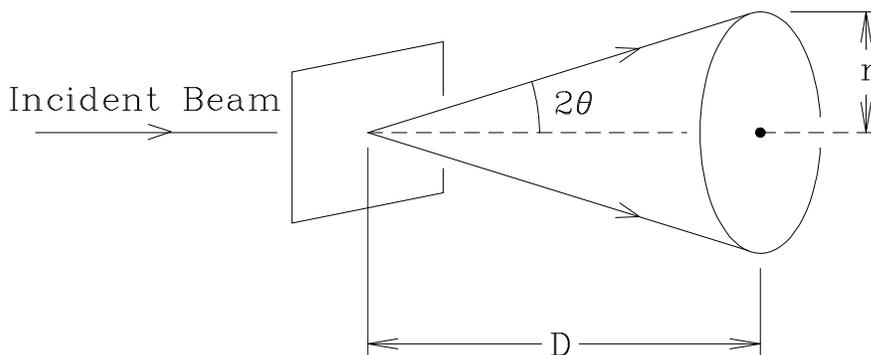
$$\lambda = \frac{a}{\sqrt{H^2 + K^2 + L^2}} \cdot \frac{r}{D}. \quad (3.9)$$

Finally, by equating the expression for the diffraction measurement of the electron beam wavelength (equation (3.9)) to the de Broglie expression for the electron beam wavelength written in terms of the electron tube accelerating voltage  $V$  (equation (3.3)) one obtains

$$r = \sqrt{H^2 + K^2 + L^2} \left( \frac{D}{a} \cdot \frac{h}{\sqrt{2me}} \right) \frac{1}{\sqrt{V}} \quad (3.10)$$

Equation (3.10) gives an explicit expression for the radii of a set of diffraction rings as a function of accelerating tube voltage and its validity rests on that of the de Broglie postulate.

## Experimental



The instrument is a modified cathode-ray oscilloscope with a variable accelerating potential  $V$ . The electron beam can be deflected with horizontal and vertical deflection plates till it strikes an Al foil in the tube. The diffraction pattern of the Al foil can be measured on the screen.

Obtain a diffraction pattern at  $V = 4000$  V, following the instruction sheet supplied with the instrument. Measure the diameters of the diffraction rings five times with calipers. In order to minimize the effects of ring distortion, make the five measurements on each ring in different directions. Repeat the measurements as a function of  $V$  up to  $V = 9000$  V. DO NOT EXCEED  $5 \mu\text{A}$  BEAM CURRENT.

From the data calculate average values of  $r$  for each ring, at each value of  $V$ . The value of  $D$  for this instrument is:

$$D = 18.2 \pm 0.2 \text{ cm.}$$

The value of  $a$  for aluminum is:

$$a = 4.05 \text{ \AA.}$$

Verify equation (3.10).



## Experiment 4

# Preparation of a thin film by vacuum evaporation

*Before starting the experiment, you should be familiar with the basic concepts of vacuum technique (pumps, gauges).*

### Introduction

Thin layers (“films”) of solid materials, with thicknesses  $\cong 10^{-6}$  m, are important components of many modern electronic and optical devices. They are often produced by evaporating the material in a vacuum, and condensing the vapour on a cold substrate until the desired thickness is reached. A vacuum is used in order:

- to prevent a chemical reaction of the material with air,
- to lower the boiling/sublimation temperature, and,
- to deposit the material in a smooth, even manner on the substrate.

In this experiment you will produce a thin film of the metal bismuth on a glass substrate, and measure its thickness with a Michelson interferometer. The electrical properties of the film will be measured in a subsequent experiment.

### 4.1 Vacuum Evaporator

The mechanical (rotary) pump evacuates the bell jar, via the bypass, to a pressure of  $\cong (10 \rightarrow 100)$  mTorr, and ejects the evacuated gas into the air. The Turbo pump can pump down to a limit of  $\cong (10^{-5} \rightarrow 10^{-6})$  Torr .

Units of pressure (force per area):

Units of Pressure (force per area)		
MKS unit	Pa (for Pascal)	$1 \text{ N}\cdot\text{m}^{-2}$
Practical units	Torr (for Torricelli)	760 Torr (1 standard atmosphere)
	mm of Hg (mercury): atmospheric pressure:	equal to 1 Torr 760 Torr, or $1.013 \times 10^5$ Pa.

It only operates properly when first evacuated by the rotary pump. The evaporation takes place in a glass bell jar, from a tungsten filament which acts as a boiling reservoir for a small

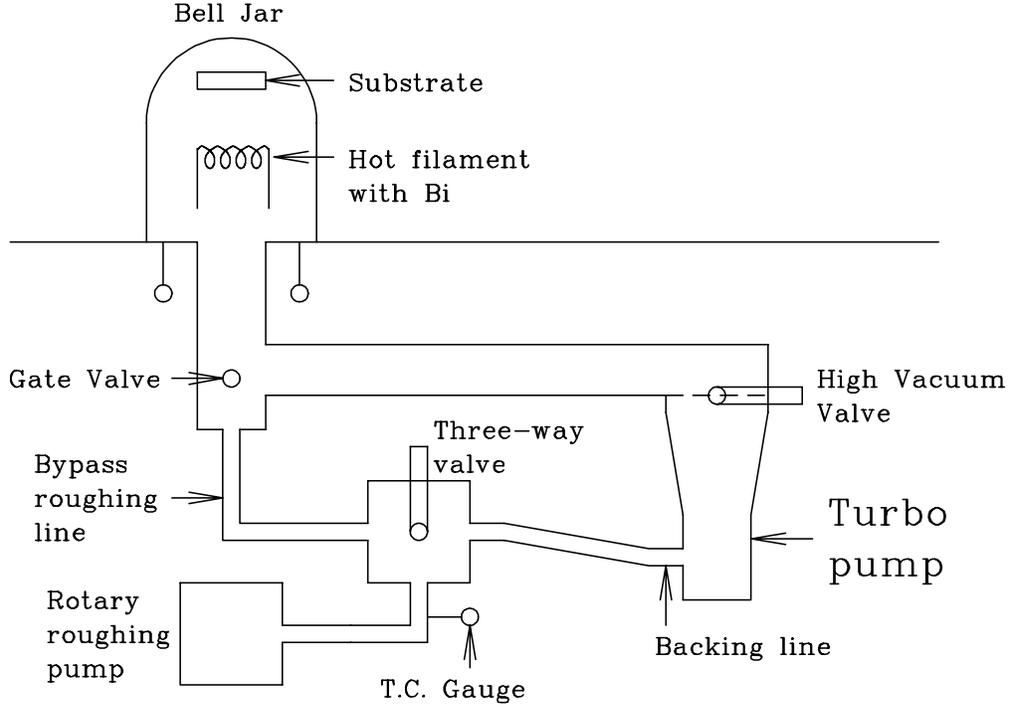


Figure 4.1: Diagram of a Vacuum Evaporator

quantity of Bi. The pressure in the bell jar is measured with either a thermocouple gauge (range  $\cong (10^{-3} \rightarrow 10^0)$  Torr), or with an ionization gauge (range  $\cong (10^{-3} \rightarrow 10^{-6})$  Torr).

Good films can be produced when the atoms/molecules of the material, in our case Bi, travel from the hot filament to the substrate directly, without colliding with residual air molecules, and when the substrate is clean and has no layers of gas, grease, etc., absorbed on its surface. In this respect the following parameters are important and can be calculated from the kinetic theory of gases:

1. the number density of gas  $N_0$ ; this is the number of gas molecules per unit volume. It can be calculated from the ideal gas law as

$$N_0 = \frac{P}{kT},$$

where  $P$  is pressure,  $T$  is temperature,  $k = \text{Boltzmann's constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$ .

2. the average speed  $\bar{V}$  of the gas molecules:

$$\bar{V} = \sqrt{\left[ \frac{8kT}{\pi m} \right]}$$

where  $m = \text{mass of the gas molecule}$ .

3. the mean free path  $\bar{L}$  of the molecules:

$$\bar{L} = \frac{1}{\sqrt{2} \cdot \pi \cdot d^2 N_0}$$

where  $d = \text{diameter of molecule}$

4. the rate at which gas molecules collide with a surface:

$$\nu = \frac{1}{4} N_0 \bar{V}$$

(collisions per unit area per unit time).

## 4.2 Exercise

Calculate, and plot versus  $P$ , the quantities  $N_0$ ,  $\bar{V}$ ,  $\bar{L}$  and  $\nu$  for  $P = 760$  Torr, 76 Torr, etc., down to  $7.6 \times 10^{-6}$  Torr. Use nitrogen at room temperature as the gas:  $d = 3.73 \times 10^{-10}$  m. Use double logarithmic scales for your graph.

At what values of  $P$  and  $T$  are:

1.  $\bar{L}$  about equal to the distance from boiling reservoir to substrate.
2. the collision rate ( $(\text{m}^2 \cdot \text{s})^{-1}$ ) about equal to the typical numbers of atoms per  $\text{m}^2$  of substrate surface ( $\cong 3 \times 10^{19} \text{ m}^{-2}$ )?

It should be clear from your calculations that a vacuum of  $< 10^{-4}$  Torr is required for proper vacuum evaporation.

Use the vacuum evaporator to produce a thin film of Bi, in the shape of a cross suitable for electrical measurements. Follow the procedure outlined below; the instructor will be present to assist you.

## 4.3 Procedure for Operating the Vacuum Evaporator

1. Preparation of substrate for film deposition:
  - (a) Wash the glass slide with detergent and water and rinse.
  - (b) Rinse with distilled water.
  - (c) Rinse with ethanol.  
These steps remove water and grease from the slide. Ethanol may dissolve grease from your fingers and leave it on the glass when it evaporates. Use gloves.
  - (d) Dry the slide in air. Check that it is clean.
2. Deposition of the bismuth film.
  - (a) Check that the ion gauge is off.
  - (b) Close the high vacuum valve.
  - (c) Vent the vacuum chamber and lift the bell jar.
  - (d) Put some bismuth into the filament basket.
  - (e) Put the slide inside the mask and place it above the filament.
  - (f) Put the bell jar back on, make sure that the vent valve is closed.
  - (g) Use the roughing pump and pump down the bell jar to below 100 m Torr pressure via the by-pass.

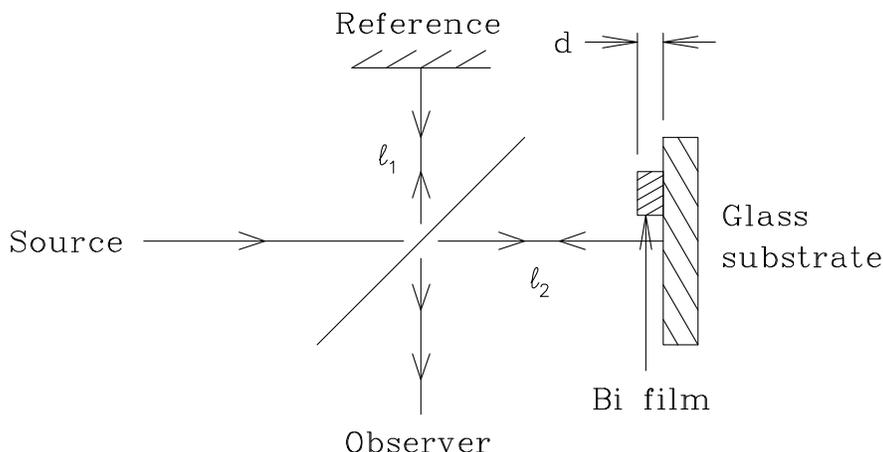


Figure 4.2: Diagram of a Michelson Interferometer

- (h) Turn on the turbo pump open high vacuum gate valve. The turbo pump will now start removing the “low pressure” residual air from the bell jar. After a few minutes the thermo-couple gauge will read  $10^{-3}$  and the cold cathode gauge switches on automatically at the  $10^{-4}$  Torr range. Wait until the pressure is  $\leq 5 \times 10^{-5}$  Torr.
- (i) Turn on the filament supply switch, increase the current until the filament starts glowing. Keep it like that for a minute to outgass it. Increase filament current, until bismuth starts evaporating. Hold filament current until all Bi has evaporated.
- (j) Shut down the filament current; turn the filament supply off.
- (k) Repeat steps (2a)–(2c).
- (l) Remove slide with bismuth film on it, replace bell jar, evacuate chamber to high vacuum as in steps (2g) and (2h).

## 4.4 Thickness Measurement

Since the thickness is of the same order of magnitude as the wavelength of visible light, we will use a method based on wave interference, the Michelson Interferometer.

When the optical paths  $\ell_1$  and  $\ell_2$  differ by  $k\lambda$  ( $k = 0, 1, 2, \dots$ ) the monochromatic waves (wavelength  $\lambda$ ) on their way to the observer are out of phase, and destructive interference produces a dark fringe in the field of view. The glass substrate plus film will form one mirror of the interferometer. The light reflecting from the Bi film travels a distance  $2d$  less than the light reflected from the glass slide, and therefore the fringe pattern over the Bi film is shifted with respect to the pattern over the glass slide. From the shift the thickness  $d$  can be easily calculated.

The actual instrument is a Michelson interferometer optically “folded-up” so that it fits over a microscope ( $\times 10$ ) objective. It contains the beam splitter and reference mirror (which can be tilted) and a means of controlling the distance  $\ell_1$ .

## 4.5 Procedure for measuring $d$ .

1. The instructor will put the interferometer over the microscope objective, and adjust  $\ell_1$ .

2. Place your slide (film up) under the microscope; turn the illuminator on; use white light.
3. Focus on a film edge by adjusting the vertical position of the microscope stage. This sets the distance  $\ell_2$ .
4. Switch to monochromatic illumination by putting a Fabry-Perot interferometer filter in the light path.
5. Gently adjust the vertical position of the film, until you see interference fringes. (Be patient).
6. Adjust the tilt screws of the reference mirror until the fringes are perpendicular to the film edge. You should see a fringe pattern as shown in figure 4.3.

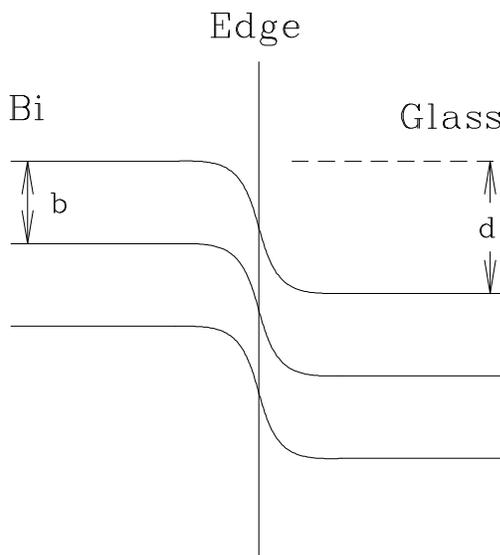


Figure 4.3: Fringe pattern

7. Estimate  $b$  (the fringe spacing) and  $a$  (the fringe shift) using a measuring eyepiece with a scale in it. The film thickness is then:

$$d = \frac{a}{b} \times \frac{\lambda}{2}.$$

8. Repeat at different positions along the edge of the film.

## References

For vacuum techniques, books such as:

- J. Yarwood, *High Vacuum Technique*.
- J. F. O'Hanlon, *A User's Guide to Vacuum Technology*.
- H. Mark and N. T. Olson, *Experiments in Modern Physics*, Ch. 4.

For the Michelson Interferometer:

- E. Hecht and A. Zajac, *Optics*, Chapter 9.10.2



## Experiment 5

# Resistivity and Hall effect of a thin Bismuth film

### Introduction

The conductivity  $\sigma$  of a solid depends on two parameters: one is the number of charge carriers (in metals, electrons of charge  $-e$ ) per unit volume,  $n$ . The other factor is the velocity with which the carriers will move through the solid when an electric field is applied. The ratio of this drift velocity (in  $\text{ms}^{-1}$ ) to the applied field (in  $\text{Vm}^{-1}$ ) is called the mobility  $\mu$  (in  $\text{m}^2\text{V}^{-1}\text{s}^{-1}$ ). The conductivity is given by

$$\sigma = en\mu \quad (5.1)$$

The conductivity (or its inverse, the resistivity  $\rho$ ) can in principle be determined by measuring the resistance of a block of length  $\ell$  and cross-section area  $A(= wxh)$ .

If a current  $i$  is passed through the block, and a potential difference  $V$  is measured at two points  $\ell$  apart then

$$\rho = \frac{V}{i} \times \frac{wxh}{\ell} \quad (5.2)$$

In practice one rarely has (or can make) a sample in this ideal shape, and a number of techniques are available to determine  $\rho$  from  $i - V$  measurements on samples of irregular shape. We will use a method developed by van der Pauw, for homogeneous samples of constant thickness  $d$ , but otherwise of arbitrary shape.

Four contacts, labelled A, B, C and D, are attached to the side of the flat sample in arbitrary positions. A current  $i_{AB}$  is passed from A to B, and the resulting voltage difference  $V_{CD}$  is measured between contacts C and D; call the ratio  $V_{CD}/i_{AB}$  a “resistance”;

$$R_{AB,CD} = R_1.$$

Next, the current is passed from B to C, and the voltage  $V_{DA}$  is measured:

$$R_{BC,DA} = R_2.$$

Similarly, determine  $V_{AB}/i_{CD}$ , which equals  $R_1$ , and  $V_{BC}/i_{DA}$  which equals  $R_2$  again. Now van der Pauw showed that  $R_1$  and  $R_2$  are related to the sample resistivity  $\rho$  and thickness  $d$  by:

$$\exp\left(-\frac{\pi d R_1}{\rho}\right) + \exp\left(-\frac{\pi d R_2}{\rho}\right) = 1 \quad (5.3)$$

Equation (5.3) cannot be solved for  $\rho$  in closed form, but it can be rewritten as:

$$\rho = \frac{\pi d}{\ln 2} \cdot \frac{(R_1 + R_2)}{2} f\left(\frac{R_1}{R_2}\right) \quad (5.4)$$

where the van der Pauw function  $f$  of the argument  $R_1/R_2$  satisfies

$$\cosh\left[\frac{\ln 2}{f} \cdot \frac{(R_1/R_2) - 1}{(R_1/R_2) + 1}\right] = \frac{1}{2} \exp\left[\frac{\ln 2}{f}\right] \quad (5.5)$$

Values of  $f(R_1/R_2)$  have been tabulated; therefore after measuring  $R_1$  and  $R_2$  and calculating  $R_1/R_2$  for the sample its resistivity  $\rho$  can be calculated from equation (5.4). Notice that the van der Pauw method requires the measurement of only one dimension ( $d$ ) of the sample; the standard method requires three measurements of sample dimensions.

The Hall effect is the sideways deflection of the moving charge carriers in the sample due to a magnetic field  $\vec{B}$  perpendicular to the current  $\vec{i}$ . The electron flowing in the (negative)  $x$ -direction experiences a Lorentz force  $q\vec{v} \times \vec{B}$  in the (negative)  $z$ -direction due to a  $\vec{B}$  field in the (positive)  $y$ -direction; the result is a potential difference, the Hall voltage  $V_H$ , developing between the top and bottom face of the sample, (positive)  $z$ -direction. The Hall coefficient  $R_H$  is now defined as:

$$R_H = \frac{V_{PQ} \cdot w}{i \cdot B} \quad (5.6)$$

It is related to the carrier concentration  $n$  from equation (5.1) by

$$n = \frac{1}{eR_H} \quad (5.7)$$

Once  $n$  is known, the mobility  $\mu$  of the charge carriers can be calculated (from equation (5.7)):

$$\mu = \frac{R_H}{\rho} \quad (5.8)$$

The van der Pauw method allows an easy determination of  $R_H$  for a flat sample: Pass a current  $i_{AC}$  between opposite contacts; it gives a voltage  $V_{BD}$ . Now put a  $\vec{B}$  field perpendicular to the sample;  $V_{BD}$  changes to a new value  $V_{BD} + \Delta V_H$ . The Hall coefficient is related to  $\Delta V_H$  by

$$R_H = \frac{\Delta V_H \cdot d}{i_{AC} \cdot B} \quad (5.9)$$

## Procedure

To determine  $\rho$ ,  $n$  and  $\mu$  for a thin Bismuth film:

1. Place the film in the sample holder and attach the four contacts A, B, C and D to the film.
2. Connect a circuit from a current source, a voltmeter and ammeter to measure  $i_{AB}$  and  $V_{CD}$ . Do the measurements, and calculate  $R_1$ . Reverse the direction of the current leads and determine  $R_1$  again. The two values of  $R_1$  should be identical unless you have bad (rectifying) contacts.
3. Similarly determine  $R_2 = I_{BC}/V_{AD}$ ; also check that indeed  $R_1$  is also equal to  $i_{CD}/V_{AB}$  and  $R_2$  to  $i_{AD}/V_{BC}$ . Calculate average values of  $R_1$ ,  $R_2$  and  $R_2/R_1$ .

4. From the graph of the van der Pauw function  $f(R_1/R_2)$  determine the appropriate  $f$  value for your sample, and calculate  $\rho$ .
5. Place the samples between the poles of the electromagnet, and measure  $i_{AC}$  and  $V_{BD}$ . Switch the  $\vec{B}$ -field on, and measure the Hall voltage  $\Delta V_H$ . Reconnect the leads to measure  $i_{BD}$ , and  $V_{AC}$ , and measure  $\Delta V_H$  again. Reverse the current leads to check for non-Ohmic contacts. Reverse the direction of the  $\vec{B}$ -field, and check that  $\Delta V_H$  changes sign.
6. Calculate  $R_H$  from the average  $\Delta V_H$  value, and calculate  $n$  and  $\mu$  for Bismuth. Compare  $n$  (the number of “free” electrons per unit volume) to the number of Bi atoms per unit volume.

## References

1. D. Halliday and R. Resnick, *Fundamentals of Physics*, Chapters 28, 30-4
2. K. Seeger, *Semiconductor Physics*, Chapter 4.3
3. E. H. Putley, *The Hall Effect and Semi-conductor Physics*, Chapters 1-2 and 2-1.



## Experiment 6

# The gamma-ray Scintillation Spectrometer

The spectrometer is designed to record and display the spectrum of gamma-rays emitted by radioactive sources. The energies cover the range from about 10 KeV (X-rays) to nearly 10 MeV. The use of energy units, rather than the wavelength or frequency of the EM waves, for the classification of the emissions from radioactive materials emphasises the particle-like interaction of the gamma-rays with matter. There is no dispersive element, such as a grating or a prism, in the spectrometer. There are no slits, lenses or mirrors.

The detector is a crystal of thallium-doped sodium iodide. When radiation, in the form of an X or gamma-ray photon, or energetic electrons, interacts with the crystal a certain fraction of the energy deposited in the crystal is transferred to the thallium activation centres and subsequently emitted as visible light. Other fractions of the energy deposit are dissipated as heat or by radiation in the invisible portions of the spectrum. The energy deposit is itself proportional to the incident gamma or X-ray photon energy. The number of visible light photons emitted by the centres is directly proportional to the amount of energy deposited and, hence, to the energy of the radiation.

The group of photons emitted by the centres appears as a short flash of light, or scintillation. The photons are detected by a photomultiplier tube (PMT) which produces a voltage pulse with height proportional to the number of photons that entered the tube. Hence, the height of the voltage pulse is proportional to the energy deposited in the crystal. If the dimensions of the crystal are such that the incident radiation is completely absorbed (i.e., the probability of absorption approaches 1), the height is proportional to the energy of the radiation. By means of electronic circuits the number of voltage pulses per second with heights lying between  $V$  and  $V+dV$  (a channel) is counted and recorded for channels having voltage  $V$  (proportional to energy) between pre-set limits. The channels are numbered in order of increasing energy. The device that performs the counting and recording function is called a multi-channel analyzer (MCA). The MCA, the detector crystal and the photomultiplier form the gamma-ray spectrometer.

If a source of monoenergetic gamma-rays, such as  $\text{Cs}^{137}$ , is examined with the spectrometer it is found that the recorded spectrum consists of a fairly well defined peak at some channel number, and a broad continuum with some smaller peaks that extends from the main peak towards lower channel numbers. The origin of these features of the spectrum is found in the nature of the interactions of the gamma-ray with the crystal.

The gamma-ray interacts with the crystal in one of three ways described below.

1. Photoelectric Absorption. The gamma-ray is annihilated and the energy of the photon is transferred to an electron bound to an atom or ion of the crystal. The kinetic energy of the electron (KeV or MeV), plus its binding energy to the atom or ion (eV), is equal to the energy of the gamma-ray photon. This photoelectron then transfers, on average, a certain fraction

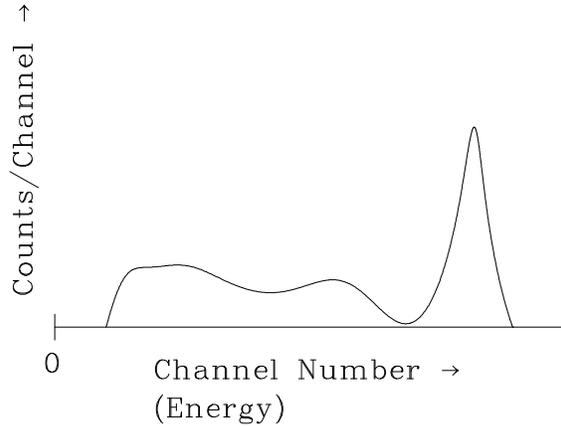


Figure 6.1: Spectrum recorded with Cesium<sup>137</sup> sources

of its energy to the thallium activation centres as it moves through the crystal. In crystals of the sizes used in practice the photoelectron loses all its kinetic energy within the crystal. The number of photons emitted by the thallium centres is proportional to the gamma-ray energy. The voltage pulse at the output of the PMT that collect the light from the NaI(Th) crystal is proportional to the gamma-ray energy.

2. Compton Scattering. The gamma-ray transfers momentum to an electron and is thereby “scattered” with lower energy and momentum. The scattered electron will excite some thallium centres and cause a voltage pulse to appear at the PMT output. The scattered gamma photon may interact by mechanism (1) and produce a PMT voltage pulse. The height of this pulse, as well as that produced by the interaction of the scattered electron with the crystal, will be less than that produced when the gamma photon energy is deposited as in (1). Since the loss of energy by the gamma photon depends on the scattering angle, there will be a range of pulse heights detected.
3. Pair Production. When a photon of energy greater than or equal to twice the rest energy of an electron interacts with matter it may be annihilated. The photon energy appears as a positron electron pair. When the positron meets another electron a gamma photon (annihilation radiation) appears. For our purposes this effect may be ignored because the gamma sources used do not have sufficient energy.

The spectrum recorded for a Cesium<sup>137</sup> source with the equipment to be used is shown in Figure 6.1. The large peak at the high end of the pulse height (gamma energy) axis arises from mechanism (1). It is called the Full Energy Peak. The energy corresponding to the channel of the maximum height is the energy of the gamma photon. The step to the lower energy side is the Compton Edge. It is produced by the most energetic electrons arising from the Compton scattering process. The secondary peak at still lower energy arises from the full energy peak of the back-scattered photons in the Compton process. Other lower energy peaks may arise from x-rays produced in the PMT shielding or from background effects. The recognition of the full energy peak and its channel number provides a means to calibrate the apparatus.

## 6.1 Calibration Procedure

The scintillation crystal and PMT are housed in a protective shield. The anode voltage of the PMT has been set to +1000 V. The gain of the amplifier that follows the PMT has been adjusted to give about 6 volt pulses for the full energy peak of the cesium. If the gain setting or the PMT voltage is altered your calibration will be invalidated and will have to be repeated.

The MCA electronics are mounted on a card in an Apple computer. The program that runs the analyzer and permits various display options for the data, as well as recording on disk, is loaded and set for 256 channels. This setting is sufficient for the energy resolution of the detector. (Read the instruction manuals or ask for help.)

Place the cesium source near the crystal end of the detector and record the spectrum until you have a clean looking spectrum. Note the channel for the full energy peak. Print out a spectrum and a channel count around the peak. Identify the backscatter peak and the Compton edge and verify that the energy value of the feature corresponds to theory in each case.

Repeat the measurements for the sodium<sup>22</sup>, barium<sup>133</sup> and cobalt<sup>57</sup> sources.

The energies are:

Source	Energy
Cs <sup>137</sup>	0.662 MeV
Na <sup>22</sup>	0.511 MeV
Ba <sup>133</sup>	0.356 MeV
Co <sup>57</sup>	0.1222 MeV

❗ Plot a graph of Channel Number vs. Energy in MeV and determine the slope so that you may measure other gamma energy values.

❓ Why is the full energy peak not sharp?

## References

A discussion of the scintillation detector is given in: *Radiotracer Methodology in Biological, Environmental and Physical Sciences*, C. H. Wang, D. L. Willis and W. D. Loveland (Prentice-Hall), Chapters 4, 6, 7, 12. Call Number QC 795.42 W36 1975.

*The electronic equipment described is obsolete. The physics of the processes is the important part.*

## Computer and Printer Commands

### Equipment activation procedure

1. Turn on "AC" toggle switch on Regulated High Voltage Power Supply
2. Turn dial from "HV Off" to "1000"
3. Turn on CI Power Supply (lower far right) toggle switch
4. Turn on Oscilloscope
5. Turn on printer and set to "Draft" mode (see section "Printer Commands" on page 28).

6. Turn on Apple monitor followed by the Apple computer which computer will display:

Dos Version 3.3 Apple II Plus

```
alternate slot? (Y/N)    type y
alternate slot #(3 or 5) type 5
printer slot #          type 1
```

**(DO NOT TOUCH ANY OTHER DIALS !)**

### Multi-channel Analyser commands:

Key(s)	Function
f	full screen display (spectrum broken into 2048 channels)
h1, h2	half screen display
q1, q2, q3, q4	quarter screen display
←, →	continous scrolling of spectrum (left, right)
a	accumulate
d	halt accumulation
c + [return]	clear screen
[esc]	quit
< , >	change (decrease, increase) vertical scale
p	print

For use with paddle: (halt accumulation first)

r            paddles control two pointers, the total counts between  
             pointers and the channel numbers are displayed

### Printer Commands

#### Draft Mode

- with the printer on, press “ON LINE” to take the printer offline (the light next to “ON LINE” should now be off)
- press “PRINT MODE” twice (until only the “HS” (**H**igh **S**peed) light is on) to set the printer to draft mode
- press “ON LINE” to put the printer online

#### Line Feed/Form Feed

- press “ON LINE” to take the printer offline
  - to perform a “line feed”, press and release “LF/FF” to move the paper forward one line
  - to perform a “form feed”, press and hold “LF/FF” to eject the page
- to put the printer online, press “ON LINE”

## Experiment 7

# The Compton effect

The Compton effect can be studied and the scattering formula verified. The apparatus has been set up as shown in Figure 7.1. Measure the energy,  $E'$ , of the scattered gamma photons as a function of their scattering angle  $\theta$ .

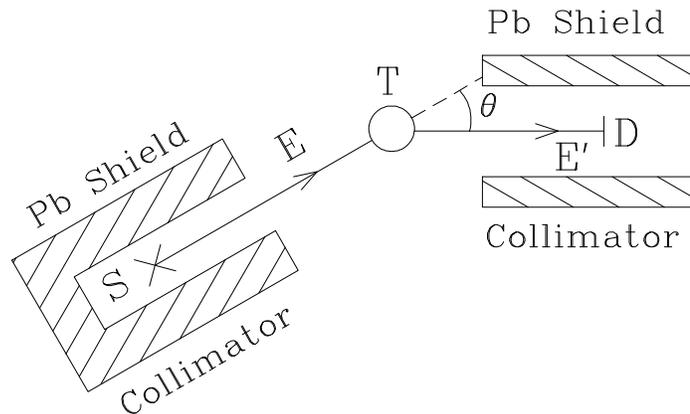


Figure 7.1: Compton scattering arrangement

**S**  $\text{Cs}^{137}$  gamma source; Energy 0.661 MeV; Strength 25 millicuries.

**E** Direction of the collimated gamma photon beam of energy  $E$ .

Note: The 25 millicurie source produces an intense collimated beam. DO NOT stand in the direct line  $E$ .

**T** Aluminum target that contains the free electrons from which the gamma rays will be scattered.

**E'** Direction of the scattered photons of energy  $E'$ .

**D** NaI gamma ray detector.

Both  $S$  and  $D$  are well shielded with lead. The source shielding minimizes the radiation intensity in all directions except within a small solid angle centred on the beam direction,  $E$ . The detector shielding is designed to admit the scattered photons,  $E'$ , and to exclude entrance to photons incident from other directions. The scattering angle,  $\theta$ , is varied by rotating the source assembly,  $S$ , about

the target position,  $T$ . To measure  $E'$  use the energy calibration that you made earlier. Check it out if there has been an interval between laboratory sessions!

Accumulate the pulse height spectrum of the scattered photons as a function of the scattering angle and so determine  $E'(\theta)$ . The Compton formula is usually given in terms of incident wavelength  $\lambda$  and scattered wavelength  $\lambda'$ :

$$\lambda' - \lambda = \frac{h}{m_0c}(1 - \cos \theta).$$

Show that in terms of photon energies  $E'$  and  $E$  this formula becomes

$$\frac{1}{E'} - \frac{1}{E} = \frac{1}{m_0c^2} \cdot (1 - \cos \theta)$$

Plot your experimental data as  $1/E'$  versus  $(1 - \cos \theta)$  and from the results determine  $E$  and the electron rest mass energy  $m_0c^2$ .

The minimum value of the scattering angle  $\theta$  at which the Compton scattering energy shift is observable will be determined by the energy resolution of the detector and the effectiveness of the lead collimator which defines the solid angle of emergence of the cesium photon beam. If the scattering angle is made too small, the direct shine of the cesium gamma photons into the detector will completely mask the small Compton energy shift. Only a small fraction of the photons are scattered in the target. The intensity of the scattered beam is much less than that of the incident beam. The recording time required to obtain a satisfactory spectrum may be many minutes. If there is any direct shine into the detector the recording time will be seconds rather than minutes!

## Experiment 8

# Planck's Constant—Method 1

### Introduction

The constant  $h$  ( $= 6.62559 \times 10^{-34}$ Js) was introduced by Planck in 1900 when he provided the first satisfactory theoretical basis for the temperature and wavelength dependence of the radiation from a black body. A black body, by definition, absorbs all the radiation that falls upon it. The rate of energy emission by the body summed over all wavelengths is proportional to the fourth power of the thermodynamic temperature  $T$ . The spectral distribution of the radiation depends on the temperature. There is a maximum in the emission versus wavelength curve so that the product of the wavelength of the maximum emission and the temperature is a constant. These relations are known as Stefan's Law and Wien's Law, respectively. In 1893, W. Wien showed from thermodynamic considerations that the form of the radiation curve was given by

$$E_\lambda = \frac{1}{\lambda^5} \cdot f\left(\frac{1}{\lambda T}\right)$$

where  $E_\lambda$  is the energy emitted in range  $d\lambda$  at wavelength  $\lambda$  and  $f$  is some function to be determined. The experimental data were obtained by Coblentz, who examined the spectral distribution of the body maintained at temperature  $T$ . Such a system provides an excellent approximation to a black body because radiation that enters the hole from outside has an exceedingly small chance to escape before it is absorbed at the walls of the cavity. The form of the radiation curve is shown in Figure 8.1. The curves are independent of the nature of the material that forms the walls of the cavity. Today, for convenience, the cavity is a hollow metal cylinder, blackened inside, and completely closed except for a narrow slit in one end. At the temperatures available in the laboratory the shortest wavelength radiation of measurable intensity is in the near ultra-violet.

In 1900, Planck introduced the assumption that each mode of vibration of frequency  $\nu$  of the electromagnetic field in the cavity could change its energy by a definite amount  $h\nu$  (where  $h$  was a constant to be determined) rather than by continuously variable amounts. He was then able to derive the explicit form of the radiation curve and to show that Stefan's Law and the Wien Law followed from his form. Moreover, the form was identical to that determined by Coblentz if the constant  $h$  was given a value close to that quoted above. Planck's assumption was the first use of the idea of quantization in the radiation field and, later, in the interaction of radiation with atoms.

Some years later, Einstein introduced the idea of light quanta in the explanation of the photo electric effect. The electromagnetic wave of frequency  $\nu$  that is the light beam gives energy to the electrons in the material in units of  $h\nu$ . The kinetic energy of the emitted electron is  $h\nu - W$ , where  $W$  is the energy required to remove the electron from the material.

The constant  $h$  appeared again in the Bohr theory of the hydrogen atom in 1912 and eventually became an essential part of modern quantum mechanics. Its value is important and much effort

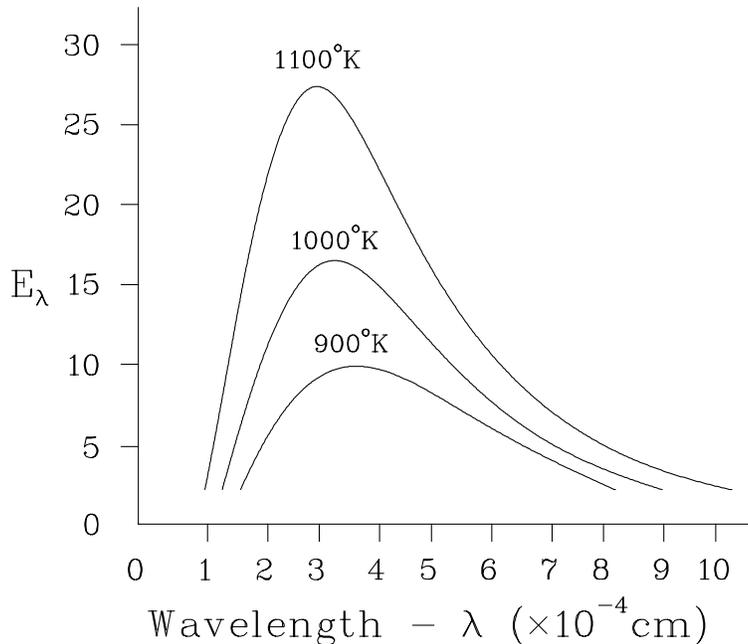


Figure 8.1: Temperature dependent energy emission by a blackbody

has been devoted to the measurement. In two experiments you will obtain estimates for  $h$ . The first method is based on the black body radiation curve. The second is a direct experimental examination of Einstein's photo electric equation introduced above.

## Method

Planck's formula for the power radiated by a black body at frequency and temperature  $T$  is

$$P = 2\pi \left(\frac{\nu}{c}\right)^2 \left(\frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right) - 1}\right)$$

where  $P$  is the energy radiated per unit area of surface per second per unit frequency range at frequency  $\nu$ ,  $k$  is Boltzmann's constant and  $c$  is the velocity of light. If a body is not a black body radiator, so that it does not absorb all radiation that falls upon it, the power radiated is less than the formula value. If the absorption coefficient at  $\nu$  is independent of the temperature  $P$  will be reduced by a constant factor for all temperatures. If the frequency and temperature are such that  $\exp(h\nu/kT) \gg 1$  then  $P$  is proportional to

$$\exp\left(-\frac{h\nu}{kT}\right).$$

Thus a graph of  $\ln P$  versus  $1/T$  should be a straight line of slope  $h\nu/k$ . The frequency is determined by the experimental conditions and  $k$  is known independently of this experiment. Thus a value of  $h$  can be obtained.

In the experiment we use an oven cavity as a black body radiation source which uses  $\text{MoSi}_2$  as a heater element. This oven can reach temperature as high as  $1500^\circ\text{C}$ . The Temperature of the oven

will be measured using two different methods. In the first method the temperature is measured directly using S-type thermocouple inside the oven. In the second method you use the Infra red technique to measure the temperature of the oven.

**Determination of  $h$**  Increase the current to the oven heater until a noticeable light output  $P$  is registered on the detector output meter. Measure and record the temperature of the oven when it is stable as well as the light output power  $P$ . Correct, if necessary,  $P$  for the zero-light-output, and plot  $\ln P$  versus  $1/T$ .

Repeat at ten different temperatures, by increasing current to the oven. The data points  $(\ln P, 1/T)$  should fall on a straight line, with slope  $h\nu/k$ . Calculate  $h$  from the slope; the Fabry-Perot filter passes only wavelengths in a narrow band at  $\lambda = 650$  nm or as indicated by the instrument.



## Experiment 9

# Planck's Constant—Method 2

The Einstein photoelectric equation relates the kinetic energy  $K$  of the emitted electron to the frequency of the radiation falling on the emitter and the energy required to extract the electron from the material

$$K = h\nu - eV_0,$$

where the extraction energy is written as the product of the magnitude of the electron charge and a potential  $V_0$  known as the work function of the material. Unless

$$h\nu > eV_0$$

no electrons will be emitted. If the emitter is used as the cathode of a vacuum cell and connected to the other electrode, or anode, a current will exist in the circuit as long as light of sufficiently short wavelength falls on the cathode. If a retarding potential is applied between anode and cathode in the external circuit the current will be reduced. The current will be zero if the retarding potential is set at the value  $K/e$ . In the experiment you will measure the retarding potential required to stop the photoelectric current as a function of frequency, or reciprocal wavelength. A value of  $h$  will be calculated from the graph of the “stopping” potential versus reciprocal wavelength.

The photocell has a potassium cathode and a ring anode set so that light can reach the cathode without striking the anode. The cell is connected to a low leakage small value capacitor so that the capacitor charges as the photo current is generated. The potential difference between the capacitor plates approaches the stopping potential, the photo current is reduced by the electric field between anode and cathode, and eventually the voltage across the capacitor reaches a steady value. This value is the stopping potential plus any contact EMFs arising from the connections between circuit elements. These unknown voltages only affect the intercept of the graph and can be ignored.

The incident light is provided by a microscope illuminating lamp and condensing lens. The frequency is selected with a narrow band pass Fabry-Perot filters. Care must be taken to ensure that the light does not strike the anode and cause the emission of electrons from its material or from small amounts of potassium acquired in the manufacturing of the cell. The cell must also be shielded from room light.

The steady potential reached by the capacitor is measured by an electrometer rather than a voltmeter. The input impedance of the electrometer is so high that it does not provide any significant drain for the charge on the capacitor. Such is not the case for everyday voltmeters. The potentials measured will lie in the range 0.3 to 1.2 volts (approximately). Use filters for wavelengths 650, 600, 550, 500, 450 nm with the microscope lamp. Then substitute a mercury lamp without filter to get the stopping potential for the 404.7 nm line. (The glass lenses prevent the ultraviolet lines of mercury from reaching the cell.) Plot a graph of stopping potential versus reciprocal wavelength and determine Planck's constant and the apparent work function of potassium. Record

the stopping potential for the microscope lamp without any filters and use your other results to determine the short wavelength limit of the emission from the lamp.